Rotation and Radiation Effects on MHD Flow through Porous Medium Past a Vertical Plate with Heat and Mass Transfer

Uday Singh Rajput, Gaurav Kumar

Effects of rotation and radiation on unsteady MHD flow past a vertical plate with variable wall temperature and mass diffusion in the presence of Hall current is studied here. Earlier we studied chemical reaction effect on unsteady MHD flow past an exponentially accelerated inclined plate with variable temperature and mass diffusion in the presence of Hall current. We had obtained the results which were in agreement with the desired flow phenomenon. To study further, we are changing the model by considering radiation effect on fluid, and changing the geometry of the model. Here in this paper we are taking the plate positioned vertically upward and rotating with velocity $\Omega$. Further, medium of the flow is taken as porous. The plate temperature and the concentration level near the plate increase linearly with time. The governing system of partial differential equations is transformed to dimensionless equations using dimensionless variables. The dimensionless equations under consideration have been solved by Laplace transform technique. The model contains equations of motion, diffusion equation and equation of energy. To analyze the solution of the model, desirable sets of the values of the parameters have been considered. The governing equations involved in the flow model are solved by the Laplace-transform technique. The results obtained have been analyzed with the help of graphs drawn for different parameters. The numerical values obtained for the drag at boundary and Nusselt number have been tabulated. We found that the values obtained for velocity, concentration and temperature are in concurrence with the actual flow of the fluid.

Keywords: MHD flow, rotation, radiation, Hall current.
1. Introduction

The MHD flow past a flat plate through porous medium is one of the classical problems in the fluid dynamics. Applications of the study arise in magnetic field controlled materials processing systems, planetary and solar plasma fluid dynamics systems, and rotating MHD induction machine energy generators etc. Radiation effect on combined convection over a vertical flat plate embedded in a porous medium of variable porosity was analyzed by Pal and Mondal [4]. Prasad et al [2] have worked on radiation and mass transfer effects on two dimensional flow past an impulsively started infinite vertical plate. Radiative MHD flow over a non isothermal stretching sheet in a porous medium was considered by Vyas and Srivastava [5]. Chauhan and Rastogi [3] have investigated Hall current and heat transfer effects on MHD flow in a channel partially filled with a porous medium in a rotating system. Guchhait et al. [6] have examined combined effects of Hall current and radiation on MHD free convective flow in a vertical channel with an oscillatory wall temperature. Prasada et al. [1] have studied Hall effect on free and forced convective flow in a rotating channel. Hall effect on unsteady MHD free convection flow over a stretching sheet with variable viscosity and viscous dissipation was analyzed by Srinivas and Naikoti [7]. Rajput and Kanaujia [8] have worked on combined effects of Hall current and chemical reaction on unsteady MHD flow past an impulsively started vertical plate with constant wall temperature and mass diffusion. Chemical reaction effect on unsteady MHD flow past an exponentially accelerated inclined plate with variable temperature and mass diffusion in the presence of Hall current was studied by us [9]. The present study is carried out to examine the effects of rotation and radiation on unsteady MHD flow through porous medium past a vertical plate with variable temperature and mass diffusion in the presence of Hall current. The problem is solved by the Laplace transform technique. A selected set of graphical results illustrating the effects of various parameters involved in the problem are presented and discussed. The numerical values of skin-friction and Nusselt number have been tabulated.


The geometrical model of the problem is shown in figure-1
The x axis is taken along the vertical plate and z axis is normal to it. Thus the z axis lies in the horizontal plane. The fluid and the plate rotate as a rigid body with a uniform angular velocity $\Omega$ about z-axis. The magnetic field $B_0$ of uniform strength is applied perpendicular to the flow. Since the fluid is electrically conducting whose magnetic Reynolds number is very small, therefore the induced magnetic field produced by the fluid motion is negligible in comparison to the applied one. Initially it has been considered that the plate as well as the fluid is at the same temperature $T_\infty$. The species concentration in the fluid is taken as $C_\infty$. At time $t > 0$, the plate starts moving with a velocity $u_0$ in its own plane, and temperature of the plate is raised to $T_w$. The concentration $C_w$ near the plate is raised linearly with respect to time.

The flow modal is as under:

$$\frac{\partial u}{\partial t} - 2\Omega v = \nu \frac{\partial^2 u}{\partial z^2} + g\beta(T - T_\infty) + g\beta' (C - C_\infty) - \frac{\sigma B_0^2 (u + mv)}{\rho(1 + m)} - \frac{v u}{K}. \tag{1}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} + \frac{\sigma B_0^2 (mu - v)}{\rho(1 + m)} - \frac{v v}{K}. \tag{2}$$

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial z^2}. \tag{3}$$

$$\rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - \frac{\partial q_v}{\partial z}. \tag{4}$$
The initial and boundary conditions are
\[ t \leq 0: u = 0, \ v = 0, \ T = T_{\infty}, \ C = C_{\infty}, \text{for every } z, \]
\[ t > 0: u = u_0, \ v = 0, \ T = T_{\infty} + (T_w - T_{\infty})A_0, \ C = C_{\infty} + (C_w - C_{\infty})A_0, \text{ at } z = 0, \]
\[ u \to 0, \ v \to 0, \ T \to T_{\infty}, \ C \to C_{\infty} \text{ as } z \to \infty. \] (5)

Here \( u \) is the primary velocity, \( v \)-the secondary velocity, \( g \)-the acceleration due to gravity, \( \beta \)-volumetric coefficient of thermal expansion, \( t \)-time, \( m(= \omega \tau) \) is the Hall current parameter with \( \omega \)-cyclotron frequency of electrons and \( \tau_e \)-electron collision time, \( T \)-temperature of the fluid, \( \beta^* \)-volumetric coefficient of concentration expansion, \( C \)-species concentration in the fluid, \( \nu \)-the kinematic viscosity, \( \rho \)-the density, \( C_p \)-the specific heat at constant pressure, \( k \)-thermal conductivity of the fluid, \( D \)-the mass diffusion coefficient, \( K \)-the permeability parameter, \( T_w \)-temperature of the plate at \( z = 0 \), \( C_w \)-species concentration at the plate \( z = 0 \), \( B_0 \)-the uniform magnetic field, \( \sigma \)-electrical conductivity.

The local radiant for the case of an optically thin gray gas is expressed by
\[ \frac{\partial q_r}{\partial z} = -4a^* \sigma(T_{\infty}^4 - T^4), \] (6)
where \( a^* \) is absorption constant. The temperature difference within the flow is considered sufficiently small, hence \( T^4 \) can be expressed as the linear function of temperature. This is accomplished by expanding \( T^4 \) in a Taylor series about \( T_{\infty} \) and neglecting higher-order terms
\[ T^4 \equiv 4T_{\infty}^3T - 3T_{\infty}^4. \] (7)

Using equations (6) and (7), equation (4) becomes
\[ \rho C_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial z^2} - 16a^* \sigma T_{\infty}^3(T - T_{\infty}). \] (8)

The following non-dimensional quantities are introduced to transform equations (1), (2), (3) and (8) into dimensionless form:
The symbols in dimensionless form are as under:

$u$ - the primary velocity, $v$ - the secondary velocity, $t$ - time, $θ$ - the temperature, $C$ - the concentration, $G_r$ - thermal Grashof number, $G_m$ - mass Grashof number, $µ$ - the coefficient of viscosity, $R$ - Radiation parameter, $K$ - the permeability parameter, $P_r$ - the Prandtl number, $S_c$ - the Schmidt number, $M$ - the magnetic parameter.

The model now becomes

\[
\frac{∂ū}{∂t} - 2Ωv = \frac{∂^2 u}{∂z^2} + G_r + G_m \frac{C}{u_t} - \frac{M(ū + m\bar{v})}{(1 + m^2)} - \frac{l}{K} \bar{u}. \tag{10}
\]

\[
\frac{∂v}{∂t} + 2Ω\bar{u} = \frac{∂^2 u}{∂z^2} + \frac{M(mū - \bar{v})}{(1 + m^2)} - \frac{l}{K} \bar{v}. \tag{11}
\]

\[
\frac{∂C}{∂t} = \frac{1}{S_c} \frac{∂^2 C}{∂z^2}. \tag{12}
\]

\[
\frac{∂θ}{∂t} = \frac{1}{P_r} \frac{∂^2 θ}{∂z^2} - \frac{Rθ}{P_r}. \tag{13}
\]

The corresponding boundary conditions (5) become:

\[
\bar{t} \leq 0 : \bar{u} = 0, \bar{v} = 0, \bar{θ} = 0, \bar{C} = 0, \text{ for every } \bar{z},
\]

\[
\bar{t} > 0 : \bar{u} = 1, \bar{v} = 0, \bar{θ} = \bar{t}, \bar{C} = \bar{t}, \text{ at } \bar{z} = 0,
\]

\[
\bar{u} \to 0, \bar{v} \to 0, \bar{θ} \to 0, \bar{C} \to 0, \text{ as } \bar{z} \to \infty. \tag{14}
\]

Dropping bars in the above equations, we get
\[
\frac{\partial u}{\partial t} - 2\Omega v = \frac{\partial^2 u}{\partial z^2} + G, \ \theta + G_m C - \frac{M(u + mv)}{(1 + m^2)} - \frac{I}{K} u. \quad (15)
\]
\[
\frac{\partial v}{\partial t} + 2\Omega u = \frac{\partial^2 v}{\partial z^2} + \frac{M(mu - v)}{(1 + m^2)} - \frac{I}{K} v. \quad (16)
\]
\[
\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}. \quad (17)
\]
\[
\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2} - \frac{R \theta}{P_r}. \quad (18)
\]

The following boundary conditions become
\[
\begin{align*}
t \leq 0: & \quad u = 0, \ v = 0, \ \theta = 0, \ C = 0, \ for \ every \ z, \\
t > 0: & \quad u = 1, \ v = 0, \ \theta = t, \ C = 0, \ at \ z=0, \\
u \to 0, & \ v \to 0, \ \theta \to 0, \ C \to 0, \ as \ z \to \infty. \quad (19)
\end{align*}
\]

Writing the equations (15) and (16) in combined form (using \( q = u + i v \))
\[
\frac{\partial q}{\partial t} = \frac{\partial^2 q}{\partial z^2} + G, \ \theta + G_m C - q\left(\frac{M(1-im)}{1+m^2} + \frac{1}{K}\right) + 2i\Omega, \quad (20)
\]
\[
\frac{\partial C}{\partial t} = \frac{1}{S_c} \frac{\partial^2 C}{\partial z^2}, \quad (21)
\]
\[
\frac{\partial \theta}{\partial t} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial z^2} - \frac{R \theta}{P_r}. \quad (22)
\]

Finally, the boundary conditions become:
\[
\begin{align*}
t \leq 0: & \quad q = 0, \ \theta = 0, \ C = 0, \ for \ all \ z, \\
t > 0: & \quad q = 1, \ \theta = t, \ C = t, \ at \ z=0, \\
qu \to 0, & \ \theta \to 0, \ C \to 0, \ as \ z \to \infty. \quad (23)
\end{align*}
\]

The dimensionless governing equations (20) to (22), subject to the boundary conditions (23), are solved by the usual Laplace - transform technique. The solution obtained is as under:
\[ C = t \left\{ 1 + \frac{z^2 S_c}{2t} \right\} \left[ e r f c \left( \sqrt{\frac{S_c}{2t}} \right) - \frac{z}{\sqrt{\pi t}} e^{-\frac{z^2}{4t}} S_c \right] \],

\[ \theta = e^{\sqrt{\frac{S_c}{t}}} \left\{ e r f c \left( \frac{-2\sqrt{R}t + zP}{\sqrt{P_t}} \right) - \frac{2\sqrt{R}t + zP}{\sqrt{P_t}} \right\} \left( 2\sqrt{R}t - zR \right) + e^{2\sqrt{S_c}} \left\{ e r f c \left( \frac{2\sqrt{R}t + zP}{\sqrt{P_t}} \right) \right\} \left( 2\sqrt{R}t + zR \right) \},

\[ q = \frac{1}{2} e^{-\sqrt{S_c}} \left[ 2e^{-\sqrt{S_c}} (A_1 + P, A_2) + 2iA_3 e^{-\sqrt{S_c}} (a - R) + zA_4 e^{-\sqrt{S_c}} (\sqrt{a - \sqrt{S_c}}) \right]

\[ + \frac{G_m}{\sqrt{a}} \left[ 2e^{-\sqrt{S_c}} (2A_1 + 2\sqrt{a} A_2) + 2e^{-\sqrt{S_c}} A_2 (S_c + at) \right]

\[ + 2A_3 A_2 (1 - S_c) \left[ \frac{P_G}{2\sqrt{\pi (a - R)^2}} A_6 A_0 \sqrt{\pi z (at - 1 - R + P_0)} + A_1 \sqrt{\pi z (1 - P_0)} \right]

\[ + \frac{1}{2} \left[ \frac{P_G}{R} A_6 A_0 A_1 \sqrt{\pi z (a - R)} \right] - \frac{G_m}{2a^2 \sqrt{\pi}} \left[ 2az \sqrt{S_c} e^{-\sqrt{S_c}} \sqrt{\pi} + A_3 \sqrt{\pi} (az^2 S_c + 2at + 2S_c - 2) + A_3 \sqrt{\pi} (A_6 + A_0 S_c) \right] \]

The expressions for the constants involved in the above equations are given in the appendix.

3. Skin friction

The dimensionless skin friction at the plate \( z=0 \) is obtained by

\[ \left( \frac{dq}{dz} \right)_{z=0} = \tau_x + i \tau_y \]

The numerical values of \( \sigma_x \) and \( \sigma_y \) are given in table 1 for different parameters.

4. Nusselt Number

The dimensionless Nusselt number is given by

\[ Nu = \left( \frac{\partial \theta}{\partial z} \right)_{z=0} = \left[ e r f c \left( \frac{\sqrt{R}t}{\sqrt{P_t}} \right) - \frac{\sqrt{R}t}{2} + \frac{P_0}{4\sqrt{R}} \right] - \left[ e r f c \left( \frac{\sqrt{R}t}{\sqrt{P_t}} \right) - \frac{\sqrt{R}t}{2} + \frac{P_0}{4\sqrt{R}} \right] - \frac{R_0}{\sqrt{\pi}} \]

The numerical values of \( Nu \) are given in table-2 for different parameters.
5. Results and Discussion

In this paper we have studied the effects of rotation, radiation and permeability of porous medium on the flow. The behavior of other parameters like magnetic field, Hall current and thermal buoyancy is almost similar to the earlier model studied by us [9]. The analytical results are shown graphically in figures 2 to 10. The numerical values of skin-friction and Nusselt number are presented in table-1 and table-2 respectively. From figures 3 and 6, it is observed that primary and secondary velocities increase with permeability parameter ($K$). This result is due to the fact that increases in the value of ($K$) results in reducing the drag force, and hence increasing the fluid velocity. Effect of rotation on fluid flow behavior is shown by figures 4 and 7. It is observed that an increase in rotation parameter $\Omega$ primary velocity decreases throughout the boundary layer region whereas secondary velocity increases continuously near the surface of the plate. This implies that rotation tends to accelerate secondary velocity whereas it retards the primary velocity in the boundary layer region. Figures 2 and 5 indicates the effect of radiation parameter $R$ on both components of the velocity; and it is observed that it retards the flow. As the radiation parameter increases the temperature of the system decreases; as a result fluid flow becomes slow. Further, it is observed that the temperature decreases when Prandtl number and radiation parameter are increased (figures 8 and 9). However, from figure 10 it is observed that the temperature increases with time. This is due to the heat which is transported to the system continuously.

Skin friction is given in table1. The values of $\tau_x$ and $\tau_y$ decreases with the increases in radiation and permeability parameter. If rotation parameter is increased then $\tau_x$ decreases but $\tau_y$ increases.

Nusselt number is given in table2. The value of $Nu$ decreases with increase in Prandtl number, radiation parameter and time.

![Figure 2. Velocity $u$ for different values of $R$](image)
Figure 3. Velocity $u$ for different values of $K$

Figure 4. Velocity $u$ for different values of $\Omega$

Figure 5. Velocity $v$ for different values of $R$
Figure 6. Velocity $v$ for different values of $K$.

Figure 7. Velocity $v$ for different values of $\Omega$.

Figure 8. Temperature $\theta$ for different values of $Pr$. 

$M = 2, m = 1, P_r = 0.71, S_c = 201, G_m = 100, G_0 = 10, K = 2, \Omega = 5, R = 2, t = 0.3$

$M = 2, m = 1, P_r = 0.71, S_c = 201, G_m = 100, G_0 = 10, K = 5, \Omega = 3, R = 2, t = 0.5$

$R = 2$

$P_r = 0.71, 7$

$t = 0.4$
Table 1. Skin friction for different Parameters.

<table>
<thead>
<tr>
<th>$M$</th>
<th>$m$</th>
<th>$Pr$</th>
<th>$Sc$</th>
<th>$Gm$</th>
<th>$Gr$</th>
<th>$R$</th>
<th>$\Omega$</th>
<th>$K$</th>
<th>$t$</th>
<th>$\tau_x$</th>
<th>$\tau_y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>3</td>
<td>5</td>
<td>0.2</td>
<td>0.3</td>
<td>128.755</td>
<td>-282.116</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>5</td>
<td>5</td>
<td>0.2</td>
<td>0.3</td>
<td>084.850</td>
<td>-319.835</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>2</td>
<td>3</td>
<td>0.2</td>
<td>0.3</td>
<td>311.498</td>
<td>-299.057</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>2</td>
<td>7</td>
<td>0.2</td>
<td>0.3</td>
<td>078.997</td>
<td>-205.095</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>2</td>
<td>5</td>
<td>0.5</td>
<td>0.3</td>
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<tr>
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<td>0.71</td>
<td>2.01</td>
<td>100</td>
<td>10</td>
<td>2</td>
<td>5</td>
<td>1.0</td>
<td>0.3</td>
<td>074.198</td>
<td>-332.323</td>
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</tbody>
</table>
Table 2. Nusselt number for different parameter.

<table>
<thead>
<tr>
<th>Pr</th>
<th>R</th>
<th>t</th>
<th>Nu</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.71</td>
<td>2</td>
<td>0.4</td>
<td>-0.80527</td>
</tr>
<tr>
<td>7.00</td>
<td>2</td>
<td>0.4</td>
<td>-1.95926</td>
</tr>
<tr>
<td>0.71</td>
<td>3</td>
<td>0.4</td>
<td>-0.89401</td>
</tr>
<tr>
<td>0.71</td>
<td>2</td>
<td>0.5</td>
<td>-0.97608</td>
</tr>
<tr>
<td>0.71</td>
<td>2</td>
<td>0.6</td>
<td>-1.09494</td>
</tr>
</tbody>
</table>

4. Conclusion

In this paper a theoretical analysis has been done to study the unsteady MHD flow through porous medium past a moving vertical plate with heat and mass transfer in the presence of Hall current, rotation and radiation. The results obtained are in agreement with the usual flow which strengthens the fluid model. It has been found that the velocity in the boundary layer region increases with the values of permeability parameter. But trend is reversed with radiation parameter; that is the velocity decreases when radiation parameter is increased. The rotation parameter retards the primary flow whereas it accelerates the secondary flow. It is also observed that radiation and permeability parameters decrease the drag at the plate surface. Nusselt number decreases with increase in radiation parameter, Prandtl number and time. The results obtained will have applications in the research related to the solar physics dealing with the sunspot development, the structure of rotating magnetic stars, cooling of electronic components of a nuclear reactor, bed thermal storage and heat sink in the turbine blades.

Appendix

\[
a = \frac{M(1 - im)}{1 + m^2} + \frac{1}{K} + 2i\Omega, \quad A_0 = \frac{u_0^2 t}{\nu}, \quad A_1 = 1 + e^{\sqrt{\pi^2}}(1 - A_{18}) - A_{17}, \quad A_2 = -A_1, \]
\[
A_3 = 1 - e^{\sqrt{\pi^2}}(1 - A_{18}) - A_{17}, \quad A_4 = -1 + A_0 + A_{10}(A_{20} - 1), \quad A_5 = -1 + A_{22} + A_{24}(A_{22} - 1),
\]
\[
A_6 = -1 + A_{23} + A_{26}(A_{23} - 1), \quad A_7 = -1 + A_{29} + A_{27}(A_{29} - 1), \quad A_8 = -1 + A_{33} + A_{36}(A_{33} - 1),
\]
\[
A_9 = -1 - A_{24} - A_{28}(1 - A_{28}), \quad A_{10} = -A_0, \quad A_{11} = \text{Abs}[z].\text{Abs}[\mathcal{P}],
\]
\[
A_{12} = e^{\mathcal{P}_{12} - 1}, \quad A_{13} = e^{\frac{a_{13}}{\mathcal{S}_{13} - 1}}, \quad A_{12} = e^{\mathcal{P}_{12} - 1}.\text{Abs}[\mathcal{P}],
\]

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\[ A_{15} = -1 + \text{erf} \left[ \frac{z \sqrt{S}}{2 \sqrt{t}} \right], \quad A_{16} = e^{Abs [z] \sqrt{P_a R}}, \quad A_{17} = \text{erf} \left[ \frac{2 \sqrt{a t} - z}{2 \sqrt{t}} \right]. \]

\[ A_{18} = \text{erf} \left[ \frac{2 \sqrt{a t} + z}{2 \sqrt{t}} \right], \quad A_{19} = \text{erf} \left[ \frac{z - 2t \frac{a P e - R}{P - 1}}{2t} \right], \quad A_{20} = \text{erf} \left[ \frac{z + 2t \frac{a P e - R}{P - 1}}{2t} \right]. \]

\[ A_{21} = \text{erf} \left[ \frac{z - 2t \frac{a S}{S - 1}}{2t} \right], \quad A_{22} = \text{erf} \left[ \frac{z + 2t \frac{a S}{S - 1}}{2t} \right], \quad A_{23} = \text{erf} \left[ \frac{Abs [z] Abs [P]}{2 \sqrt{t}} - \frac{t R}{P} \right]. \]

\[ A_{24} = \text{erf} \left[ \frac{2 \sqrt{a t}}{2t} \right], \quad A_{25} = \text{erf} \left[ \frac{2 \sqrt{a t} + 2 \sqrt{S}}{2t} \right], \quad A_{26} = e^{Abs [z] \sqrt{P_a R}}, \]

\[ A_{27} = e^{-2t \frac{a S}{S - 1}}, \quad A_{28} = e^{-2t \frac{a S}{S - 1}}, \quad A_{29} = \text{erf} \left[ \frac{Abs [z] Abs [P]}{2 \sqrt{t}} \right] - \frac{t (R - a P_{-1})}{P_{-1} (1 - P_{-1})}. \]

\[ A_{30} = e^{-2t \frac{a P e - R}{P - 1}}, \quad A_{31} = \text{erf} \left[ \frac{Abs [z] Abs [P]}{2 \sqrt{t}} + \frac{t R}{P} \right], \quad A_{32} = \text{erf} \left[ \frac{Abs [z] Abs [P]}{2 \sqrt{t}} + \frac{t (R - a P_{-1})}{P_{-1} (1 - P_{-1})} \right]. \]

\[ A_{33} = 1 + A_{31} + e^{2 \sqrt{a t}} A_{34}, \quad A_{34} = \text{erf} \left[ \frac{2 \sqrt{a t} + z}{2 \sqrt{t}} \right]. \]

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