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A sinc-function based method for frequency evaluation

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Accurate frequency assessment is nowadays required in the mechanical engineering applications. Because in the standard methods that allow finding the real frequency occur errors, which come from the fact that they depend largely on the acquisition time, we set out to improve these methods. The purpose of this work is to find the maximum amplitude of the interpolation curve that passes through two amplitude peaks on two adjacent spectral lines. The method presented in the paper ensures a high precision when determining the frequency indicated by the maximizer.

Keywords: *Frequency estimation, interpolation, sinc function*

1. Introduction

The standard methods for determining the resonance frequencies have shown a low sensitivity due to the direct influence of the acquisition time on the accuracy of the result [1-3]. In essence, the frequency resolution, that is the distance between two consecutive values belonging to the frequency spectrum, is the lower that is the longer the acquisition time and the length of the acquired signal.

The signal processing by standard methods involves the use of interpolation to obtain the waveform, starting from a limited number of point values. At present, signal processing in order to identify the frequency value, using interpolation, implies the analysis of the maximizer and at most two other neighboring spectral lines [4-10].

It was found that although by performing a frequency-amplitude coupled analysis, within the same spectrum, the accuracy of the result increases, depending on the length of the acquired signal respectively on the acquisition time remains at the same level [11, 12].

Another direction of analysis to determine the frequency of the signal is the fragmentation of the signal generated in three distinct spectra followed by the concomitant processing of the maximizer in each obtained spectrum [13-15].

A different method of identifying, precisely, the frequency of a signal is the generation of a spectrum based on maximizer of the different spectra belonging to the analyzed signal, spectra obtained at different acquisition times. This method practically eliminates the direct dependence on acquisition times, respectively the signal length, the errors induced by the process being very small compared to those generated by using the standard methods. Using this method it is observed that the distribution of the maximizer is done through a pseudo-sinc function. The identified pseudo-sinc function is asymmetric but generates frequencies close to the real one and thus can be used to perform spectral analysis [16].

In this paper we introduce a method to use the sinc function for interpolation, which eliminates the disadvantage of the pseudo-sinc function asymmetry, and permits accurately evaluating the frequencies of a signal.

2. Determining the correct frequency

We consider a continuous harmonic signal having the known amplitude A and the frequency f. The expression describing the discrete form of the continuous signal has, according to [17], the form $\{x\}=\{x[0], x[1], ..., x[k], ..., x[N-1]\}$. Here, N is the number of samples used to describe the discrete signal. It is shown that for each sequence element of the mentioned expression, to which corresponds a real coefficient a_j and an imaginary coefficient b_j, a module X_j can be associated. This module X_j can be determined for each desired signal time length t_S.

The coefficients a_j and b_j for the signal having the time length t_S are defined by the expressions:

$$a_j = \frac{2}{t_s}\int_0^{t_S} \cos(2\pi f t)\cos(2\pi j \Delta f t)dt \qquad (1)$$

$$b_j = \frac{2}{t_s}\int_0^{t_S} \cos(2\pi f t)\sin(2\pi j \Delta f t)dt \qquad (2)$$

where $j = 0...N-1$ and $\Delta f = 1 / t_S$ is the frequency resolution. Due to the spectrum symmetry, it is sufficient to take $N/2$ spectral lines to define the signal in the frequency domain.

By integrating the expressions of the coefficients a_j and b_j , we obtain:

$$a_j = \frac{\sin[2\pi(f - j\Delta f)t_s]}{2\pi(f - j\Delta f)t_s} + \frac{\sin[2\pi(f + j\Delta f)t_s]}{2\pi(f + j\Delta f)t_s} \qquad (3)$$

$$b_j = \frac{1-\cos[2\pi(f - j\Delta f)t_s]}{2\pi(f - j\Delta f)t_s} - \frac{1-\cos[2\pi(f + j\Delta f)t_s]}{2\pi(f + j\Delta f)t_s} \qquad (4)$$

Because the term $f + j\Delta f$ from the denominator is much larger than the numerator, the fractions containing this term can be neglected.

Thus, the coefficients become:

$$a_j = \frac{\sin[2\pi(f - j\Delta f)t_s]}{2\pi(f - j\Delta f)t_s} \tag{5}$$

$$b_j = \frac{1 - \cos[2\pi(f - j\Delta f)t_s]}{2\pi(f - j\Delta f)t_s} \tag{6}$$

$$X_j = \sqrt{a_j^2 + b_j^2} = \mathrm{sinc}[(f - j\Delta f)t_s] \tag{7}$$

where the function *sinc* is by definition, in the digital signal processing theory:

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \tag{8}$$

If the difference $f - j\Delta f \to 0$ that is the product $j\Delta f$ approaches the frequency f of the analyzed signal, the *sinc* function will be close to 1 and the maximizer X_j will approach to the true value of the amplitude of the signal. If the frequency f of the analyzed signal departs as the measured frequency $j\Delta f$ the value of the maximizer X_j determined with the *sinc* function decreases and its amplitude and frequency are not in line with reality. In the literature, this phenomenon is known as leakage.

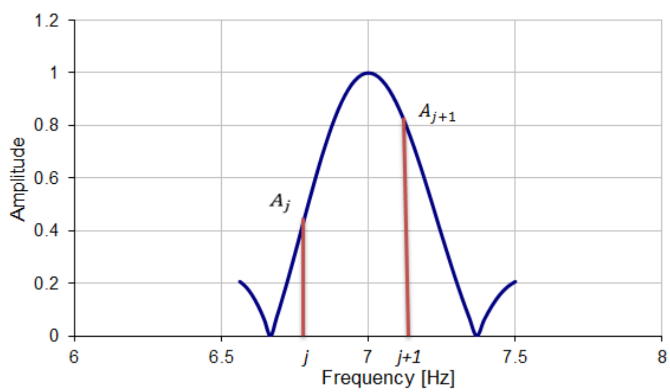

Figure 1. The spectrum indicating the amplitudes for the sinc function

If we note $f_m = j\Delta f$ and choose two points on two neighboring spectral lines having the amplitudes A_j and A_{j+1} that belong to the main lobe of the *sinc* function

(Figure 1), we can determine the estimated frequency at the position where the sinc function get maxima. We note this frequency f_e.

To find f_e, we solve the system of equations below, written for the two amplitudes A_j and A_{j+1} found on the spectral lines j and $j + 1$, located at a distance Δf one from each other:

$$\frac{\sin \pi t_s (f_e - f_m)}{\pi t_s (f_e - f_m)} = A_j \tag{9}$$

$$\frac{\sin \pi t_s [f_e - (f_m + \Delta f)]}{\pi t_s [f_e - (f_m + \Delta f)]} = A_{j+1} \tag{10}$$

From relation (10) we have that:

$$\frac{\sin[\pi t_s (f_e - f_m) - \pi t_s \Delta f]}{\pi t_s (f_e - f_m) - \pi t_s \Delta f} = A_{j+1} \tag{11}$$

Solving the system formed by the equations (9) and (11), knowing that $\pi t_s \Delta f = \pi$ and $\sin(x - \pi) = -\sin x$, we obtained:

$$f_e = \frac{(A_j + A_{j+1}) f_m t_s + A_{j+1}}{(A_j + A_{j+1}) t_s} \tag{12}$$

or, if expressing in accordance with the frequency measured at the first spectral line of the main lobe:

$$f_e = f_m + \delta \tag{13}$$

where $\delta = \dfrac{A_{j+1}}{(A_j + A_{j+1}) t_s}$ is a correction term.

This formula allows us to determine the estimated frequency when we know the measured frequency for the spectral line j and the amplitudes A_j and A_{j+1} for two considered points, taken from the main lobe.

3. Experimental testing

In our tests we wanted to show that, unlike the standard DFT, the method we propose ensures better accuracy when calculating the frequency of a signal. Thus, we studied the influence of the length of the analysis time on the accuracy of determining the frequency. For this purpose, we have generated, using the PyFEST program, a signal having the frequency f_G = 7 Hz and the amplitude A_G = 1 m/s^2, and the different scenarios analyzed are presented in Table 1.

Table 1. Different scenarios for the generated signal

Sampling rate	***r***	1000						
Number of samples	***N***	343	486	629	771	914	1057	1200
Number of cycles	***n***	2.4	3.4	4.4	5.4	6.4	7.4	8.4
Time of analysis	t_s	0.34285	0.48571	0.62857	0.77142	0.91428	1.05714	1.2

By varying the analysis time, implicitly the number of cycles, for each frequency measured f_m we considered two points on the main lobe, of amplitude A_j and A_{j+1} and based on (13) we determined the estimated frequency f_e . The results are presented in the Table 2. It is found that with the increase of the number of cycles, the value of the estimated frequency became very close to the real one. Thus, after considering 8 cycles, the error becomes very small and it is no longer necessary to increase the signal length in order to obtain a frequency value very close to the real one.

Table 2. Values used to estimate the frequency

n	2.4	3.4	4.4	5.4	6.4	7.4	8.4
t_s	0.34285	0.48571	0.62857	0.77142	0.91428	1.05714	1.2
f_m	5.848	6.1856	6.3694	6.4935	6.5717	6.6288	6.6722
A_j	0.8113	0.79301	0.78301	0.78234	0.77785	0.7743	0.7713
A_{j+1}	0.46357	0.12159	0.48312	0.48039	0.48511	0.4907	0.4950
f_e	6.9107	6.9588	6.9775	6.9874	6.9921	6.9949	6.9966

Through the tests performed it was found that if the acquisition time is greater than 1s, the frequency can be estimated with very good accuracy. Thus, it is found that the errors are very small if we use the proposed method, as opposed to the DFT standard, as can be seen from Figure 2.

In table 3 we present the errors achieved if estimating the frequency with the standard DFT and the proposed method. One can observe that these errors are less than 17% if the standard DFT is utilized, while significantly lower error values result for the estimation with the proposed method (less than 1.3%).

Table 3. Errors achieved by involving the standard DFT and the proposed method

n	2.4	3.4	4.4	5.4	6.4	7.4	8.4
t_s (s)	0.34285	0.48571	0.62857	0.77142	0.91428	1.05714	1.2
f_G (Hz)	7	7	7	7	7	7	7
f_m (Hz)	5.848	6.1856	6.3694	6.4935	6.5717	6.6288	6.6722
ε_m (%)	16.4571	11.6342	9.0085	7.23571	6.1185	5.3028	4.6828
f_e (Hz)	0.8113	0.79301	0.78301	0.78234	0.77785	0.7743	0.7713
ε_e (%)	1.2757	0.5885	0.3214	0.18	0.1128	0.07285	0.0485

The method is simple to be used and can be implemented in a program to obtain an accurate frequency estimation in real time.

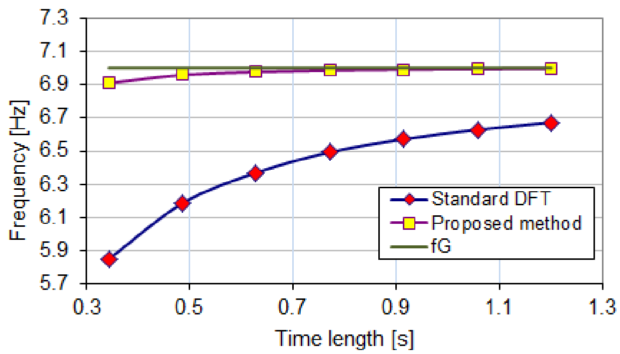

Figure 2. The estimated frequencies using the standard DFT and the proposed method

The frequencies estimated by involving this algorithm permitted observing the occurrence of damage in early stage [18-20], which qualifies the vibration-based methods for damage detection purposes.

4. Conclusion

The proposed method and the formula found allow us to estimate the frequency with high accuracy at a fairly small number of cycles, as opposed to the standard DFT, which needs over 1000 cycles to obtain the same precision that we obtained with this method, using only 8 cycles.

Thus, by knowing the frequency $f_m = j\Delta f$ measured at the spectral line j, and the two amplitudes associated with the two spectral lines j and $j + 1$, we can determine the estimated frequency. The achieved results show that the errors are small enough to permit observing small frequency changes. In consequence, by applying this method for estimating the natural frequencies of structures we can assess damages in early state.

References

[1] Gillich G.R., Furdui H., Wahab M.A., Korka Z.I.**,** A robust damage detection method based on multi-modal analysis in variable temperature conditions**,** *Mechanical Systems and Signal Processing***,** 115, 2019, pp. 361-379**.**

[2] Ostachowicz W.M., Krawczuk C., Analysis of the effect of cracks on the natural frequencies of a cantilever beam, *Journal of Sound and Vibration*, 150(2), 1991, pp. 191–201.

[3] Tufoi M., Hatiegan C., Vasile O., Gillich G.R., Dynamic Analysis of Thin Plates with Defects by Experimental and FEM Methods, *Romanian Journal of Acoustics and Vibration,*10(2), 2013, pp. 83-88.

[4] Djukanović S., Popović T., Mitrović A., Precise sinusoid frequency estimation based on parabolic interpolation, *In Proceedings of the 24th Telecommunications Forum TELFOR*, 2016, pp. 1-4.

[5] Grandke T., Interpolation Algorithms for Discrete Fourier Transforms of Weighted Signals, *IEEE Transactions on Instrumentation and Measurement*, 32, 1983, pp. 350-355.

[6] Quinn B.G., Estimating Frequency by Interpolation Using Fourier Coefficients, *IEEE Transactions on Signal Processing*, 42, 1994, pp. 1264-1268.

[7] Jain V.K., Collins W.L., Davis D.C., High-Accuracy Analog Measurements via Interpolated FFT, *IEEE Transactions on Instrumentation and Measurement*, 28, 1979, pp. 113-122.

[8] Ding K., Zheng C., Yang Z., Frequency Estimation Accuracy Analysis and Improvement of Energy Barycenter Correction Method for Discrete Spectrum, *Journal of Mechanical Engineering*, 46(5), 2010, pp. 43-48.

[9] Voglewede P., Parabola approximation for peak determination, *Global DSP Magazine*, 3(5), 2004, pp. 13-17.

[10] Jacobsen E., Kootsookos P., Fast, accurate frequency estimators, *IEEE Signal Processing Magazine*, 24(3), 2007, pp. 123-125.

[11] Minda A.A., Gillich N., Mituletu I.C., Ntakpe J.L., Manescu T., Negru I., Accurate frequency evaluation of vibration signals by multi-windowing analysis, *Applied Mechanics and Materials*, 801, 2015, pp. 328-332.

[12] Minda A.A., Gillich G.R., Sinc Function based Interpolation Method to Accurate Evaluate the Natural Frequencies, *Analele Universitatii "Eftimie Murgu" Resita*, *Fascicula de Inginerie,* 24(1), 2017, pp. 211-218.
[13] Gillich G.R., Mituletu I.C., Praisach Z.I., Negru I., Tufoi M., Method to Enhance the Frequency Readability for Detecting Incipient Structural Damage, *Iranian Journal of Science and Technology, Transactions of Mechanical Engineering*, 41(3), 2017, pp. 233–242.
[14] Gillich G.R., Mituletu I.C., Negru I., Tufoi M., Iancu V., Muntean F., A Method to Enhance Frequency Readability for Early Damage Detection, *Journal of Vibration Engineering & Technologies*, 3(5), 2015, pp. 637-652.
[15] Ntakpe J.L., Gillich G.R., Mituletu I.C., Praisach Z.I., Gillich N., An Accurate Frequency Estimation Algorithm with Application in Modal Analysis, *Romanian Journal of Acoustics and Vibration*, 13(2), 2016, pp. 98-103.
[16] Gillich G.R., Minda A.A., Korka Z.I., Precise estimation of the resonant frequencies of mechanical structures involving a pseudo-sinc based technique, *Journal of Engineering Science and Innovation*, 2(4), 2017, pp.37-48.
[17] Gillich G.R., Mituletu I.C., Signal Post-processing for Accurate Evaluation of the Natural Frequencies, *In: R. Yan, X. Chen, S. Mukhopadhyay (eds), Structural Health Monitoring. Smart Sensors, Measurement and Instrumentation*, 26, Springer Cham, 2017, pp. 13-37.
[18] Minda P.F., Praisach Z.I., Gillich N., Minda A.A., Gillich G.R., On the Efficiency of Different Dissimilarity Estimators Used in Damage Detection, Romanian *Journal of Acoustics and Vibration,* 10(1), 2013, pp. 15-18.
[19] Gillich G.R., Minda P.F., Praisach Z.I., Minda A.A., Natural frequencies of damaged beams – a new approach*, Romanian Journal of Acoustics and Vibration*, 9(2), 2012, pp. 101-108.
[20] Gillich G.R., Maia N., Mituletu I.C., Tufoi M., Iancu V., Korka Z., A new approach for severity estimation of transversal cracks in multi-layered beams, *Latin American Journal of Solids and Structures*, 13(8), 2016, pp. 1526-1544.

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