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## **Dynamic Research of the Flexible Wheel of a Double Harmonic Gear Transmission**

*The paper presents the results of a dynamic research of the flexible wheel of the double harmonic gear transmission, by determining the state of strain and stress of its wall, in cases the wheel is deformed by a mechanical waves generator with: two rolls, two eccentric discs and cam. The dynamic research involves modelling and the numerical simulation of flexible wheel, by using the finite element method, with the help of SolidWorks Simulation program in elastic range.*

**Keywords:** *flexible wheel, double harmonic gear transmission, simulation, displacement, stress*

### **1. Introduction**

The functional performance and durability of the harmonic gear transmission are greatly influenced by the durability of the flexible toothed wheel, as it is the most powerfully requested element of the transmission.

The dynamic behaviour of the flexible toothed wheel depends on the law of deformation of the wheel, which in turn is influenced by: the type of waves generator, the geometrical form of the wheel and its mode of coupling with the output shaft of the transmission, [1], [2], [3], [4], [7].

The functioning principle of the harmonic gear transmission is essentially different from the classic gear, because the transmission of the rotary movement is accomplished by means of elastic deformation, which is propagated by the harmonic law, in the periphery of one of its elements called flexible toothed wheel.

The deformation of the flexible toothed wheel is achieved by forced mounting of the waves generator in the inside of the wheel, so that in the wheel body will appear a complex state of stress.

The strain and stress in the wall of the flexible toothed wheel is influenced by many factors such as: the geometric parameters of the waves generator and of the teeth of the wheel, the flexible wheel body geometry, the coupling of the flexible wheel with the output shaft, the waves generator speed, the torque transmitted.

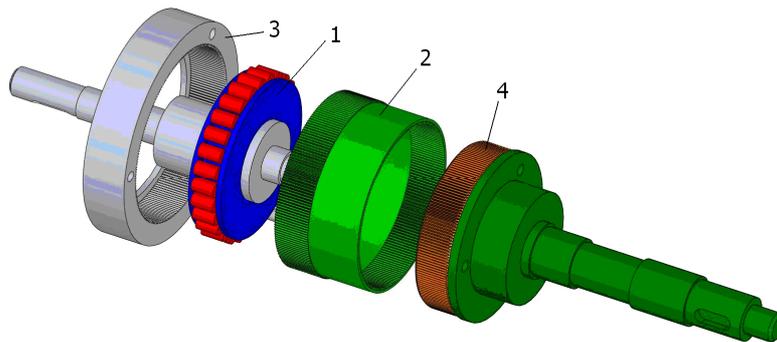
The main problem that needs to be improved in the future, of the harmonic gear transmissions, is the increase durability flexible wheel. This may be achieved by using materials with the upper resistance or by modifying the form of the flexible toothed wheel, conducive to the less severe stress.

In the paper presented the research of dynamical status of the flexible toothed wheel of a double harmonic gear transmission, by performing numerical simulations using SolidWorks Simulation software.

## 2. Numerical simulation of the flexible toothed wheel

Dynamics testing of the flexible toothed wheel was made for double harmonic gear transmission shown in Figure 1.

This transmission is composed of: a waves generator (1) as input element, a short flexible toothed wheel (2) with the external and internal teeth, the fixed rigid wheel (3) and the mobile rigid wheel (4) as output element, [2, 5, 6].



**Figure 1.** The scheme of a double harmonic gear transmission.

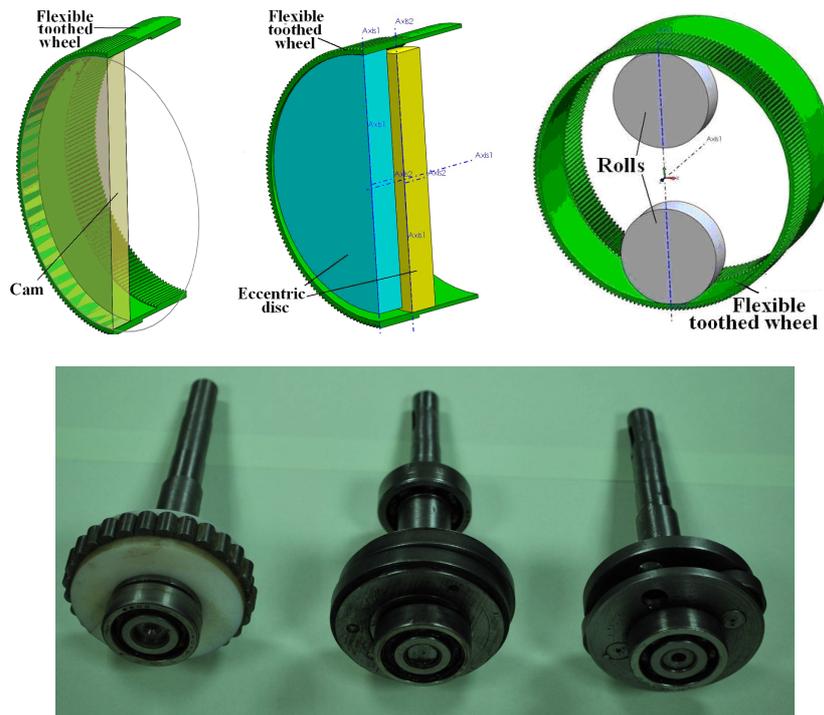
In order to investigate the state of strains and stress of the flexible toothed wheel of the double harmonic gear transmission and to achieve numerical simulation using the finite element method, with the help of SolidWorks Simulation module, the following steps were required, [5]:

- 3D geometry modeling of the flexible wheel of the three waves generators (with: two rolls, two eccentric discs and cam) in SolidWorks CAD software;
- defining case analysis;
- defining material for the flexible wheel;
- definition restrictions and applying of of the tasks;
- finite element mesh of the model tested;
- calculating the strains and the stress by SolidWorks Simulation module;
- view and analyze the results.

In numerical calculus, the flexible toothed wheel of the double harmonic gear transmission was modeled by a cylinder opened at both ends, defined by the

radius,  $r = 29,3$  mm, length,  $l = 30$  mm and the constant wall thickness,  $s = 0.6$  mm, which is provided with two teeth (outer and inner) of width 12 mm.

The analysis of the dynamic behavior of the flexible gear has been made for the case of three models of the waves generator (Figure 2). The analytical models are composed of a flexible toothed wheel of the double harmonic gear transmission and of a waves generator.



**Figure 2.** The models of a waves generator.

SolidWorks Simulation operate with the concept of "Study", pointing out specific characteristics of the analysis: analysis type and associated options, materials, set task and boundary conditions and meshing the model analyzed.

The geometry of the flexible toothed wheel of the double harmonic gear transmission was modeled in "solid" and its numerical simulation consisted of a linear static analysis, meshing was performed using spatial finite elements. finite element meshing was performed spatial. Selected material is flexible toothed wheel steel (Alloy Steel), with the following characteristics: Young's modulus,  $E = 2,1 \times 10^{11}$  N/m<sup>2</sup>; Poisson's ratio,  $\nu = 0,28$  and yield strength,  $\sigma_c = 620,422$  MPa.

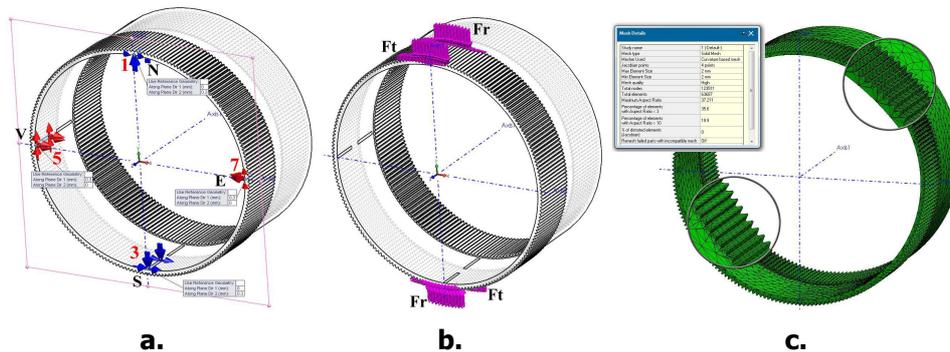
The geometry of the flexible toothed wheel has the origin in the point O of its symmetry, the Oz - axis being oriented along the generatrix, the xOy plane being

oriented NSVE, the Ox - axis being positively oriented from V to E and the Oy - axis from S to N (Figure 3. a).

The contact between the flexible wheel and the waves generator, is produced on the inner side of the wheel, in the areas N1, S3, V5 and E7. In these four contact areas the following restrictions will be applied to the flexible wheel:

- two restriction of value 0 which will cancel the movement of the contact zones N1 respectively S3, in the direction Ox, and two restrictions of value 0.3 mm applied to the exterior of the wheel, which materialize the deformation in the direction Oy of the wheel due to the action of the generator on the major axis of the ellipse;
- two restriction of value 0 which will cancel the movement of the contact zones V5 respectively E7, in the direction Oy, and two restrictions of value 0.3 mm applied to the inside of the flexible wheel, which materialize the deformation in the direction Ox of the wheel due to the action of the generator on the small axis of the ellipse.

Also, in simulation will be applied a restriction type Roller/Slider, to the side which is parallel and opposed to the NSVE side. For this type of restriction, the points belonging to this plane side can move freely in their plane, but are not able to move perpendicular to this plane.



**Figure 3.** Restrictions applied to the tasks and meshing to the flexible wheel.

The tasks applied to the flexible wheel consisted in the only two components of the normal forces (tangential force  $F_t$  and radial force  $F_r$ ), developed in the first stage of harmonic gear (Figure 3. b). This is because the effect of the interaction between the wave generator and flexible wheel has already been taken into account by imposing elastic deformation produced by the generator inside the flexible wheel. The maximum values of tangential and radial forces meshing harmonic were determined for the steps of torque a transmission,  $M_{t4} = (0, 100, 200, 300, 400, 500)$  Nm.

In order to simulate the flexible toothed wheel, meshing to use solid type (Figure 3. c) was used, which generated a total number of 63.687 finite elements

and a number of 123.511 nodes. After following the above mentioned stages, the analyses calculus was performed by the simulation of the deformation of the flexible toothed wheel, in order to determine and display graphically the state of stress and strains.

After the numerical processing of the simulation of the behavior of flexible toothed wheel, can view the results, that can be viewed graphically (charts and color maps) or analytically (numerical values for von Mises stress and displacements).

### 3. Numerical simulation results

In the cases of numerical analysis that was performed using SolidWorks Simulation program, there were studied the variations of the displacements and stresses (von Mises), in the body of the flexible toothed wheel, according to the torque moment of the double harmonic gear transmission.

Thus, by successive runs of the numerical analysis program, maximum values of displacements and von Mises stress were recorded for six steps of the torque ( $M_{t4} = 0, 100, 200, 300, 400, 500 \text{ N}\cdot\text{m}$ ).

The results obtained after applying numerical simulations are summarized in Table 1 - for maximum von Mises stress and in Table 2 - for the resultant displacement  $\Delta$  [mm] of the nodes located on the generatrix direction N or E of the flexible wheel.

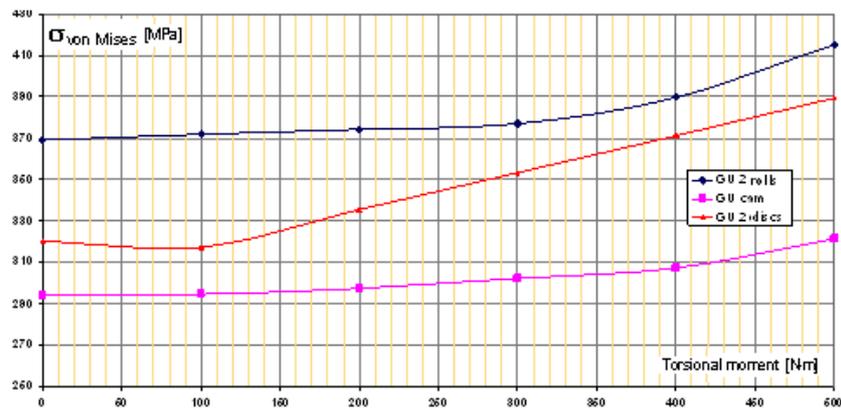
**Table 1.**

Torque $M_{t4}$ [N·m]	Tangential force $F_t$ [N]	Radial force $F_r$ [N]	Stress $\sigma_v$ Mises max [MPa]		
			2 rolls	2 discs	cam
0	0	0	369.1	320.3	294,1
100	52,33	19,04	371.8	317.3	294,5
200	104,66	38,09	374.4	335.4	297,4
300	156,99	57,13	377.2	353.3	302.3
400	209,33	76,18	389.8	371.4	307.3
500	261,66	95,23	415.4	389.3	321.5

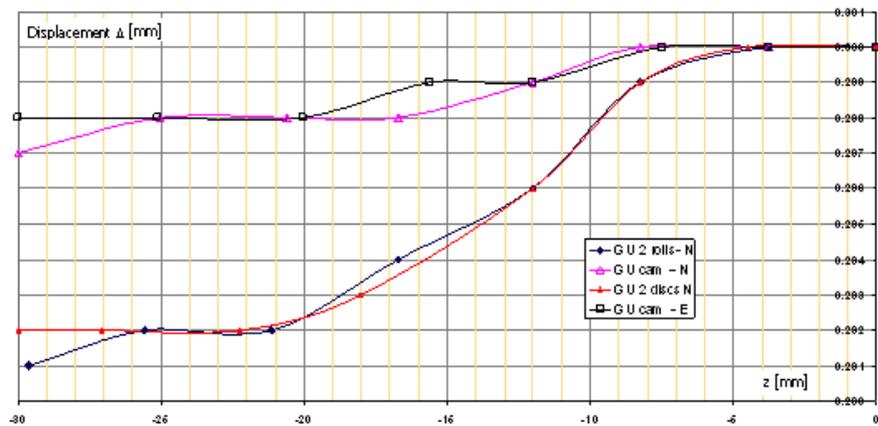
**Table 2.**

No. node	Positioning $Z_{nod}$ [mm]	Displacement $\Delta$ [mm]			
		2 rolls (N)	2 discs (N)	cam (N)	cam (E)
1	0	0,3	0,3	0,3	0,3
2	-3,75	0,3	0,3	0,3	0,3
3	-8,25	0,299	0,299	0,3	0,3
4	-12	0,296	0,296	0,299	0,299
5	-16,725	0,294	0,293	0,298	0,299
6	-20,597	0,292	0,292	0,298	0,298
7	-25,054	0,292	0,292	0,298	0,298
8	-30	0,291	0,292	0,297	0,298

In the Figures 4 and 5 present their variation diagrams for all three types of waves generator analyzed, [5].



**Figure 4.** Maximum von Mises stress diagram,  $\sigma_{VM} = \sigma_{VM}(M_{t4})$



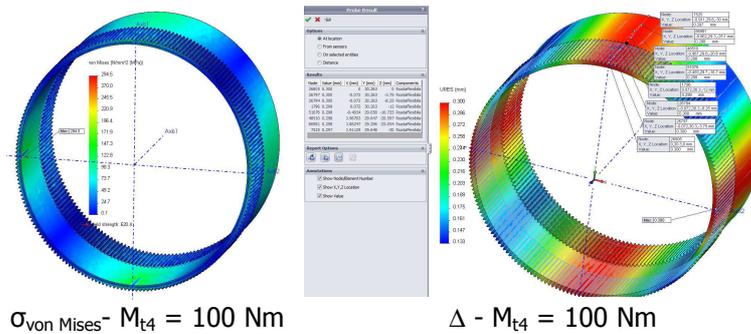
**Figure 5.** Resultant displacement diagram,  $\Delta = \Delta(z)$

We can observe that Z coordinate values, that establish the positions of nodes of the flexible wheels give negative results, because Oz-axis has positive orientation axis in the opposite direction.

In order to research and appreciate more correctly the deformation mode of the flexible toothed wheel there were visualized the stress and the resulting displacements in the form of color maps.

Figure 6 shows the von Mises stress distribution and the resulting displacement of the characteristics nodes, from the finite elements located in the

direction of the generatrix N of the flexible wheel, in the case the deformation of the flexible wheel where the cam generator, to a torque of  $M_{t4} = 100 \text{ N}\cdot\text{m}$ .



**Figure 6.** The von Mises stress ( $\sigma_{vMmax}$ ) and the resultant displacement ( $\Delta$ ).

From the analysis of the variation diagram of the resultant displacements  $\Delta = \Delta(z)$ , one can see that in the proximity of the point where one applies the deflection force on the flexible toothed wheel (corresponding to cote  $z = 0$  on the N generator), it is found for the value of the resultant displacement the exact value of the maximum radial elastic deformation of the flexible wheel,  $\Delta(0) = 0.3 \text{ mm}$ , for all three types of waves generators considered.

#### 4. Conclusions

The paper presents the results of numerical simulation of the flexible toothed wheel of the double harmonic gear transmission allowing to evaluate the dynamic behavior of the flexible wheel.

The analysis of these results reveals the following conclusions:

- confirmation of the state of stress and strain dependence of charging moment, we observed an increase von Mises stress ( $\sigma_{vM}$ ) with increasing torque ( $M_{t4}$ );
- von Mises stress in the wall of the flexible toothed wheel presenting a slightly increasing with increasing torque ( $M_{t4} \in [0; 500] \text{ N}\cdot\text{m}$ ), for all three cases considered analysis (generator with 2 rolls -  $\sigma_{vMmax} \in [369,1; 415,4] \text{ MPa}$ , generator with 2 eccentric discs -  $\sigma_{vMmax} \in [320,3; 389,3] \text{ MPa}$ , generator with cam -  $\sigma_{vMmax} \in [294,1; 321,5] \text{ MPa}$ );
- maximum value of the stress occurs in the immediate proximity of the point where we apply the force of elastic deformation on the flexible toothed wheel, in the area of the waves generator, and this stress is well below the yield strength of the material of the wheel ( $\sigma_c$ );
- maximum stress variation is insignificant at low torque of the transmission ( $M_{t4} \leq 100 \text{ N}\cdot\text{m}$ ), for all three cases considered;

- resultant displacement of the nodes located on the generator N has a slightly decreasing character, once we remove the nodes from the NSVE side of the flexible, a fact that was confirmed by the results of experimental research conducted

The experimental research has shown the superiority of the waves generator with the cam to generators with 2 roller or 2 eccentric disc, in terms of deformation of the flexible toothed wheel.

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