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A Practical Method to Increase the Frequency Readability for Vibration Signals

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Damage detection and nondestructive evaluation of mechanical and civil engineering structures are nowadays very important to assess the integrity and ensure the reliability of structures. Thus, frequency evaluation becomes a crucial issue, since this modal parameter is mainly used in structural integrity assessment. The herein presented study highligts the possibility of increasing the frequency readability by involving a simple and cost-effective method.

Keywords*: frequency estimation, power spectrum, polynomial curves*

1. Introduction

Frequency estimation of real-life signals is becoming an important issue in various technical domains, among which is worth to mention structural health monitoring [1]. This emerging domain bases on analysis of vibration signals, most of them acquired for rapid damping structures [2]-[4]. A short acquisition time results due to the damping, which makes frequency evaluation difficult [5]. Several methods are used to improve the frequency readability, from complicated time- and resources-consuming procedures to simple algorithms [6]-[7].

In this paper a frequency estimation method is introduced and the achieved results are compared with that obtained from standard frequency evaluation in order to find the accuracy of the proposed method.

2. Problems in standard frequency evaluation

Multidimensional signals are in general linear combination of sinusoids. Frequency evaluation makes use of this property, it being in fact the decomposition of the signal in harmonic components. This is usually made by the Fourier Transform or derivate procedures, i.e. Discret Fourier Transform (DFT), Power Spectrum (PS), Power Spectral density (PSD) and so on.

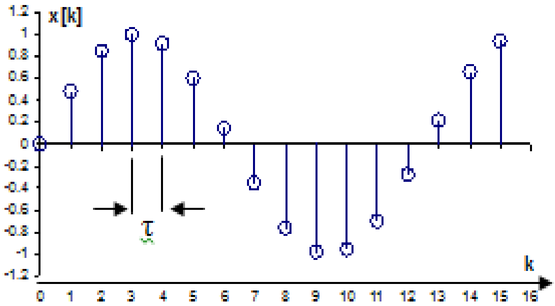

Figure 1. Digital signal.

The problem arising from this procedure is the sinusoids' frequencies. These have values fixed by the time of analysis [5], the difference between two consecutive sinusoids being in the relation:

$$\Delta f = \frac{j}{(N-1)\tau} = \frac{1}{T_S} \tag{1}$$

where Δf is the frequency resolution, j is the number of the sinusoid placed at the j–th spectral line, N is the total number of samples in the signal, τ is the sampling time and T_S is the signal length in time.

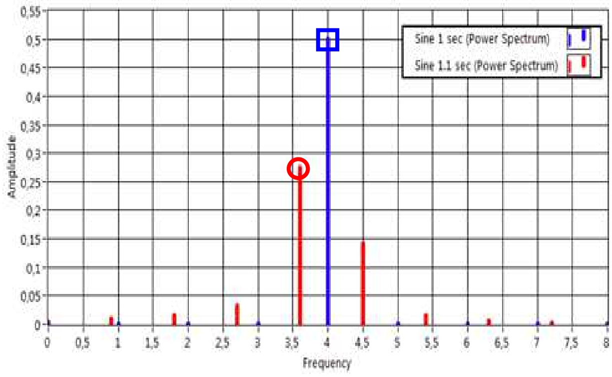

Figure 2. Power spectral representation of a harmonic signal for two analysis time lengths.

Having a look onto figure 2 one can remark that different pick values are indicated for the two analysis cases, obviously associated with different frequencies. This shows that even for harmonic signals the standard frequency evaluation method is not suitable is short or medium analysis time is involved.

The real result is achieved for the analysis time for which the signal (sinusoid, in this case) contains entire number of cycles. For multidimensional signals the harmonic components has to fulfill individually this condition. This shows that the analysis time length for a multidimensional signal has to be adapted for each comprised harmonic component. The real frequency is always indicated by the highest amplitude in the spectra located in the neighborhood of the evaluated frequency, if different analysis times are involved. Based on this fact, in order to avoid numerous spectral analyses, a simple method to overcome this inconvenient is to use spline functions to find the pick amplitude-frequency pair. This concept is presented in the following section.

3. Polynomial regression curves used in the frequency estimation process

Spline functions are based on polynomials, which are functions involving only non-negative integer powers of x. The standard way to write a polynomial function is:

$$P(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \ldots + c_i x^i \ldots + c_n x^n \tag{2}$$

where the c's are real numbers called coefficients and n is the degree of the polynomial. Also, for practical reasons, $P(x_i)$ is sometimes denoted y_i.

It is possible to find the curve passing trough a number of fixed points; the degree of the polynomial is determined by the number of support points. Following procedure is used o find the coefficients c. From points $\mathrm{A}_1(x_1, y_1)$ to $\mathrm{A}_n(x_n, y_n)$ result n pair values $x_i \leftrightarrow y_i$ and the next algebraic system:

$$\begin{cases} y_1 = c_0 + c_1 x_1 + c_2 x_1^{\;2} + c_3 x_1^{\;3} + \ldots + c_{i-1} x_1^{\;i} \ldots + c_{n-1} x_1^{\;n} \\ y_2 = c_0 + c_1 x_2 + c_2 x_2^{\;2} + c_3 x_2^{\;3} + \ldots + c_{i-1} x_2^{\;i} \ldots + c_{n-1} x_2^{\;n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_i = c_0 + c_1 x_i + c_2 x_i^{\;2} + c_3 x_i^{\;3} + \ldots + c_{i-1} x_i^{\;i} \ldots + c_{n-1} x_i^{\;n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ y_n = c_0 + c_1 x_n + c_2 x_n^{\;2} + c_3 x_n^{\;3} + \ldots + c_{i-1} x_n^{\;i} \ldots + c_{n-1} x_n^{\;n} \end{cases} \tag{3}$$

There are many ways to solve the system and find the coefficients c, analytically or by means of algorithms implemented in software. Herein the system for the quadratic polynomial, i.e. a second degree polynomial constructed by using three points, is resolved analytically.

4. Frequency estimation by involving several amplitudes from the spectra

The first analyzed case concerns the frequency estimation from three points located around the evaluated frequency, as shown in figure 3. Note that the difference between any two consecutive frequency values indicated at the spectral lines is $x_i - x_{i-1} = \Delta f$, which actually the frequency resolution is. For simplicity, the abscissa of the first considered point x_1 (that is frequency f_{j-1}) is taken zero; this is possible by the shifting the vertical axis with f_{j-1} to the $j-1$-th spectral line. The next two points are thus located at Δf respectively $2\Delta f$. The three amplitudes A_{j-1}, A_j and A_{j+1} confer the ordinates y_1, y_2 and y_3. This is illustrated in figure 3.

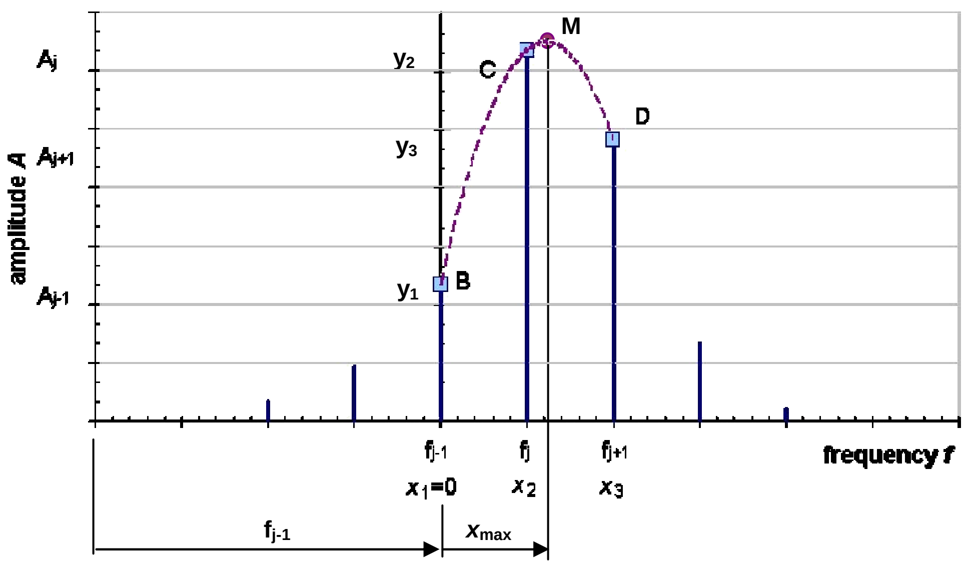

Figure 3. Power spectral representation of a sinusoid with highlighting the points used in frequency evaluation and the vertical axis shift.

The three points have therefore the coordinates: $\mathrm{B}(0, y_1)$, $\mathrm{C}(x_2, y_2)$ and $\mathrm{D}(x_3, y_3)$. The point with biggest amplitude is always centrally located. From eq.(3), the coefficients result as:

$$c_0 = y_1 \qquad c_1 = \frac{-y_3 + 4y_2 - 3y_1}{2\Delta f} \qquad c_2 = \frac{y_3 - 2y_2 + y_1}{2\Delta f^2} \tag{4}$$

The regression curve will indicate a pick $\mathrm{M}(x_{\max}, y_{\max})$ between the two biggest amplitudes that is, in the case of figure 3, between points C and D. The abscissa $x_{\max}$ is used to correct the read frequency and attain a value closer to the real frequency. This is made by adding $x_{\max}$ to f_{j-1} which represents the vertical axis shift. The corrected frequency results as:

$$f_{corr} = f_{j-1} + x_{\max} \tag{5}$$

The $x_{\max}$ coordinate of point M is found at the location where the derivative of the regression function $P(x)$ is null. For

$$P'(x) = c_1 + 2c_2 x = 0 \tag{6}$$

which has the solution

$$x_{\max} = -c_1 / 2c_2 \tag{7}$$

An exemplification for a signal having the real frequency $f_{\mathrm{real}} = 4\mathrm{Hz}$ is presented below. The signal has an amplitude $A = 1$ and was created for a sampling rate $r_s = 1000\,\mathrm{Hz}$. Two cases differing the number of samples is presented. In the first analyzed case the number of samples is $N_{\mathrm{I}} = 11300$, and on two spectral lines the amplitudes have close values. For the second case the number of samples is $N_{\mathrm{II}} = 11600$; here one point has the amplitude much higher as the other two points. For both cases, the resulted time length, frequency resolution and position of the first 6 spectral lines are presented in table 1.

Table 1. Signal parameters for the two considered cases

N	T_s	Δf	f_0	f_1	f_2	f_3	f_5	f_6
11300	1.13	0.8849	0	0.8849	1.7699	2.6548	3.5398	4.4247
116200	1.16	0.8620	0	0.8620	1.7241	2.5862	3.4482	4.3103

Table 2 presents the coordinates for the three points used to plot the polynomial curve. The frequency-amplitude pairs are indicated by coordinates x_i respecyively y_i. Note that $x_1 = 0$, thus the free term of the polynomial function is y_1. These points are used to calculate the coefficients c which are indicated in table 3, togheder with the x-coordinate of point M, namely $x_{\max}$.

Figures 4 and 5 illustrate the regresion curves for the two analyzed cases.

Table 2. Coordinates of the points considered to find the polynomial curve

Case number	x_1	x_2	x_3	y_1	y_2	y_3
1	0	0.8849	1.7699	0.03559	0.237207	0.223348
2	0	0.862069	1.724138	0.129233	0.35403	0.022186

Table 3. Coefficients of the polynomial curves and the abscissa of point M

Case number	c_0	c_1	c_3	$x_{\max} = -c_1 / 2c_2$
1	0.03559	0.349571	-0.13757	1.270515
2	0.1292	0.5836	-0.37451	0.779177

Applying now eq. (5) to find the corrected frequency one obtain:

- for case 1, the corrected frequency is $f_{corr} = 2.6548 + 1.2705 = 3.9253\,\text{Hz}$ is improved comparing to $f_{read} = 3.5398\,\text{Hz}$ or $f_{read} = 4.4247\,\text{Hz}$;
- for case 2, the corrected frequency is $f_{corr} = 3.4482 + 0.7791 = 4.2273\,\text{Hz}$ in stand of $f_{read} = 4.3103\,\text{Hz}$.

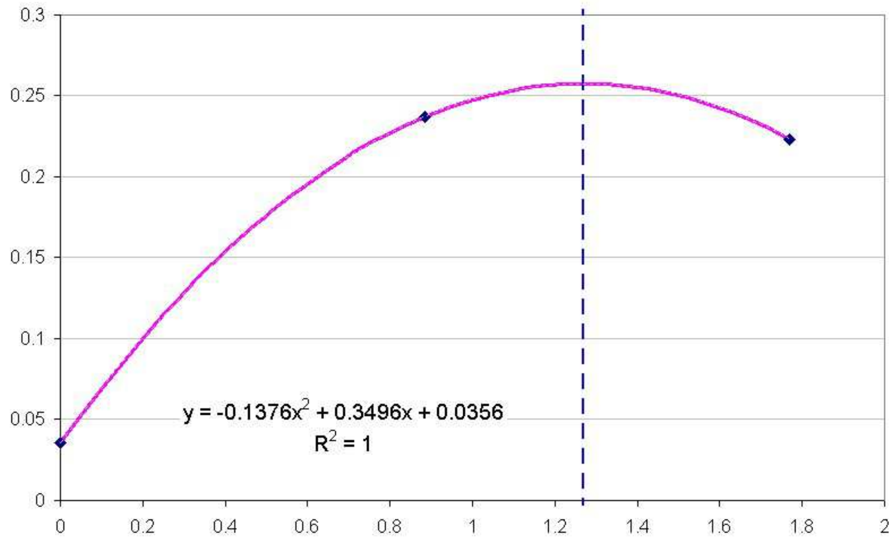

Figure 4. Polynomial function used to find the local maximum – case 1

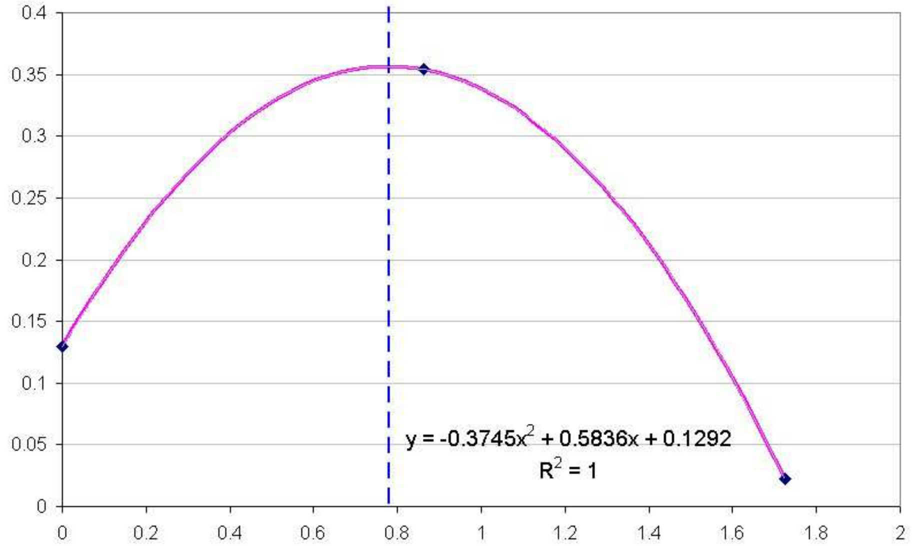

Figure 5. Polynomial function used to find the local maximum – case 2

The frequencies evaluated involving the standard procedure in comparison with that obtained using the proposed method is presented in table 4. It is shown that the method definitely improves the frequency readability, but the level of improvement depends on the relative position (in terms of amplitudes) of the involved points.

Table 4. Frequency evaluated by standard procedure and involving the method proposed in this paper

Case number	f_{read} $[Hz]$	f_{real} $[Hz]$	Error [%]	$f_{corr}[Hz]$	Error [%]
1.1	3.5398	4	-11.505	3.9253	-1.867
1.2	4.4247		10.617		
2	4.3013	4	7.532	4.2274	5.685

Good results are achieved in the case that two points have closely-big amplitudes the correction, the error being reduced from around 10% to less than 2%. In contrast, for the case when one point has much bigger amplitude as the other two, the correction is insignificant, from 7.5% to 5.5%.

4. Conclusion

A method to increase the frequency readability is presented in this paper, and the results are compared with that obtained from standard evaluation. It is found that the proposed method increases the frequency readability, diminishing the maximum achieved error from 11% to 5%.

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