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Research Concerning Deformation of Flexible Wheel of the Double Harmonic Gear Transmission

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The paper presents a study of the dynamic behaviour of flexible toothed wheel whit combined radial teeth (exterior, respectively interior), of a double harmonic gear transmission (DHGT). We present a comparative-critical analysis of two deformation laws of the flexible toothed wheel (one elliptical and other cosine type), using a set of computer programs of own conception, written in Visual Basic. We demonstrate a similarity between cosine deformation law of the flexible wheel and the elliptical law, proving that the cosine deformation law can approximate the elliptical law with a high degree of accuracy.

Keywords: *double harmonic gear transmission, flexible toothed wheel, deformation law, dynamic behaviour, dynamic reference fiber.*

1. Introduction

The development and modernization of machinery industry has led to the continuous improvement of existing drive systems, as well as to the appearance of new and more efficient systems.

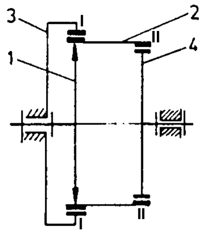
In this context, there appeared a series of new gear mechanical transmissions (called unconventional), which do not comply with the fundamental law of gearing in the form known for traditional gears (cylindrical, conical and worm gears) [1], [2], [3], [7], [12], [17].

The harmonic gear transmission can be included in the category of unconventional gear; it is a relatively recent technical creation, based on an entirely different function principle.

The harmonic gear transmissions allows the transmission of rotational motion from the driving element to the driven, due to the propagation of elastic deformations (waves) on the periphery of the flexible toothed wheel, with a specific frequency [4], [6], [8], [9]. Through constructive and functional diversifications of harmonic gear transmissions, a new variant was created, named the double harmonic gear transmission (DHGT) [5], [11], [13], [14].

2. The construction of a double harmonic gear transmission

DHGT is composed of four principal elements (Figure 1): the waves generator (1) is a driving element, the rigid wheel with interior teeth (3) is a fixed element, the short flexible toothed wheel (2) with the outer teeth (section I-I), respectively the inner teeth (section II-II), is an intermediate element, the mobile rigid wheel (4) with the outer teeth, as a driven element [5].



1 – waves generator

2 – flexible toothed wheel

3 – fixed rigid wheel

4 – mobile rigid wheel

Figure 1. Structural scheme of a DHGT

The functional performance and durability of DHGT are greatly influenced by the durability of flexible toothed wheel, which is the most stressed element in the transmission.

From a constructional point of view, the flexible toothed wheel has the form of a short circular tube with thin wall, open at both ends and having at each end a crown gear (outer, respectively inner - Figure 2).

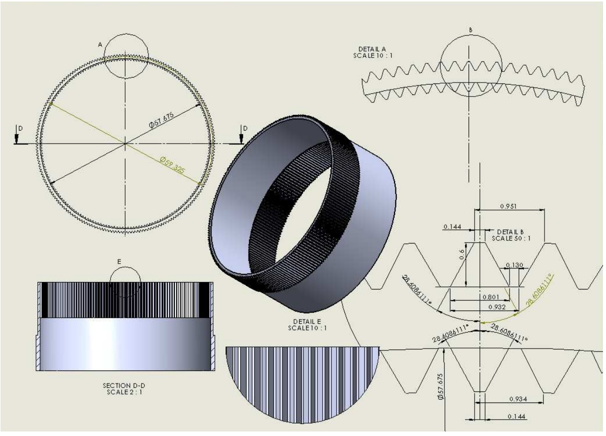


Figure 2. Flexible toothed wheel

During operation of the DHGT, the flexible toothed wheel is in a complex stress state and elastic deformation, depending on many factors such as: the type of waves generator, the geometric shape of the flexible wheel, the torque transmitted and the coupling of the flexible wheel with the output shaft [15, 16].

These aspects have imposed the need for research of the deformation state of the flexible toothed wheel, in assessing the dynamic behavior of flexible wheel.

In this paper we present a detailed study of two deformation laws of the flexible toothed wheel in the two front sections.

3. The deformation law of the flexible toothed wheel

The functioning and the durability of DHGT are largely depending on the geometrical shape and the precision execution of the flexible toothed wheel, and also its deformation law [5, 10].

For the study of the dynamic behavior of the flexible toothed wheel we adopt the simplifying hypothesis that the dynamic reference fiber of the deformed flexible wheel keeps its elliptical shape and constant length in any cross-section of the flexible wheel.

By forced mounting of the waves generator inside the flexible toothed wheel, this will pass from undeformed status (the circular shape) in the deformed status (the elliptical shape). The dynamic reference fiber of the flexible wheel is an ellipse, and the dynamic reference fibers of rigid fixed wheel (3) and mobile rigid wheel (4) are concentric circles [15, 16]. In Figure 3 we represented the dynamic reference fibers of the conjugated wheels.

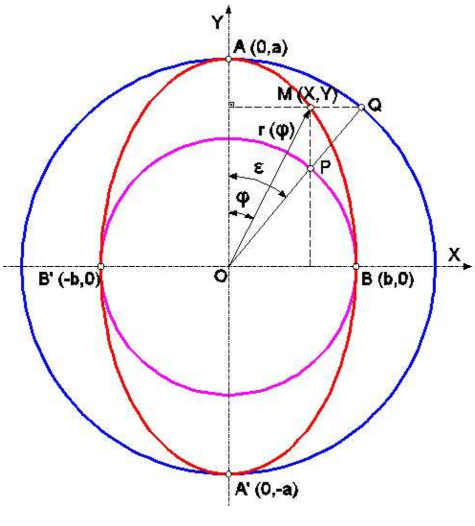


Figure 3. Dynamic model of DHGT

It is observed that the dynamic reference fiber of the flexible wheel is an ellipse that intersects the OX axis in the points B (b, 0) and B' (- b, 0), respectively the OY axis in the points A(0,a) and A'(0,-a), which are precisely the extreme points of the ellipse.

Using the usual definition of the ellipse, the parametric equations of dynamic reference fiber of the flexible wheel can be expressed through coordinates of a point M situated on the ellipse, in the mobile system of coordinate axes S_1(XOY).

$$\begin{cases} X = b \cdot \sin\varepsilon = \dfrac{a \cdot b \cdot \sin\varphi}{(a^2 \cdot \sin^2\varphi + b^2 \cdot \cos^2\varphi)^{1/2}} \\ Y = a \cdot \cos\varepsilon = \dfrac{a \cdot b \cdot \cos\varphi}{\left(a^2 \cdot \sin^2\varphi + b^2 \cdot \cos^2\varphi\right)^{1/2}} \end{cases} \tag{1}$$

where: a - is semi-major axis of the ellipse;

b - small semi axis of the ellipse;

ε - the angle formed between the radius OQ of the big circle and ordinate axis OY;

φ - the angular parameter for positioning of the considered section, with reference to the OY axis.

It can be seen from Figure 2, that between the ε angle at centre and φ angular parameter there is the following mathematics dependence:

$$tg\varphi = \frac{X}{Y} = \frac{b}{a} \cdot tg\varepsilon \tag{2}$$

From the relation (1) we can write the implied equation of the ellipse expressed in polar coordinates:

$$r(\varphi) = \sqrt{X^2 + Y^2} = \frac{a \cdot b}{\left(a^2 \cdot \sin^2\varphi + b^2 \cdot \cos^2\varphi\right)^{1/2}} \tag{3}$$

where: $a = r_{02} + w_0$; $b = r_{02} - w_0$, for the front section I-I;

$a = r_{02} + w'_0$; $b = r_{02} - w'_0$, for the front section II-II;

r_{02} - the radius of median fiber of the undeformed flexible wheel;

w_0, w'_0 - the maximum radial deformation in sections I-I, and II-II.

The law of elliptical deformation (3) can be approximated with good accuracy through the simple cosine law of the deformation of the flexible wheel:

$$r(\varphi) = a_0 + a_2 \cdot \cos 2\varphi \tag{4}$$

where: $a_0 = r_{02}$; $a_2 = w_0$, for the front section I-I;

$a_2 = w'_0$, for the front section II-II.

For any type of deformation law (elliptical or cosine) of the flexible toothed wheel, the r(φ) vector rays of the points situated on the dynamic reference fiber of the deformed wheel, in the two front sections, can be calculated using the following relations:

$$r_{I-I}(\varphi) = r_{02} + w(\varphi);\ r_{II-II}(\varphi) = r_{02} + w'(\varphi) \qquad (5)$$

where: w(φ), w'(φ) – are the radial deformation in frontal sections I-I and II-II.

In order to study the degree of approximation of the elliptical deformation law (3) by the cosine law (4), which can be materialized by means of a wave generator with cam, we developed a set of programs written in Visual Basic.

By running these programs it was possible to highlight the static state of deformation of the flexible toothed wheel, by numerical calculation of r(φ) vector rays of the points situated on the dynamic reference fibers from the front sections of the flexible element, using the relations (3) and (4). We considered the real case of DHGT characterized by the following structural and functional parameters: $i_{14}^{(3)}$ = 48,5; m = 0,3 mm; r_{02} = 29,3 mm; w_0 = 0,3 mm; w'_0 = 0,27 mm; n = 90° and i = 5°.

The results obtained after implementation of these programs are presented in Table 1, and in Figure 4 and Figure 5 are represented the diagrams of variation of the vector rays in the two front sections (I-I, respectively II-II) of the deformed flexible toothed wheel.

Table 1.

Angle φ [°]	Vector radius, r(φ) [mm]			
	$r_{I-I}(\varphi)$ / section I-I		$r_{II-II}(\varphi)$ / section II –II	
	Elliptical law (3)	Cosine law (4)	Elliptical law (3)	Cosine law (4)
0	29,6000	29,6000	29,5700	29,5700
5	29,5953	29,5954	29,5658	29,5659
10	29,5814	29,5820	29,5533	29,5538
15	29,5588	29,5600	29,5331	29,5340
20	29,5283	29,5302	29,5056	29,5072
25	29,4907	29,4934	29,4719	29,4741
30	29,4473	29,4508	29,4329	29,4357
35	29,3995	29,4036	29,3899	29,3932
40	29,3488	29,3533	29,3443	29,3480
45	29,2968	29,3014	29,2975	29,3013
50	29,2450	29,2494	29,2509	29,2545
55	29,1949	29,1990	29,2058	29,2091
60	29,1482	29,1516	29,1637	29,1665
65	29,1060	29,1087	29,1256	29,1278
70	29,0697	29,0716	29,0929	29,0944
75	29,0402	29,0414	29,0663	29,0672
80	29,0184	29,0189	29,0463	29,0470
85	29,0048	29,0050	29,0344	29,0345
90	29,0000	29,0000	29,0300	29,0300

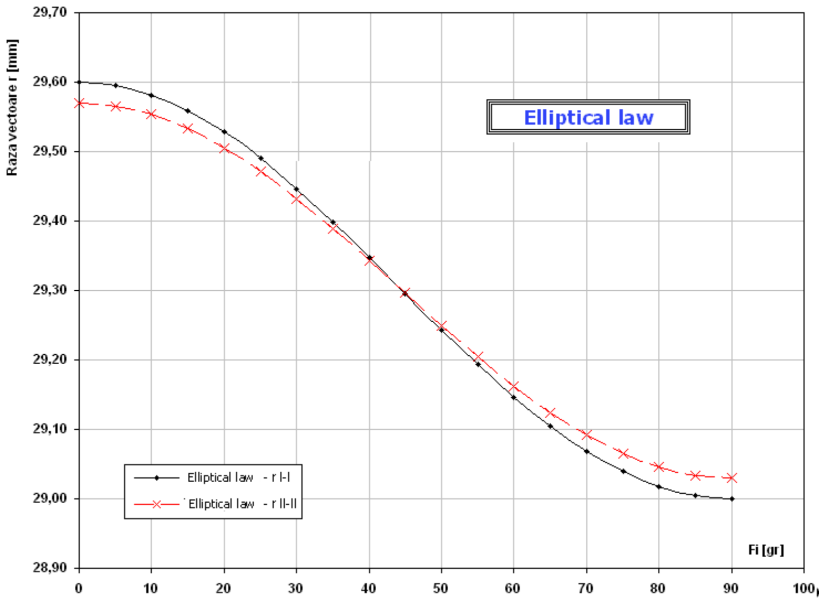


Figure 4. Diagrams of vector rays of deformed flexible wheel - elliptical law

i

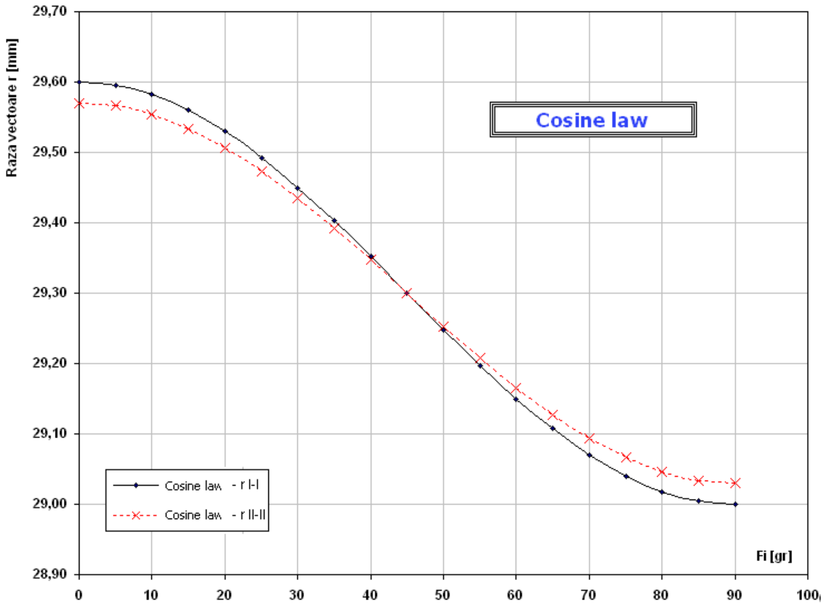


Figure 5. Diagrams of vector rays of deformed flexible wheel - cosine law

The deformation of a flexible toothed wheel of the DHGT depends on the size of w_0 radial movement of a point situated on the dynamic reference fiber, measured in the direction of the vector radius.

From the analysis of the obtained results, it can be observed that the two deformation laws are very close. The forms of variation curves of the vector rays in the two front sections of the deformed flexible toothed wheel (corresponding to those two steps of the harmonic drive) are very similar. In the front section II-II of the flexible wheel, the sizes of the suitable vector rays are lower than in the front section I-I.

4. Conclusions

In this paper we presented an original study of the dynamic behaviour of flexible toothed wheel of the particular DHGT.

The theoretical research conducted on the study of deformation of a flexible wheel of the particular DHGT, emphasizes the following conclusions:

- formulating the premises for studying the dynamic behaviour of the flexible toothed wheel, with the combined radial teeth (exterior, respectively interior), which is regarded as a thin cylindrical tube, of constant thickness and small elastic deformations (w_0 = 0,3 mm);
- the comparative-critical presentation of a deformation laws of the flexible toothed wheel by conceiving and running an original program, written in Visual Basic;
- optimizing the law of deformation of a flexible toothed wheel by demonstrating a similarity between cosine deformation law and the elliptical law.

The optimal materialization of a elliptical deformation law of a flexible toothed wheel of the DHGT may be achieved by suitable choice of the type and the constructive form of the waves generator.

References

[1] Anghel Ş., Ianici S., *Design of mechanic transmission,* Vol. I, Lito. Traian Vuia Politehnic Institute, Timişoara, 1993, pp. 265.

[2] Anghel Ş., Ianici S., Anghel C.V., *Îndrumar de proiectare a mecanismelor,* Universitatea Eftimie Murgu Reşiţa, 1994, pp. 120.

[3] Anghel Ş., Ianici S., *Proiectarea transmisiilor mecanice,* Litografia Universitatea Tehnică Timişoara, 1993, pp. 498.

[4] Anghel Ş., Ianici S., *Design of mechanical transmissions,* Vol.II, Lito. Traian Vuia Politehnic Institute, Timişoara, 1994, pp. 233.

[5] Ianici S., *Contributions to the synthesis of the transmission with deformable elements,* Ph.D. thesis, U.P. Timişoara, 1998, pp. 214.

[6] Ianici S., Ianici D., *Testing of mechanical systems,* Editura Eftimie Murgu Reşiţa, 2010, pp. 121.

[7] Ianici S., Ianici D., *Elemente de inginerie mecanică,* Lito. UEM Reşiţa, 2015, pp. 251.

[8] Ianici S., Ianici D., *Design of mechanical systems,* Editura Eftimie Murgu Reşiţa, 2010, pp. 342.

[9] Ianici S., Ianici D., Potoceanu N., *Design of double harmonic transmission*, Proceedings of the 6th International Conference of DAAAM, Tallinn, 2008, Estonia, pp.95-100.

[10] Ianici S., *Contributions au calcul des deformations et tensions dans l´element flexible de la transmission harmonique hermetique*, Eighth IToMM Symposion SYROM 2001, Bucharest, pp. 175-180.

[11] Ianici S., *Contributions on determining the forces distributions in the toothed harmonic drive (THD)*, The 8-th International Symposium MTM, Timişoara, 2000, pp. 165-168.

[12] Ianici S., Ianici D., a.o., *Double harmonic transmission used in industrial robotic drives*, Proceedings of International Conference on Robotics, No.1/2004, University of Timişoara, pp. 89-90.

[13] Ianici S., Ianici D., *Constructive design and dynamic testing of the double harmonic gear transmission,* Analele Universităţii „Eftimie Murgu" Reşiţa, 2015, pp. 231-238.

[14] Ianici D., Ianici S., *Dynamic research of the flexible wheel of a double harmonic gear transmission,* Analele Universităţii „Eftimie Murgu" Reşiţa, 2015, pp. 223-230.

[15] Ianici D., Ianici S., *Simulation of dynamic behavior of the flexible wheel of the double harmonic gear transmission,* Journal Robotica & Management no. 19(1), Reşiţa, 2014, pp. 17-20.

[16] Ianici S., Miltenovic V., Ianici D., *Numerical simulation of stress and strain state of the flexible wheel of the double harmonic gear transmission,* The 8th International Symposium Machine and Industrial Design KOD 2014, Novi Sad, Serbia, pp. 135-138.

[17] Kuzmanovic S., Ianici S., Rackov M., *Analysis of typical method of connection of electric motor-gear unit in the frame of universal motor gear reducer*, Conference Machine Design, N. Sad, 2010, Serbia, pp. 141-146.

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