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# Solving Systems of Equations with Techniques from Artificial Intelligence

*A frequent problem in numerical analysis is solving the systems of equations. That problem has generated in time a great interest among mathematicians and computer scientists, as evidenced by the large number of numerical methods developed. Besides the classical numerical methods, in the last years were proposed methods inspired by techniques from artificial intelligence. Hybrid methods have been also proposed along the time [15, 19]. The goal of this study is to make a survey of methods inspired from artificial intelligence for solving systems of equations.*

***Keywords***: *systems of equations, genetic algorithms, particle swarm optimization, artificial glowworm swarm algorithm, memetic algorithms*

## 1. Introduction

### 1.1. Solving system of equations by classical methods

The classical methods are usually divided into direct and iterative methods. In direct methods, like Cramer, Gaussian elimination [24], Gauss-Jordan elimination, LU factorization, QR decomposition etc., the solution is obtained after a fixed number of operations, a number that is directly proportional to system size. The solution is affected by rounding errors at each step, fact which not recommend the use of these methods for medium and large systems. In case of large systems are preferred iterative methods. These methods are based on an iterative process that starts from an initial approximation of the solution. A major advantage of iterative methods is that in practice rounding errors and truncation errors can be insignificant sometimes even eliminated. The basic process in iterative methods consists in building a sequence $x_k$ that converges to the exact solution of the system. The iterative process is stopped when reaching a specific precision imposed by the user or a maximal number of iterations. The most popular iterative methods are: the

Conjugate Gradient (CG) [1], the Newton-type methods [23] , Broyden [2, 4], the Chebyshev method [7, 12, 17].

### 1.2 Brief descriptions of the techniques used from AI in system equations solving

**Genetic Algorithms (GA)** are computer programs which create an environment where populations of data can compete and only the fittest survive, sort of evolution on a computer. GAs is very good for all tasks requiring optimization and produce a large number of possible outputs.

A genetic algorithm is a problem solving method that is inspired by its genetics as its model of problem solving. A genetic algorithm (GA) is a search heuristic that mimics the process of natural evolution [25]. This heuristic is routinely used to generate useful solutions to optimization and search problems. Individual solution is represented through a chromosome, which is just a summarization of representation. So, to find the best solution, it is necessary to perform certain operations of each optimum solution. At the beginning it is generating an initial population of chromosomes. This population must offer a wide diversity of genetic materials and the gene pool should be as large as possible so that any solution of the search space can be propagated. Then, the GA loops over an iteration process to make the population evolve. Each iteration consists of selection, crossover, mutation and replacement.

Genetic algorithms are search methods that can be used for both solving problems and modeling evolutionary systems. GAs differs from other heuristic methods in several ways. The first important difference is that it works on a population of possible solutions, while other heuristic methods use a single solution in their iterations. Another important difference is that GAs is not a deterministic but a probabilistic one.

The strength of genetic algorithms is that they rapidly converge to near-optimal solutions.

**Particle Swarm Optimization** [10] is a population based algorithm constructed on swarm behavior of fish schooling and bird flocking to find an optimum solution of a problem. The algorithm has some similarities with Genetic Algorithms, but it is much simpler because it does not need crossover or mutation operator [20]. Each candidate solution, named as particle, flies around the solution space and lands on the optimal position. Particles in the swarm adjust their position by their own experience and experience of neighbouring particles. A big advantage of PSO is that each agent has a memory which keeps track of its previous best position with its respective fitness value. Agent with the best fitness value in the swarm is called global best which holds the optimum solution for the generation.

Another advantage of PSO is that, throughout the execution of the algorithm the initial population is fixed.

In 2005, Krishnanand and Ghose put forward **glowworm swarm optimization (GSO) algorithm** [9]. GSO algorithm is a swarm intelligence bionic algorithm and it has good capacity to search for global extremum and more extremums of multimodal optimization problems.

AGSO algorithm researches the behaviors of glowworms in the nature which glow to attract mates or prey. It is a swarm intelligence algorithm, and its basic principle is as follows: Luciferin induces glowworm to glow to attract mates or prey. The brighter the glow more is the attraction, meanwhile the higher of the luciferin, then glowworm moves towards the position having hight luciferin. The luciferin value is corresponding to the fitness function value, so glowworm looks for the position having highest luciferin value to determine the optimal value of the fitness function in dynamic decision domains.

The artificial GSO does not depend on the initial points and derivative to solve objective function and the problem of solving system of nonlinear equations can be transformed into a function optimization problem.

**Memetic Algorithms (MAs)** are computational intelligence structures combining multiple and various operators in order to address optimization problems [16].

The Memetic Algorithm combine evolutionary techniques with other classical or intelligent optimization techniques and use a double optimization: local and global.

It represents a particular class of evolutionary algorithms that apply a local search in order to refine the current approximation of the global optimum.

A detailed description of memetic algorithms is made by Ferrante Neri et. all in *Handbook of Memetic Algorithms* [16].

Some aspects are very important in terms of designing a good memetic algorithm:

- the choice of recombination operators in the evolutionary process must take into consideration the three basic properties of these: purity, assortment and transmission [5];
- a clear separation between local search/optimization and global search/optimization must be made. At the same time a balance between local and global search is required to avoid premature convergence and the waste of computing resources;
- in case of population convergence, the population must be refreshed in order to avoid the exploration of the same search space, which would lead to obtaining the same solutions and unnecessary waste of processing time;
- regarding the initial population, when is possible, is recommended a non-random initialization, which can direct the search into a particular regions that contain good or appropriate solutions. This can be achieved mainly by the inclusion in the initial population of good solutions previously known or by a process selected from a large population generated randomly (selection based on fitness);
- the local search operators must be different from recombination and mutation operators of the evolutionary process;

- using the knowledge acquired in previous optimization stages to guide the current phase of optimization is also an important aspect to be considered in designing a memetic algorithm.

## 2. Methods inspired from artificial intelligence for system equations solving

### 2.1. Solving systems of equations with genetic algorithms

In paper [3] is proposed a new paradigm for solving systems of nonlinear equations through using Genetic Algorithm (GA) techniques. So, the authors applied Gauss-Legendre integration as a technique to solve the system of nonlinear equations. Then it was used genetic algorithm (GA) to find the results without converting the nonlinear equations to linear equations and the obtained result, that is achieved by using GA was compared with the exact solution that is obtained by numerical methods. The results indicate that a GA is efficient to solve the systems of nonlinear equations.

The main idea of the new perspective proposed in [6] is that a system of equations can be viewed as a multiobjective optimization problem: every equation represents an objective function whose goal is to minimize difference between the right and left term of the corresponding equation in the system. To solve the problem obtained by transforming the system of nonlinear equations into a multiobjective problem it is used an evolutionary computation technique.

The important information needed in genetic algorithm include objective function, and the representation scheme in a form of a coded string, containing information about the possible solutions. Evaluation of a possible solution is done after every set of genetic operations.

In paper [8] were solved seven different systems of simultaneous linear equations with two different methods. So, the conclusion of this study was that conventional numerical methods always produce a set of solutions for a particular system of simultaneous linear equations, but the GA from this paper produced more than one set of solutions for certain systems of equations.

### 2.2. Solving systems of equations with particle swarm optimization algorithm

In [22] is used a particle swarm optimization algorithm and an artificial fish swarm algorithm for solving a linear system of equations, especially designed for ill conditioned linear systems equations.

A system of equations is considered to be ill-conditioned if a small change in the coefficient matrix or a small change in the right hand side results in a large change in the solution vector.

Ill-Conditioned Systems are very sensitive to roundoff errors and, therefore, may pose problems during computation of the solution [1]. During computing process, these errors induce small changes in the coefficients which, in turn, result a large error in the solution [1].

In paper [21] were presented several advantages of the swarm intelligent algorithm: their does not require of a "good" initial point to perform the search, and the search space can be bounded by lower and upper values for each decision variable.

### 2.3. Solving systems of equations with artificial glowworm swarm algorithm

In paper [18] is proposed an algorithm that uses a artificial glowworm swarm approach to solve linear systems of equations.

The proposed algorithm is able to solve the problem that it's difficult for traditional algorithms to select initial values, improves the convergence speed, ensures the global convergence and for solving system of nonlinear equations is a new effective optimization algorithm.

In paper [11] is proposed an algorithm inspired from glowworm swarm for solving systems of linear equations. The proposed algorithm can solve multi-equations at a time, and it offers an effective way to solve system of nonlinear equations.

### 2.4. Solving systems of equations with memetic algorithms

In paper [14] is proposed an algorithm inspired from memetics for solving systems of linear equations.

The proposed method is able to find all the solutions inside of a given interval for a linear system of equations that has infinite solutions.

So, the method proposed in this paper is a combined approach: an enumeration problem (finding a lot or all solution) and an optimization problem (finding a solution by minimized a given objective function). The algorithm knows how good a particular solution is with the help of fitness functions $\mathrm{abs}(f_i(X))$, for i=1, 2,...,n.

In paper [13] is proposed a metaheuristic inspired from human brainstorming combined with concepts from graph theory for solving system of equations. The proposed method can find solutions of a given system of equations, even in cases where traditional methods fail and in cases where no accurate solution for a linear system of equations can not be determined, an approximate solution is obtained.

## 4. Conclusions

In this paper, it is made a survey of methods inspired from artificial intelligence for solving systems of equations.

So, the strength of genetic algorithms is that they rapidly converge to near-optimal solutions.

Particle Swarm Optimization has some similarities with Genetic algorithms and ant algorithms, but it is much simpler since it does not need crossover or mutation operator.

The memetic algorithm use a double optimization: local and global and represent a particular class of evolutionary algorithms that apply a local search in order to refine the current approximation of the global optimum.

As advantage of the memetic algorithm, it can be specified that in situations when a linear system of equations has multiple solutions, memetic algorithm finds as many solutions as possible, inside of a given interval. In cases where no accurate solution for a linear system of equations exists, an approximate solution it can be obtained by the memetic algorithm.

The study of convergence of these intelligent methods will be our main direction of study. A comparative study as performance with classical methods it is also required. Parallelization of these methods will be also an important direction of our study.

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