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Sinc Function based Interpolation Method to Accurate Evaluate the Natural Frequencies

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Due to leakage phenomena the frequencies and amplitude of the vibration signals given by the sinc function are not indicated correctly. There are interpolation methods that solve the problem of frequency evaluation but which sometimes present significant errors. This paper presents how the real value can be found if the amplitudes are known on 3 spectral lines that are in the vicinity of the actual frequency.

Keywords: *leakage, sinc function, interpolation*

1. Introduction

The spectral leakage phenomenon occurs when truncated signals with finite length are used. This is why other frequency components are obtained besides the original frequency. To reduce the effects of this phenomenon we can take a different window function or choose more samples. Especially for short-time signals the phenomenon of spectral leakage occurs [1].

For a precise frequency determination, advanced algorithms must be applied. In [2] a study regarding spectral leakage and aliasing using biwavenumber transmission function is presented.

In damage detection, we want to improve the frequency readability using interpolation methods. The methods used are based on analyzing two or three points belonging to a spectrum obtained from vibration signal measurements.

A number of papers present different interpolation methods. Among those who have developed interpolation methods based on the study of two spectral lines, we mention Grandke who gives an efficient method that involves one DFT peak and its bigest neighbor, Quinn who proposed a method in which two interpolations are made, both using only two amplitudes [4]. Another similar interpolation method is given by Jain et al. [5]

Interpolation methods based on the use of three spectral lines and thus three amplitudes have been developed by: Ding, which proposed a barycentric method

[6]; Voglewede with a quadratic method [7]; Jacobsen with a new quadratic estimator [8]; Çandan [9] who gives a relation to improve Jaconsen's correction estimator.

Using an iterative interpolation method based on a sequential removal of strong sinc functions in the spatial frequency domain an elimination of spectral leakage is reported in [10]. In [11] there are shown the advantages of using the Gaussian method of interpolation.

A frequency estimation method based on polynomial interpolation is introduced in [12] and the obtained results are compared with that from other frequency evaluation methods in order to find its accuracy.

In this paper we want to develop an interpolation method based on sinc function. For this we study the way in which the energies are distributed on the spectral lines, if they are conform to the sinc function.

2. Theoretical background

A continuous signal $x(t)$ has a discrete representation [16]:

$$\{x\} = \{x[0], x[1], ..., x[k], ..., x[N-1]\} \tag{1}$$

Knowing the sampling time τ, the number of samples N and the sampling rate F_S the signal time length is calculated:

$$T_S = (N-1)\tau = \frac{(N-1)}{F_S} \tag{2}$$

The sequence {x} from (1) is expressed as a sum of sinusoids

$$x[k] = \sum_{j=0}^{N-1} a_j e^{i2\pi \frac{k}{N-1} j} \tag{3}$$

where the coefficients a_j are defined as

$$a_j = \frac{1}{N} \sum_{j=0}^{N-1} x[k] e^{-i2\pi \frac{k}{N-1} j} \tag{4}$$

and f_j is the frequency of the j-th component, given by

$$f_j = \frac{j}{(N-1)\tau} = \Delta f \cdot j \tag{5}$$

Thence we have

$$\Delta f = \frac{1}{(N-1)\tau} = \frac{1}{T_S} \tag{6}$$

The frequencies f_j are equidistantly distributed in the spectrum and there appear the spectral lines. The distance Δf between two consecutive spectral lines is the frequency resolution. The power of the function cos is leaked out from its real frequency component into the components of the Fourier series representation. We calculate the real coefficients

$$a_j = \frac{2}{\tau}\int_0^{\tau} \cos(\omega t)\cos(j\omega_0 t)dt \qquad (7)$$

We have

$$a_j = \frac{2}{\tau}\left[\frac{\sin(\omega - j\omega_0)t}{2(\omega - j\omega_0)} + \frac{\sin(\omega + j\omega_0)t}{2(\omega + j\omega_0)}\right]_0^{\tau} =$$

$$= \frac{1}{\tau}\left[\frac{\sin(\omega - j\omega_0)\tau}{(\omega - j\omega_0)} + \frac{\sin(\omega + j\omega_0)\tau}{(\omega + j\omega_0)}\right]$$

If ω is large enough we can consider

$$a_j = \frac{\sin 2\pi(f_m - j\Delta f)\tau}{2\pi(f_m - j\Delta f)\tau} = \mathrm{sinc}(2\pi(f_m - j\Delta f)\tau) \qquad (8)$$

where $\omega_0 = 2\pi\Delta f$ and $\omega = 2\pi f_m$.

The power spectrum is a powerful tool for frequency evaluation [16] and is given by:

$$P(f_j) = \frac{1}{2}\left|\frac{\sin \pi(f_m - j\Delta f)\tau}{\pi(f_m - j\Delta f)\tau}\right|^2 = \frac{1}{2}\mathrm{sinc}^2(\pi(f_m - j\Delta f)\tau) \qquad (9)$$

where $\mathrm{sinc}(x) = \sin x / x$ is the sinc function.

It can thus be concluded that the total power of the signal $x(t)$ is concentrated around the frequency f_m and the spectral component at any frequency f_j includes a leaked contribution from f_m. So the function sinc^2 determines the size of this contribution.

3. Numerical verification

In order to determine the real frequency we need to find a curve that best fits with some points obtained from DFT. We consider a harmonic signal that we have cut, processed and analyzed for various lengths of time. We have f_m=5Hz, N=4,8, the sampling time τ=0.2s, t=T*N=0.96s. Starting with the observation time length $T_S = 1.1\mathrm{s}$ we considered ten observation times with the 0.2 s step. Different frequency resolutions are achieved for the different time lengths.

Table 1. The coordinates of three considered points for several observation times

T_s [s]	Δf [Hz]	f_{j-1} [Hz]	A_{j-1} [mm/s^2]	f_j [Hz]	A_j [mm/s^2]	f_{j+1} [Hz]	A_{j+1} [mm/s^2]
1.1	0.90909	3.63636	0.24571	4.54545	0.66693	5.45454	0.60894
1.08	0.92592	3.70370	0.24297	4.62963	0.78051	5.55555	0.48337
1.06	0.94339	3.77358	0.20824	4.71698	0.86637	5.66037	0.36156
1.04	0.96153	3.84615	0.15081	4.80769	0.92993	5.76923	0.23961
1.02	0.98039	3.92156	0.08090	4.90196	0.97575	5.88235	0.11659
1	1	4	0	5	1	6	0
0.98	1.02040	4.08163	0.10058	5.10204	0.99166	6.12244	0.09685
0.96	1.04166	4.16666	0.22826	5.20833	0.94150	6.25	0.16205
0.94	1.06383	4.25531	0.37816	5.31914	0.85048	6.38297	0.19194
0.92	1.08695	4.34782	0.53349	5.43478	0.73149	6.52173	0.19380

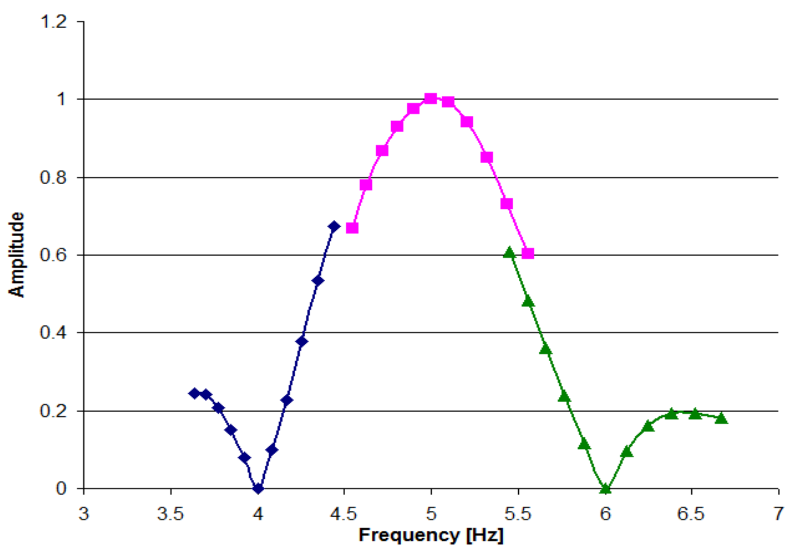

Figure 1. Graphical representation of the data obtained from the measurements

If we consider three points we want to interpolate, for the spectral lines j-1, j and j + 1 we will have the amplitudes A_{j-1}, A_j and A_{j+1}. In table 1 we have this observation time lengths, the coordinates for the three considered points and the frequency resolution.

In figure 1 we have the graphical representation of the data obtained from the measurements more precisely, we plotted the amplitudes, from table 1, depending of frequencies. As can be seen from figure 1, we can not find a curve going through these points. For this, a correction is needed. For this we multiply the column A_{j-1} with 0.86 and the column A_{j+1} by 1/0.86= 1.16, thus obtaining the graph in figure 2.

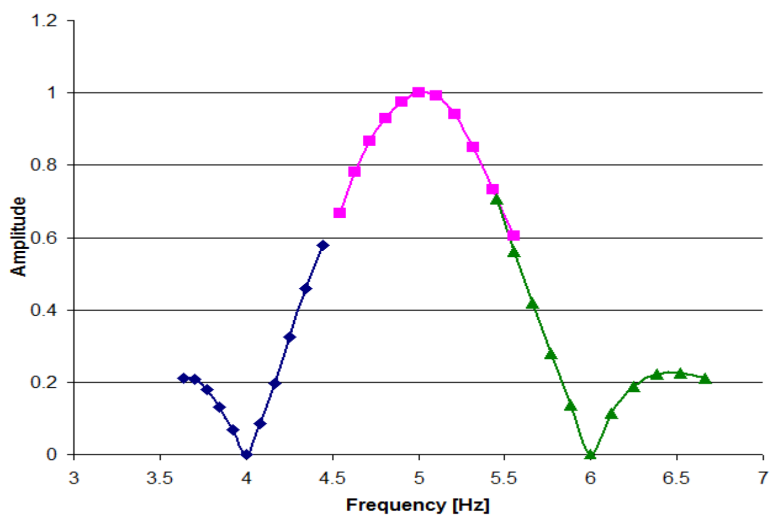

Figure 2. The corrected graph

We want to see if the sinc function defined by (10) best fits with the points obtained from DFT.

$$g(f_j) = \frac{\sin(\pi * (f_m - f_j))}{\pi * (f_m - f_j)} = \text{sinc}(\pi * (f_m - f_j)) \quad (10)$$

Figure 3 presents a comparison between the amplitudes obtained after the correction and the sinc function given by (10). With the adjustment we made we find that a sinc function fits pretty well with the corrected data. In Table 2 the error between sinc function and corrected data as well as relative error in percentages are presented.

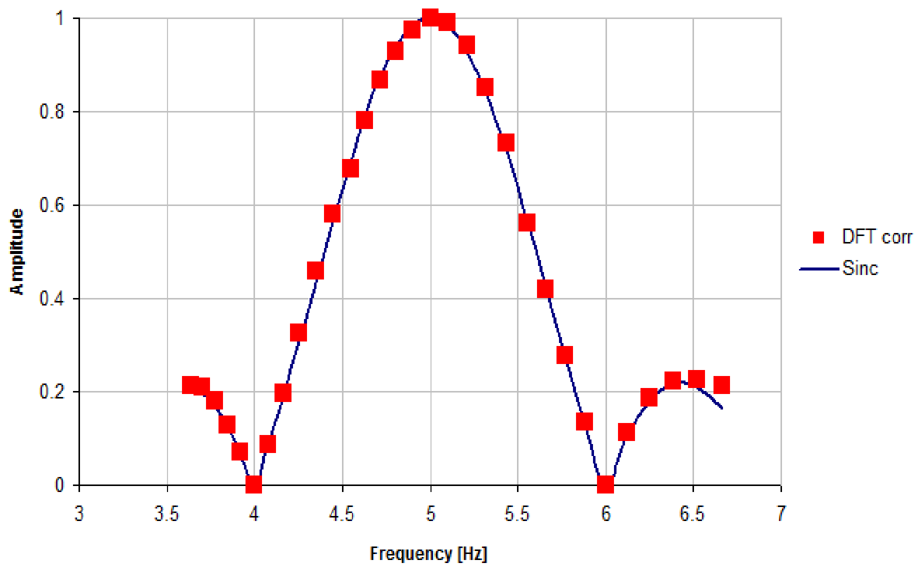

Figure 3. Graphical representation of the corrected data and the sinc function

Table 2. Comparison between the corrected amplitudes and the sinc function

Sinc	Amplitude	error	error %	Sinc	Amplitude	error	error %
0.212333	0.211313	-0.00102	-0.48022	0.984264	0.975751	-0.00851	-0.86494
0.196964	0.208959	0.011995	6.090161	1	1	0	0
0.169437	0.179086	0.00965	5.69519	0.98296	0.991669	0.008709	0.885983
0.128202	0.129699	0.001497	1.167501	0.930119	0.941507	0.011388	1.224341
0.071993	0.069577	-0.00242	-3.35565	0.840676	0.850485	0.009809	1.166772
3.9E-17	0	-3.9E-17	0	0.716799	0.73149	0.014691	2.049462
0.087918	0.086501	-0.00142	-1.61143	0.564253	0.562059	-0.00219	-0.38872
0.190986	0.196308	0.005322	2.786349	0.422115	0.420419	-0.0017	-0.40196
0.307257	0.325222	0.017965	5.846804	0.274402	0.278617	0.004216	1.536325
0.433355	0.458803	0.025449	5.872447	0.130318	0.135573	0.005255	4.032404
0.564253	0.579698	0.015446	2.737389	3.9E-17	0	-3.9E-17	0
0.693154	0.676935	-0.01622	-2.33995	0.10642	0.112619	0.006198	5.82449
0.789149	0.780512	-0.00864	-1.09447	0.180063	0.18844	0.008376	4.651849
0.873352	0.866372	-0.00698	-0.79926	0.214783	0.223197	0.008413	3.917098
0.940267	0.92993	-0.01034	-1.09938	0.208687	0.22535	0.016663	7.984449

4. Conclusion

As can be seen, the data obtained fit well enough to the sinc function, only requiring a small adjustment of the values. The worst results are obtained for the lateral lobes. If there is no need for high precision we can use the sinc function, but for high precision we have to find another adapted function, based on this function.

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