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Numerical Modeling of Transient Heat Transfer in Longitudinal Fin

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The main objective of the present numerical study is to investigate the transient heat transfer in one kind of all-purpose longitudinal fin with the triangular profile. The lateral surface of the concerned fin and the tip of it are subjected to general situations included heat flux at the base and insulation on the tip. For this study developed a one dimensional in house code written by Fortran 90 programming language by using finite difference method with an implicit scheme in unsteady state condition. Generally, the result of this study in time variation state after 700 seconds is steady. The results also show the fin efficiency by increasing the time of study decreases due to a reduction in the total heat transfer which is happened in the fin. The grid independence study shows that for the number of nodes greater than 20 the result will not be changed and same as before. Finally, the result of Fortran code verified by commercial CFD code which relies on finite difference method and it was shown have a consistent agreement.

Keywords: *Longitudinal fin; Triangular profile; Fortran code; Fin efficiency; Commercial CFD code.*

1. Introduction.

Fins or Extended surfaces have a very wide range of industrial and traditional applications. By using fins the rate of heat transfer between two objects increases due to enlarging the heat transfer area. A. D. Kraus et al. [11] had done a foremost investigation about three kinds of all-purpose fins which are longitudinal fins, radial or circumferential fins, and pin fins or spines. They investigated about the fins under different heat transfer situations same as radiation, boiling, condensation, and convection and reported the efficiency of different kinds of fins under different boundary condition in steady state time condition by using the analytical method. They also showed that between rectangular, triangular, convex and concave profile fin, rectangular and concave respectively, has a highest and lowest efficiency. D. Joglekar and M. Joglekar [10] utilized a numerical method based on residue minimization to solve the nonlinear differential equation of straight convective fins having temperature-dependent thermal conductivity. S. Mosayebidorcheh et al. [13] studied transient thermal analysis of longitudinal fins with internal heat generation considering temperature-dependent properties and different fin profiles. They numerically illustrated that the fin surface temperature is greatly dependent on the type of the heat transfer. The laminar film boiling and radiation heat transfer respectively have the minimum and maximum fin surface temperature.

In this paper, unsteady one-dimensional (1-D) numerical modeling of heat transfer in the longitudinal fin of a triangular profile by finite difference method with Dirichlet and Neumann boundary conditions has been done. Finally, the result of in house written code by Fortran 90 verified by commercial CFD code which relies on finite difference method.

2. Problem description

As shown in Figure 1 a longitudinal fin with triangular profile considered which is one-dimensional convection-conduction heat transfer happened in it. By assuming an axisymmetric unsteady condition, and also considering the suitable time and space marching solved.

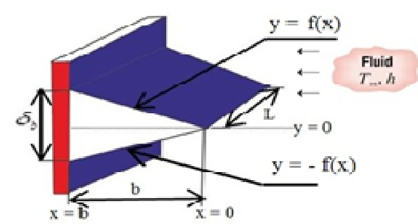

Figure 1. Longitudinal fin with triangular profile

In this study considered some limiting assumptions, which are as following:

- The fin material is homogeneous, its thermal conductivity is the same in all directions, and it remains constant.
- The convective heat transfer coefficient on the faces of the fin is constant and uniform over the entire surface of the fin.
- The temperature of the medium surrounding the fin is uniform.
- The fin thickness is small, compared with its height and length, so that temperature gradients across the fin thickness may be neglected.
- The temperature at the base of the fin is uniform.
- There is no contact resistance where the base of the fin joins the prime surface.

- ❖ There are no heat sources within the fin itself.
- ❖ Heat transfer to or from the fin is proportional to the temperature excess between the fin and the surrounding medium.

And after some mathematics simplification the final governing equation for concerned fin obtained as a following:

$$\rho C f\left(x_P\right)\frac{dT}{dt} = K\frac{d}{dx}\left[f\left(x_P\right)\frac{dT}{dx}\right] - h\left(T - T_\infty\right) \qquad 1)$$

Where the fin profile curve is $f\left(x_p\right) = \frac{\delta_b}{2}\frac{x_p}{b}$ and $0 \le x_p \le b$, $\delta_b = 2f(x_p)$

The boundary condition at the fin base, located at x = b, specified as T_{base} and another boundary condition considered as an insulated (zero heat flux) wall, located at x = 0 and the specifical temperature T_{ip} .

3. Solution procedure

This problem is a parabolic equation and several numerical methods same as finite difference method, finite volume method and finite element method exists for solving it. In this study for solving the equation using the finite difference method in space and the implicit method uses in time. Generally in the implicit method as an information of each node propagates instantaneously in the entire of computational domain and updated simultaneously in every iteration makes this method for the finer mesh is more appropriate than explicit method and also more quickly because needs less computer memory. The governing Eq. (1) discretize by using the implicit method as an Eq. (2)

$$\left[1 + \frac{\beta\Delta t}{x_P} + 2\xi\right]T_P^{n+1} = \left(\xi - \eta\right)T_{P-1}^{n+1} + \left(\xi + \eta\right)T_{P+1}^{n+1} + T_P^n + \frac{\beta\Delta t}{\alpha_P}T_\infty \qquad (2)$$

Where, $\beta = \frac{2bh}{\rho C\delta_b}$, $\eta = \frac{\alpha\Delta t}{2x_p\Delta x}$, $\xi = \frac{\alpha\Delta t}{\Delta x^2}$

The grid used as shown in Figure 2 is not irregular to the left and the right fin because of capturing even the small variation of temperature at this points which are close to boundaries. For interior nodal points 2, 3, before the Last node, we write, using general Eq. (3).

$$a_P T_P^{n+1} = a_W T_{P-1}^{n+1} + a_E T_{P+1}^{n+1} + a_p^n T_P^n + S_u \qquad (3)$$

Figure 2. Illustration of grid point used

The discretization coefficients for interior nodes are shown in table 1.

Table 1. Discretization coefficients for interior nodes

a_w	a_E	a_p^n	a_P	S_p	S_u
$\xi-\eta$	$\eta+\xi$	1	$a_w+a_E+a_p^n-S_p$	$-\beta\Delta t/x_p$	$\frac{\beta\Delta t}{x_p}T_\infty$

The first node:

Using the Eq. (1) to discretized the coefficients at boundary first node, which is insulated (zero heat flux across this boundary) is illustrated in table 2.

Table 2. Discretization coefficients for the first node

a_w	a_E	a_p^n	a_P	S_p	S_u
0	$\frac{2}{3}\eta+\frac{4}{3}\xi$	1	$a_w+a_E+a_p^n-S_p$	$-\beta\Delta t/x_p$	$\frac{\beta\Delta t}{x_p}T_\infty$

The last node:

Similarly, as shown in table 3, using the Eq. (1) to discretize the coefficient at boundary the last node which is kept at specified temperature.

Table 3. Discretization Coefficients for the first node

a_w	a_E	a_p^n	a_P	S_p	S_u
$-\frac{2}{3}\eta+\frac{4}{3}\xi$	0	1	$a_w+a_E+a_p^n-S_p$	$-\left(\frac{8}{3}\xi+\frac{8}{3}\eta\right)-\beta\Delta t/x_p$	$\frac{\beta\Delta t}{x_p}T_\infty+\left(\frac{8}{3}\eta+\frac{8}{3}\xi\right)*T_{base}$

4. Fin performance parameter

The actual conduction heat transfer in the concerned fin is given by:

$$q_f = KA\,\frac{dT}{dx} \tag{4}$$

Where $A\left(x_p\right) = 2Lf(x_p)$ is, the fin cross section, the derivative of Eq.(4) evaluated at x_p. The heat flow dissipated by convection is

$$q = hP(T - T_\infty) \tag{5}$$

Where P is the perimeter of the fin, $P = 2L\left(b^2+\delta(x_p)^2\right)^{1/2}$, the fin thickness is $\delta\left(x_p\right) = 2f\left(x_p\right)$

The fin efficiency is the ratio of the actual heat flow given by equation

$$\eta_f = \frac{q_{f,x_p=b}}{q_{f,\max}} \tag{6}$$

$$q_{f,x_p=b} = KA\left(x_p\right)\frac{dT}{dx} \tag{7}$$

At x_p=b, the cross section equal $A(x_p)=L\delta_b$ and the perimeter $P = 2L\left(b^2 + \delta_b^{\,2}\right)^{1/2}$

$$q_{f,\max} = hP\left(T_{base} - T_\infty\right) \tag{8}$$

5. Results and discussion

As illustrating in Figure 3 the results of in-house Fortran written code by increasing the time changes until 700 seconds and in this time the variation be steady and not change anymore. The temperature gradient near the base of the fin is always high, and then it decreases to reach a minimum value in the fin tip as illustrated in Figure 3.

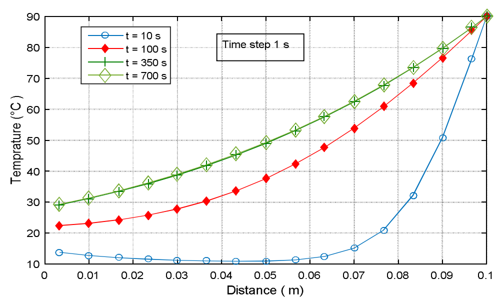

Figure 3. Temperature distribution in a longitudinal triangular fin

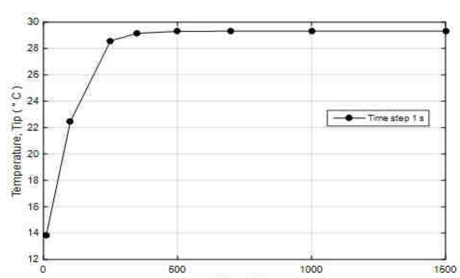

Figure 4. The Tip temperature versus time

Figure 4 displays the graphs of the fin temperature in different times for triangular profile, the max value is tip 29.15 ° C and the results do not change in this temperature.

Figure 5 illustrates the effect of conduction heat transfer coefficient at a base temperature on temperature distribution. Heat dissipated by convection heat transfer as shown in the Figure 6 increases by growing the time until reaches to 533 watts and then be steady**.**

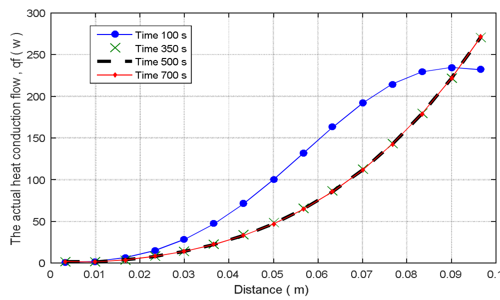

Figure 5. Distribution the actual heat conduction flow in a longitudinal triangular fin with time

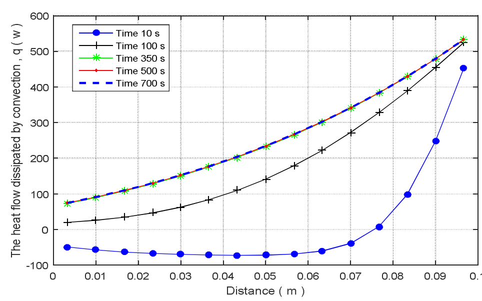

Figure 6. Distribution the heat by convection flow in a ongitudinal triangular fin with time

As shown in Figure 7 the fin efficiency along the fin changes by increasing the time and for the time more than 350 seconds as illustrated in Figure 8 not changes and be 51%.

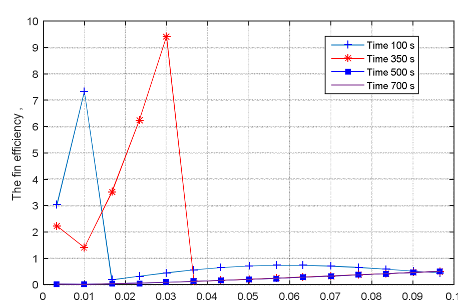

Figure 7. Plots of fin efficiency profile in triangular fin with time

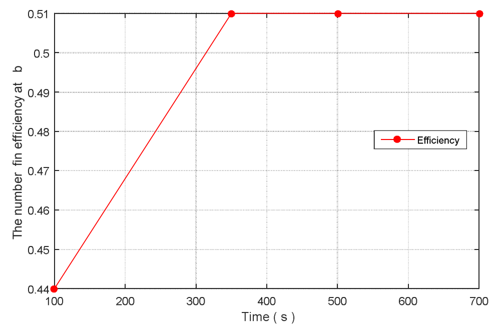

Figure 8. Plots of fin efficiency number profile in a longitudinal triangular fin with time

For the grid independence study has considered the same time and also Δt, only change the number of nodes and by changing the node number monitor the tip temperature as a following:

1) For 2 nodes, Tip temperature is 33.61 ºC
2) For 10 nodes, Tip temperature is 29.59 ºC
3) For 15 nodes, Tip temperature is 28.67 ºC
4) For 20 nodes, Tip temperature is 28.60 ºC
5) For 21 nodes, Tip temperature is 28.60 ºC

So, in this study for the number of nodes equal to 20, the result will not be changed.

6. Verification of results

For verification the results of an in house written code used one dimensional commercial CFD code which relies on finite difference method. As shown in Figure 9 in this study used 20 grid points.

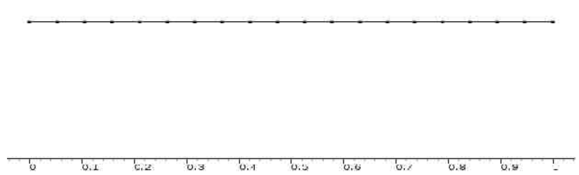

Figure 9. The computational domain of CFD study

The result of commercial CFD code study as shown in Figure 10, the steady condition happened in 700 seconds and this value exactly same as in house written code but the value of tip node temperature is same as exact value and about 1% less than in house written code.

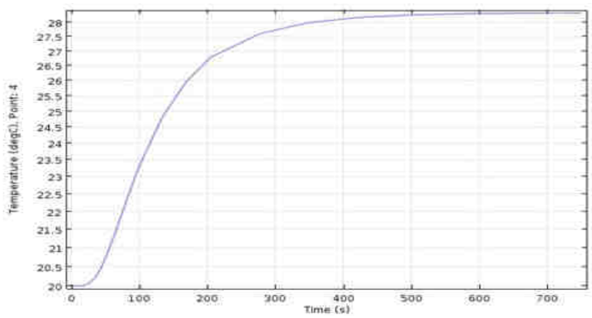

Figure 10. The tip temperature versus time

7. Conclusion

Triangular fin pattern have been numerical studied in the present work and the result of this study showed for the determined boundary conditions after a specific time the obtained results be steady for example fin efficiency at 350 seconds not change and be 51%. For grid independence study used different node numbers and for the number of nodes greater than 20 the tip temperature not changed. Finally, the results of the numerical modeling by using finite difference method and commercial CFD code which relies on finite difference method have a good agreement.

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