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Studying the Relative Positions of the Teeth of Conjugated Wheels in the Double Harmonic Transmission

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The paper presents a detailed study of the gearing processes in the two stages of the double harmonic transmission. To highlight the graphics of the relative motion of the conjugates teeth it was developed an original calculation program, written in Visual Basic. By running the calculation program, the relative successive positions of the conjugate teeth were viewed and the basic parameters of the gearing were established: the angle profile (α), the tooth height (h) and the size of the deformation of the flexible toothed wheel (w_0).

***Keywords**: double harmonic transmission, flexible wheel, tooth, relative motion, trajectory*

1. Introduction

The double harmonic transmission is part from the harmonic gear category and is based on the same operating principle as the radial harmonic transmission, but contains in addition a harmonic gear stage [1, 2, 3, 4, 5, 6, 7, 10, 11, 18].

Double harmonic transmission (Figure 1) consists of 4 main elements:

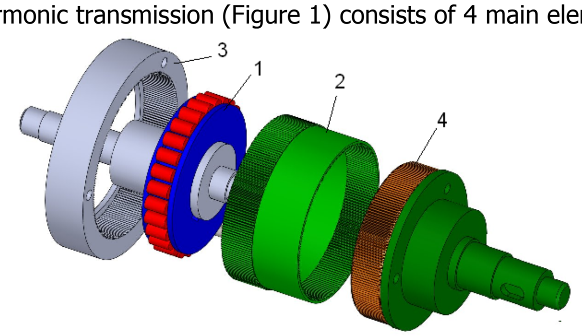

Figure 1. The double harmonic transmission

cam-wave generator (1) - as the driving element, flexible wheel (2) with outer teeth (z_2) and inner teeth (z'_2) - as an intermediate element, fixed rigid wheel (3) with internal teeth (z_3) - as a fixed element and a rigid mobile wheel (4) with external teeth (z_4) [12, 14, 15, 16, 17].

By forcefully fitting the wave generator from inside the short flexible wheel its elliptical deformation occurs. Thus, the flexible wheel will have four equidistant harmonic engagement zones: two opposite with fixed rigid wheel (step 1 - z_2 / z_3), respectively two others with a rigid mobile wheel (step 2 - z'_2 / z_4). The two pairs of opposite harmonic engagement zones are offset at an angle of 90°.

In this paper makes a detailed study of the relative movements of the conjugated teeth from the two steps of the transmission.

2. The relative positions of the teeth in step I of the double harmonic transmission

Determination of the relative positions of the conjugated teeth is based on tracing the motion trajectories of points located on the axis of symmetry of the mobile tooth [8, 13].

In Figure 2 has been represented with continuous line the relative position of the two teeth, one belonging to the fixed rigid wheel (3) and the other belonging to the deformed flexible wheel (2), which at the initial moment t = 0 are in the direction of the small axis of the wave generator. With a broken line, the position of the same tooth of the flexible wheel was highlighted when it is in an undeformed state.

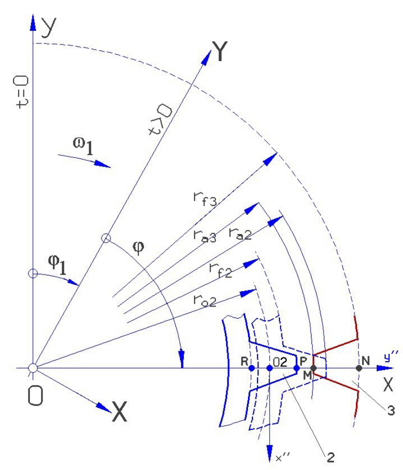

Figure 2. The initial positions of the teeth in step I of the double harmonic transmission

The mobile reference system S_2 (x "O_2y"), fixed to the flexible wheel, originates at the intersection of the minor axis of the wave generator with medium fiber of the undeformed flexible wheel (circle arc of radius r_{02}). O_2y" axis overlaps the axes of symmetry of teeth considered at the initial moment.

The initial position of the fixed rigid wheel tooth in the system S_2 (x"O_2y"), is determined by the positions of two points M and N considered on the axis of symmetry of the tooth, at the intersection with the addendum circle (index a - point M), respectively with the dedendum circle (index f - point N) of the fixed rigid wheel [9, 13].

The coordinates of these points (M, respectively N) are given by the relations:

$$\left.\begin{aligned} y''_M &= w_{a3} = r_{a3}\cdot\cos\left[(2\pi/z_3)\cdot\psi\right] - r_{02} \\ x''_M &= v_{a3} = -\,r_{a3}\cdot(2\pi/z_3)\cdot\psi \end{aligned}\right\} \qquad (1)$$

$$\left.\begin{aligned} y''_N &= w_{f3} = r_{f3}\cdot\cos\left[(2\pi/z_3)\cdot\psi\right] - r_{02} \\ x''_N &= v_{f3} = -\,r_{f3}\cdot(2\pi/z_3)\cdot\psi \end{aligned}\right\} \qquad (2)$$

For simplicity of calculations and graphical construction it is considered that the rigid wheel tooth is immobile when rotating the wave generator, and the position of the flexible wheel tooth changes successively.

Thus, successive positions of the flexible wheel tooth can be determined by successive positions of two points P and R on the axis of symmetry of the tooth, at its intersection with the addendum circle (index a - point P), respectively with the dedendum circle (index f - point R) of the flexible wheel [13[.

The coordinates of the points P and R in the mobile system S2 are given by the relations:

$$\left.\begin{aligned} y''_P &= w_{a2} = (r_{a3}+w)\cdot\cos\varphi_3 - r_{02} - w_3 \\ x''_P &= v_{a2} = v + (r_{a2}-r_{02})\cdot\theta - (r_{a2}+w)\cdot\varphi_3 - v_3 \end{aligned}\right\} \qquad (3)$$

$$\left.\begin{aligned} y''_R &= w_{f2} = (r_{f2}+w)\cdot\cos\varphi_3 - r_{02} - w_3 \\ x''_R &= v_{f2} = v + (r_{f2}-r_{02})\cdot\theta - (r_{f2}+w)\cdot\varphi_3 - v_3 \end{aligned}\right\} \qquad (4)$$

where: w, v – is the radial respectively tangential displacement; θ – the angle of rotation of the normal direction; φ_3 – the angle of relative rotation of the wheel; $w_3 = 0$; $v_3 = 0$; $\psi = 0$.

In Figure 3 indicates the successive positions of the tooth of the flexible wheel relative to the tooth of the fixed rigid wheel, obtained by running an originally designed program, for the real double harmonic transmission case characterized by the following constructive and functional parameters: transmission ratio, $i_{14}^{(3)} = 50$; module, m = 0,3 mm; radius of the dynamic fiber, r_{02} = 29,3 mm; maximum radial deformation, w_0 = 0,3 mm (for step I); w'_0 = 0,27 mm (for step II); number of teeth, z_2 = 200 teeth; z'_2 = 192 teeth; z_3 = 202 teeth; z_4 = 190 teeth; angle analysis, n = 90° and step angle, i = 10°.

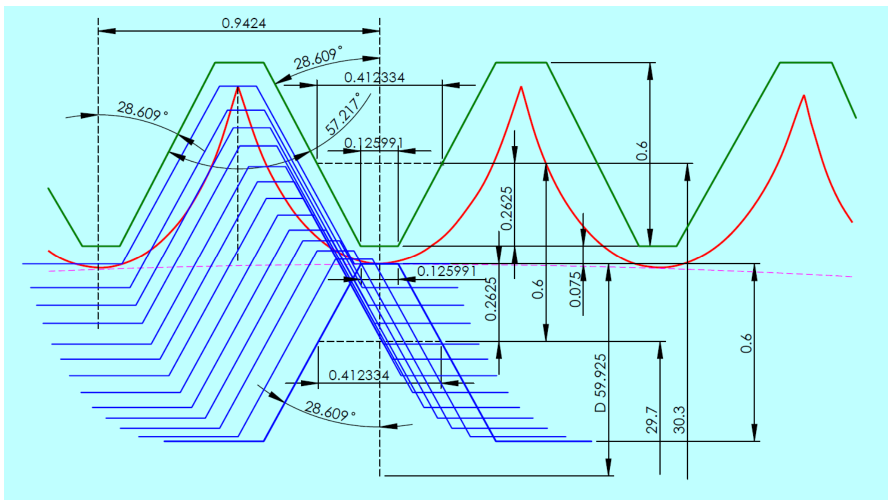

Figure 3. The relative positions of the teeth in step I of the double harmonic transmission

3. The relative positions of the teeth in step II of the double harmonic transmission

In Figure 4, the relative position of two teeth, one belonging to the mobile rigid wheel (4) and the other the deformable flexible wheel (2), was represented with a continuous line, which at the initial moment t = 0 are in the direction of the major axis of the wave generator. With a broken line, the position of the same tooth of the flexible wheel was highlighted when it is in an undeformed state.

The coordinate axis system S_2 (x"O_2y") is considered to be related to the flexible gear wheel and it is admitted that the tooth of the rigid mobile wheel is fixed and its movement is taken over by the flexible wheel tooth.

By proceeding in a similar manner as in the first steep of the double harmonic transmission, the relative positions of the two teeth in the mobile system S_2 are obtained, at the intersection with addendum circles (index a - for points M' and P'), respectively with dedendum circles (index f - for points N' and R').

Thus, the coordinates of the points M' and N' on the axis of symmetry of the tooth of the mobile rigid wheel are determined by means of the relations:

$$\left.\begin{aligned} y''_{M'} &= w_{a4} = r_{a4} \cdot \cos\left[(2\pi / z_4) \cdot \psi\right] - r_{02} \\ x''_{M'} &= v_{a4} = -r_{a4} \cdot \left[(2\pi / z_4) \cdot \psi\right] \end{aligned}\right\} \tag{5}$$

$$\left.\begin{aligned} y''_{N'} &= w_{f4} = r_{f4} \cdot \cos\left[(2\pi / z_4) \cdot \psi\right] - r_{02} \\ x''_{N'} &= v_{f4} = -r_{f4} \cdot \left[(2\pi / z_4) \cdot \psi\right] \end{aligned}\right\} \tag{6}$$

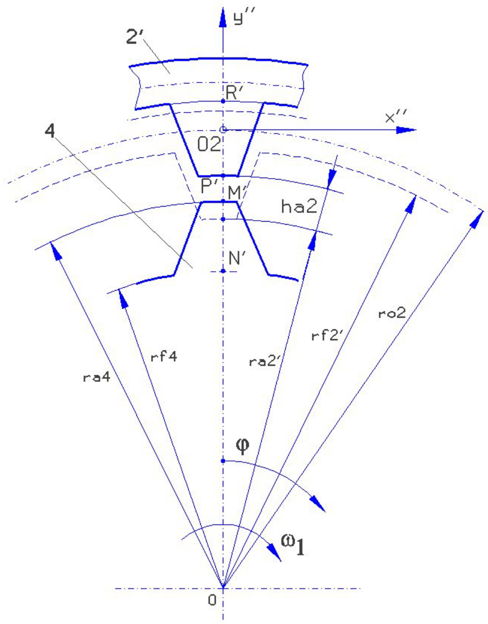

Figure 4. The initial positions of the teeth in step II
of the double harmonic transmission

The coordinates of the points P' and R' defining the relative position of the flexible wheel tooth against to the tooth of the rigid mobile wheel considered fixed, at a time t > 0, will be expressed through the following relationships:

$$\left.\begin{aligned} y''_{P'} &= w_{a2} = (r'_{a2} + w) \cdot \cos\ (\varphi_4 - \varphi_2) - r_{02} \\ x''_{P'} &= v_{a2} = v - (r'_{a2} - r_{02}) \cdot \theta + (r'_{a2} + w) \cdot (\varphi_4 - \varphi_2) \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} y''_{R'} &= w_{f2} = (r'_{f2} + w) \cdot \cos\ (\varphi_4 - \varphi_2) - r_{02} \\ x''_{R'} &= v_{f2} = v - (r'_{f2} - r_{02}) \cdot \theta + (r'_{f2} + w) \cdot (\varphi_4 - \varphi_2) \end{aligned}\right\} \tag{8}$$

The successive positions of the flexible wheel tooth (from 10° to 10°) located on its internal cylindrical surface relative to the toothed rigid wheel have been shown in Figure 5.

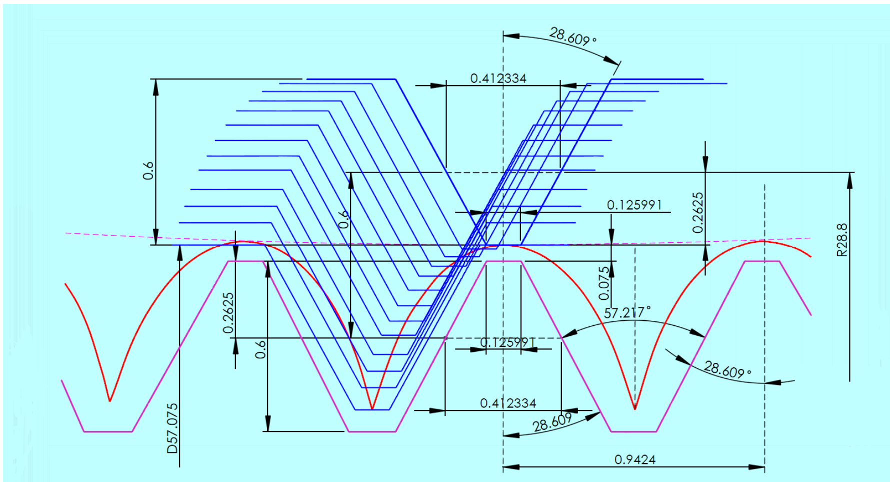

Figure 5. The relative positions of the teeth in step II of the double harmonic transmission

4. Conclusion

The paper presents a method for calculating the relative successive positions of a tooth of the short flexible wheel against a tooth of the fixed rigid wheel, respectively of the rigid mobile wheel, which led to the development of an original computer program.

The results obtained following the running of the computerized program have enabled a correct scientific interpretation of the particularities of the 2 step engagement of the double harmonic transmission, which largely depends on the transmission load.

The relative movement of the flexible wheel teeth relative to the teeth of the conjugated rigid wheels does not occur around a fixed point or axis, but there is a movement with a variable angular velocity movement occurs around a point that also has a variable position in the plane.

The gears can be considered flat, because the flexible wheel is short and the toothed crowns have reduced lengths.

The graphical constructions (Figure 3 and Figure 5) can be used to choose the basic parameters of the harmonic engagement: the angle profile (α), the tooth height (h), the clearance (c_0) and the maximum radial deformation (w_0) of the short flexible wheel, so as to avoid tooth interference.

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