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Special Cases Solution of the Transport Problem

In the present paper we are presenting an optimal way to solve the transport problems with intermediate centers, taking in consideration several approaches, using the Network Modelling Module of the WinQSB software. The transfer problem represents a special case of the transport problem, an extension of it.

Keywords*: the transportation problem, intermediate center, objective function, optimal solution.*

1. Introduction

The transportation problem is a particular problem of linear programming frequently encountered in economic applications, enounced by the following example [6], [7], [10].

We consider a number of m suppliers (supply centers, warehouses) of a certain product as well as n consumers (stores, factories etc.) for this product. The considered product is available in a_i, quantity ($1 \le i \le m$), at the supplier i, being demanded in the b_j quantity by the consumer j, ($1 \le j \le n$). If x_{ij}, ($1 \le i \le m$, $1 \le j \le n$) is the quantity of the product considered that the supplier i puts at disposal of consumer j, the unitary transportation cost from supplier i to consumer j is marked as c_{ij}.

The available amount of supplier i can be distributed to all consumers, according to:

$$x_{i1} + x_{i2} + \ldots + x_{ij} + \ldots + x_{in} = a_i, \qquad 1 \le i \le n \qquad (1)$$

Where as consumer j can receive products from all suppliers, according to relation:

$$x_{1j} + x_{2j} + \ldots + x_{ij} + \ldots + x_{mj} = b_j, \qquad 1 \le j \le m \qquad (2)$$

The total available quantity of m suppliers (the supply) is given by the relation:

$$D = \sum_{i=1}^{m} a_i \,, \quad (3)$$

and the necessary of all consumers (the demand) is equal to:

$$N = \sum_{j=1}^{n} b_j \quad (4)$$

The total expenditure for the transport activity of the considered product from the ***m*** suppliers to the ***n*** mentioned consumers is given by:

$$c_{11}x_{11} + c_{12}x_{12} + \ldots + c_{1n}x_{1n} + c_{21}x_{21} + c_{22}x_{22} + \ldots c_{2n}x_{2n} + ..$$
$$. + c_{m1}x_{m1} + c_{m2}x_{m2} + \ldots + c_{mn}x_{mn}$$

which represents the sum of all expenditure incurred for transporting the quantity x_{ij} from supplier ***i*** to consumer ***j***, equal to $c_{ij}x_{ij}$ ($1 \le i \le m; \quad 1 \le j \le n$).

The transportation problem consists in determining the optimal variant (the least expensive). Mathematically, it means minimizing the function:

$$[\min]f = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}x_{ij} \,, \quad (5)$$

in the conditions:

$$\sum_{j=1}^{n} x_{ij} = a_i ; \quad 1 \le i \le m \quad (6)$$

$$\sum_{i=1}^{m} x_{ij} = b_j ; \quad 1 \le j \le n \quad (7)$$

$$x_{ij} \ge 0 \,. \quad (8)$$

There are special cases of the Transport problem, the most important of them being the transshipment systems and the assignations. Besides the already named ones, there are also the blocked itinerary problems, the imposed solution ones or the problems with connected centers. Besides the above presented systems, in which we distinguish expedition and destination nodes, in the transshipment systems appears intermediate nods. These intermediate nodes must be transited by the expedition tasks on their way to destination [9].

For a better understanding of the transport problem with intermediate centers, is necessary a larger analysis based on an application of the WinQSB software, namely the Network Modelling Module [3].

2. Case Study

It was chosen an application in which is needed a solution for the transport problem for Supplier→Intermediate→Beneficiary as well as Supplier→Beneficiary.

An industrial enterprise specialized on manufacturing and distributing a certain product type has three supply sources for it (F_i, $i = \overline{1,3}$). During a moth, the F_1 factory produces an average of 1500 parts, F_2 factory produces up to 1200 parts and the third one, F_3 factory produces up to 2300 parts.

The enterprise distribution network disposes of three central warehouses (D_k, $k = \overline{1,3}$), situated in different cities. Four distribution centers (CD_j, $j = \overline{1,4}$) are also situated in different cities, asking in the same month 900 parts, 1500 parts, 1000 parts, respective 1600 parts.

The unit costs are presented in the tables from figure 1. We can observe that some distribution centers are supplied directly by the factories.

	D1	D2	D3
F1	1	2	2
F2	2	3	1
F3	3	1	1

	CD1	CD2	CD3	CD4
D1	5	4	8	6
D2	6	5	7	5
D3	5	4	7	7

	CD1	CD2	CD3	CD4
F1	3	-	7	-
F2	5	4	-	6
F3	-	-	7	6

Figure 1. Unit transport costs

The following research presents the solution mode for a transshipment transport problem taking in consideration several approaches:

I Determination of the optimal transport plan.

II Blocking one transport itinerary

III Stopping the activity of one factory.

I. Determination of the optimal transport plan.

The transshipment problem is an extension of the transport problem and can be solved on an extended table of the transport problem [5], showed in figure 2.

From \	D1	D2	D3	CD1	CD2	CD3	CD4	Supply
F1	1	2	2	3		7		1500
F2	2	3	1	5	4		6	1200
F3	3	1	1			7	6	2300
D1	0	M	M	5	4	8	6	5000
D2	M	0	M	6	5	7	5	5000
D3	M	M	0	5	4	7	7	5000
Demand	5000	5000	5000	900	1500	1000	1600	

Figure 2. Extended table of the transport problem

The optimal plan is presented as matric form in fig.3. The total cost of the minimal transport is 25600 m.u.

06-11-2015	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	F1	D1	300	1	300	0
2	F1	CD1	900	3	2700	0
3	F1	CD3	300	7	2100	0
4	F2	CD2	1200	4	4800	0
5	F3	D2	1600	1	1600	0
6	F3	CD3	700	7	4900	0
7	D1	D1	4700	0	0	0
8	D1	CD2	300	4	1200	0
9	D2	D2	3400	0	0	0
10	D2	CD4	1600	5	8000	0
11	D3	D3	5000	0	0	0
	Total	Objective	Function	Value =	25600	

Figure 3. Optimal solution for the extended transport problem

The problem admits multiple solutions because, after obtaining the solution presented in fig.3, appear *Obtain Alternative Solution* software command, which calculates and provides for our specific case another three optimal solutions (fig.4÷6).

06-11-2015	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	F1	CD1	900	3	2700	0
2	F1	CD3	600	7	4200	0
3	F2	CD2	1200	4	4800	0
4	F3	D2	1600	1	1600	0
5	F3	D3	300	1	300	0
6	F3	CD3	400	7	2800	0
7	D1	D1	5000	0	0	0
8	D2	D2	3400	0	0	0
9	D2	CD4	1600	5	8000	0
10	D3	D3	4700	0	0	0
11	D3	CD2	300	4	1200	0
	Total	Objective	Function	Value =	25600	

Figure 4. Second optimal solution for the extended transport problem

06-11-2015	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	F1	CD1	900	3	2700	0
2	F1	CD3	600	7	4200	0
3	F2	CD2	1200	4	4800	0
4	F3	D3	300	1	300	0
5	F3	CD3	400	7	2800	0
6	F3	CD4	1600	6	9600	0
7	D1	D1	5000	0	0	0
8	D2	D2	5000	0	0	0
9	D3	D3	4700	0	0	0
10	D3	CD2	300	4	1200	0
	Total	Objective	Function	Value =	25600	

Figure 5. Third optimal solution for the extended transport problem

06-11-2015	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	F1	D1	300	1	300	0
2	F1	CD1	900	3	2700	0
3	F1	CD3	300	7	2100	0
4	F2	CD2	1200	4	4800	0
5	F3	CD3	700	7	4900	0
6	F3	CD4	1600	6	9600	0
7	D1	D1	4700	0	0	0
8	D1	CD2	300	4	1200	0
9	D2	D2	5000	0	0	0
10	D3	D3	5000	0	0	0
	Total	Objective	Function	Value =	25600	

Figure 6. Fourth optimal solution for the extended transport problem

II. Blocking one transport itinerary.

In all the four obtained optimal transport plans F1 supply CD1 with 900 parts, and F2 supply CD2 with 1200 parts.

Therefore, it was considered that the transport itinerary (F2→CD2) is no longer available. The software provides also four optimal solutions, presented in figure 7 and figure 8.

The Network Modeling Module offers the possibility to perform „What-If" or parametrical type analysis by appealing the respective options from the Solve and Analyze menu [2]. Choosing the Perform What-If Analysis option has as effect opening a new window in which are specified the elements to be modified [3].

06-14-2015	From	To	Shipment	Unit Cost	Total Cost
1	F1	D1	300	1	300
2	F1	CD1	900	3	2700
3	F1	CD3	300	7	2100
4	F2	D3	1200	1	1200
5	F3	D2	1600	1	1600
6	F3	CD3	700	7	4900
7	D1	D1	4700	0	0
8	D1	CD2	300	4	1200
9	D2	D2	3400	0	0
10	D2	CD4	1600	5	8000
11	D3	D3	3800	0	0
12	D3	CD2	1200	4	4800
	Total	Objective	Function	Value =	26800

06-14-2015	From	To	Shipment	Unit Cost	Total Cost
1	F1	D1	600	1	600
2	F1	CD1	900	3	2700
3	F2	D3	900	1	900
4	F2	CD4	300	6	1800
5	F3	D2	1300	1	1300
6	F3	CD3	1000	7	7000
7	D1	D1	4400	0	0
8	D1	CD2	600	4	2400
9	D2	D2	3700	0	0
10	D2	CD4	1300	5	6500
11	D3	D3	4100	0	0
12	D3	CD2	900	4	3600
	Total	Objective	Function	Value =	26800

Figure 7. First two solutions for (F2→CD2) blocked itinerary

06-14-2015	From	To	Shipment	Unit Cost	Total Cost
1	F1	D1	600	1	600
2	F1	CD1	900	3	2700
3	F2	CD4	1200	6	7200
4	F3	D2	400	1	400
5	F3	D3	900	1	900
6	F3	CD3	1000	7	7000
7	D1	D1	4400	0	0
8	D1	CD2	600	4	2400
9	D2	D2	4600	0	0
10	D2	CD4	400	5	2000
11	D3	D3	4100	0	0
12	D3	CD2	900	4	3600
	Total	Objective	Function	Value =	26800

06-14-2015	From	To	Shipment	Unit Cost	Total Cost
1	F1	D1	600	1	600
2	F1	CD1	900	3	2700
3	F2	CD4	1200	6	7200
4	F3	D3	900	1	900
5	F3	CD3	1000	7	7000
6	F3	CD4	400	6	2400
7	D1	D1	4400	0	0
8	D1	CD2	600	4	2400
9	D2	D2	5000	0	0
10	D3	D3	4100	0	0
11	D3	CD2	900	4	3600
	Total	Objective	Function	Value =	26800

Figure 8. Third and fourth solution for (F2→CD2) blocked itinerary

III. Stopping the activity of one factory

In case that factory F1 stops its activity due to a strike, the optimal solution presented in fig. 9 highlights a deficit of 1000 parts, respectively 500 parts for CD3 and CD4 distribution centers.

From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
F2	CD2	1200	4	4800	0
F3	D2	1100	1	1100	0
F3	D3	1200	1	1200	0
D1	D1	5000	0	0	0
D2	D2	3900	0	0	0
D2	CD4	1100	5	5500	0
D3	D3	3800	0	0	0
D3	CD1	900	5	4500	0
D3	CD2	300	4	1200	0
Unfilled_Demand	CD3	1000	0	0	0
Unfilled_Demand	CD4	500	0	0	0
Total	Objective	Function	Value =	18300	

Figure 9. Optimal solution for stopping activity of factory F1

Parametric Analysis

After filling of the dialog box in a manner presented in figure 10, the software provides the results in matric form, showed in figure 11.

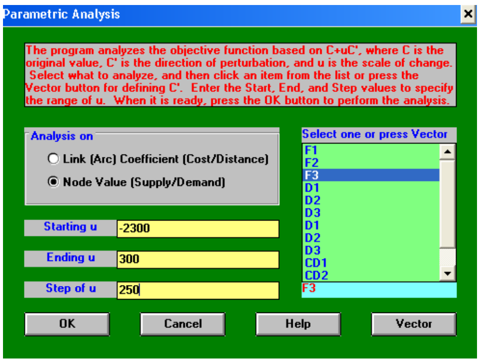

Figure 10. Parametric Analysis dialog box

F3 Supply/Demand	OBJ Value
0	11100
250	12600
500	14100
750	15600
1000	17100
1250	18600
1500	20100
1750	21750
2000	23500
2250	25250
2500	25600

Figure 11. Result of parameters analysis in matric form

It may be observed in figure 11 the objective function variation in regard to the parts number manufactured and distributed by F3.

3. Conclusion

The optimal transport plan for the proposed problem is:

- Factory F1 supplies central warehouse D1 with 300 parts, and the distribution centers CD1 and CD3 with 900 parts, respectively 300 parts;
- F2 supplies CD2 with 1200 parts;
- F3 supplies D2 and CD3 with 1600 parts, respectively 700 parts;
- Central warehouse D1 supplies CD2 with 300 parts;
- Central warehouse D2 supplies CD4 with 1600 parts;.

The problem admits another three optimal solutions. Were analyzed the following situations:

2) Blocking the transport itinerary (F2→CD2) leaded to transport program modification, which determined the objective function increment from 25600 m.u. to 26800 m.u.

3) In the case of stopping factory F1 activity, appears a deficit of 1000 parts, respectively 500 parts for CD3 and CD4 distribution centers.

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