ANALELE UNIVERSITĂȚII
"EFTIMIE MURGU" REŞIȚA
ANUL XXII, NR. 2, 2015, ISSN 1453 - 7397

Hamid Teimouri, Navid Bozorgan

Numerical Investigation of Nanofluid Mixed Convection in an Inclined Channel and Predicting Nusselt Number with Artificial Neural Networks

Artificial Neural Networks (ANNs) are used as a new approach in determination of Nusselt number of copper water nanofluid in an inclined channel with three heat sources. For training the ANNs, the simulation results are obtained by Finite Volume Method (FVM). The effects of independent parameters, including the Reynolds number, Rayleigh number, inclination angle, and the solid volume fraction of nanoparticles, on the streamlines, isotherm lines, and the average Nusselt number have been studied. Artificial neural networks (ANN) used to find a relation involve independent parameters for estimating the Nusselt number. The back propagation-learning algorithm with the tangent sigmoid transfer function is used to sequence the ANN. Finally, analytical relations for the nanofluid mixed convection in a channel are derived from the available ANN. It is shown that the coefficient of multiple determination (R^2) between the FVM and ANN predicted values is equal to 0.99866, maximum relative error is less than 5.9128% and mean square error is 1.13×10^{-3}. Results show that the obtained formulation is obviously within acceptable limits.

Keywords: *NanoFluid, Finite Volume Method, Artificial Neural Networks, Channel, Mixed Convection.*

1. Introduction

Fluid flow and heat transfer in channels has wide range of applications in transport processes. Indeed the combination of natural and forced convection (mixed convection) has been recommended for high heat dissipating electronic components where the natural convection is not able to provide effective cooling.

Especially, heat transfer from continuously heated surfaces, in relation to cooling of electronic equipment, has become an interesting subject in the last decade. Many researchers have studied the natural, mixed, and forced convection in (inclined) channels due to their practical applications such as electronic systems, heat exchangers, lubrication technologies and so on[1-3]. At more applications, the used fluids are water, ethylene glycol or propylene glycol. The main problem of these fluids is their low thermal conductivity, therefore in recent years; special fluids are presented with high thermal conductivity which are named as Nanofluids. These fluids consist of suspended Nanoparticles. Utilization of metallic Nanoparticles with high thermal conductivity will increase the effective thermal conductivity of these types of fluid remarkably. In several researches, it has been shown that with low Nanoparticles concentration (1–5% by volume), the thermal conductivity can be increased about 20%. Such enhancement depends on shape, size, concentration and thermal properties of the solid Nanoparticles [4,5]. Present work is focused on numerical heat transfer study in an inclined channel with three discrete heat sources. In addition, the ANNs method is used for formulating the Nusselt number. It is shown that the ANNs method are feasible and powerful method for investigation of heat transfer in the channel. Artificial Neural networks have been largely used in the previous decades [6-8]. Madadi et al evaluated the optimal location of three discrete heat sources which could be placed anywhere inside a ventilated cavity and cooled by forced convection [9]. Santra et al. have investigated heat transfer inside a differentially heated square cavity with using ANNs method [10]. In this study, heat transfer due to laminar mixed convection of a Newtonian copper–water Nanofluid in an inclined channel has been predicted by using Artificial Neural Network trained by Levenberg-Marquardt (LM) algorithm. The simulated results of heat transfer obtained by the finite volume method (FVM) have been used to train the ANN. Training accuracy is measured in terms of Root Mean Square Error (RMS) between target outputs and original outputs. Both the input and output data sets are normalized for achieving better accuracy. The results obtained by the ANNs method were compared with the simulated results and it is found that the convergence characteristic of LM algorithm is excellent to yield fairly comparable and accurate results.

2. Problem definition and governing equations

Mixed convection is investigated in a rectangular inclined channel, as shown in Fig. 1. The height of channel is H and the length of channel is L. Three constant heat sources q_1'', q_2'', and q_3'' of length B are placed on the bottom wall of the channel in x_1, x_2, and x_3, respectively. The lower wall except the heat sources is insulated and the upper wall in contact with the fluid is constantly cooled at temperature T_c. At the inlet of channel, the velocity and temperature profile are uniform and are U_0 and T_0, respectively. Throughout this study, the geometry is x_1=6.75 cm, x_2=14.50 cm, x_3=22.25 cm, B=H=1 cm, and L=30 cm. Temperatures

T_o and T_c are equal to zero. Also, the inclined channel is filled with Nanofluid. It is assumed that both the fluid phase and Nanoparticles are in thermal equilibrium and there is no slip between them. Except for the density, the properties of Nanoparticles and fluid are taken to be constant. Table 1 presents thermo physical properties of water and copper at the reference temperature.

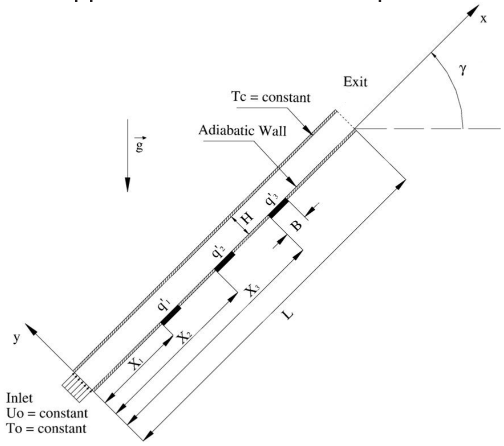

Figure 1. Schematic of the problem geometry

Table 1. Thermophysical properties of water and copper

Property	c_p (J kg^{-1} K^{-1})	ρ (kg m^{-3})	k (kW m^{-1} K^{-1})	β (K^{-1})
Water	4179	997.1	0.6	2.1×10^{-4}
Copper	383	8954	400	1.67×10^{-5}

It is assumed that the Boussinesq approximation is valid for buoyancy force. The governing equations for the steady, two-dimensional laminar flow and incompressible mixed convection flow are expressed as below. The governing equations for laminar, steady and incompressible flow including continuity, momentum and energy equations in dimensionless forms are as follows:

1. Continuity equation:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0 \quad (1)$$

2. Momentum equations:

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\frac{\rho_f}{\rho_{nf}}\frac{1}{(1-\phi)^{2.5}}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)+RaPr\frac{\rho_f}{\rho_{nf}}\left(1-\phi+\phi\frac{\rho_s\beta_s}{\rho_f\beta_f}\right)\theta\sin\gamma \quad (2)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\frac{\rho_f}{\rho_{nf}}\frac{1}{(1-\emptyset)^{2.5}}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+RaPr\frac{\rho_f}{\rho_{nf}}\left(1-\emptyset+\emptyset\frac{\rho_s\beta_s}{\rho_f\beta_f}\right)\theta\cos\gamma \tag{3}$$

3. Energy equation:

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{k_{nf}}{k_f}\frac{(\rho c_p)_f}{(\rho c_p)_{nf}}\frac{1}{Re\,Pr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right) \tag{4}$$

where:

$$X=\frac{x}{H}\ ,\qquad Y=\frac{y}{H}\ ,\qquad U=\frac{u}{U_0}\ ,\qquad V=\frac{v}{U_0} \tag{5}$$

$$P=\frac{p}{\rho_0 U_0^2}\ ,\theta=\frac{(T-T_0)k_f}{q''H},Ra=\frac{\beta_f\, g\, H^4\, q''}{k_f\,\alpha_f\,\nu_f}\ ,Re=\frac{U_0\,\rho_f\,H}{\mu_f} \tag{6}$$

The Nanofluid properties are given by Eqs. (7-10).

$$\rho_{nf}=(1-\emptyset)\rho_f+\emptyset\rho_s \tag{7}$$

$$(\rho c_p)_{nf}=(1-\emptyset)(\rho c_p)_f+\emptyset(\rho c_p)_s \tag{8}$$

$$\mu_{nf}=\frac{\mu_f}{(1-\emptyset)^{2.5}} \tag{9}$$

$$\frac{k_{eff}}{k_f}=1+\frac{k_p A_p}{k_f A_f}+c k_p Pe\frac{A_p}{k_f A_f} \tag{10}$$

where Φ is the solid volume fraction, β_s is coefficient of volume expansion of solid particles, ν_{nf} is kinematics viscosity of Nanofluid and ρ_{nf} is density of Nanofluid. Thermal diffusivity of Nanofluid is defined as $\alpha_{nf}=k_{nf}/(\rho c_p)_{nf}$. The effective thermal conductivity of Nanofluid was given by Patel et al. [11]. In Eq. 10, the coefficient 'c' is constant and must be determined experimentally. A_p/A_f and p_e are defined as:

$$\frac{A_p}{A_f}=\frac{d_f}{d_p}\frac{\emptyset}{(1-\emptyset)}\ ,Pe=\frac{u_p\,d_p}{\alpha_f} \tag{11}$$

where d_p is diameter of solid particles and in this study is assumed to be equal to 100 nm, d_f is the molecular size of liquid that is taken as 2A° for water. Also, u_p is the Brownian motion velocity of Nanoparticles which is defined as:

$$u_p=\frac{2\,k_b T}{\pi\,\mu_f\,d_p^2} \tag{12}$$

where k_b is the Boltzmann constant. The effective viscosity of Nanofluid was introduced by Brinkman [12] as below:

$$\mu_{nf}=\frac{\mu_f}{(1-\emptyset)^{2.5}} \tag{13}$$

In order to estimate the heat transfer enhancement and investigation of cooling efficiency, Nu_{ave} (average Nusselt number) have been calculated at the heat source surface by the following equation:

$$Nu_{ave} = \frac{\int Nu(x)dx}{\int dx} \tag{14}$$

where Nu(x) is defined as:

$$Nu(x) = \frac{1}{\theta_s(X)} \tag{15}$$

3. CFD solution procedure

3.1. Numerical method

The above systems of equations have been solved numerically by FVM method using SIPMLER algorithm [13]. To check the convergence of the sequential iterative solution, the sum of the absolute differences of the solution variables has been calculated between two successive iterations. When this summation falls below the convergence criterion, convergence will obtain. In this study, the convergence criterion has been chosen as 10^{-6}. In order to demonstrate independent of solution on the size of the grid, solution has been done for various mesh sizes.

3.2. Solution verification and validation

The grid independence test has been carried out. The Results of Nusselt number show that the grid system of 51×481 is fine enough to obtain accurate results.

To validate the FVM results, flow in rectangular channel with constant wall temperature has been tested for water-copper Nanofluid. In this test case, the results have been compared with those of Santra et al. [14]. The maximum deviation between the results is 2.68 percent that is acceptable.

4. Artificial Neural Networks

Artificial Neural Networks can be largely used in different fields like weather prediction, control, optimization, model detecting, information categorization and picture and sound processing [15]. The original inspiration for the term Artificial Neural Network came from examination of central nervous systems and their neurons, axons, dendrites, and synapses, which constitute the processing elements of biological neural networks investigated by neuroscience. The main part of an artificial neural network is called neuron. Basically, by receiving inputs, biological neurons confect them in some ways and give the final output by doing some non-linear operations. ANNs mainly contain one input layer, one or some hidden layers and one output layer [16].

In order to train a neural network to perform some task, we must adjust the weights of each unit in such a way that the error between the desired output and the actual output is reduced. This process requires that the neural network compute the error derivative of the weights (EW). In other words, it must calculate how the error changes as each weight is increased or decreased slightly. The back propagation algorithm is the most widely used method for determining the EW. All activation functions contain relations in linear or non-linear algebra.

4.1. The Levenberg-Marquardt Algorithm

The Levenberg-Marquardt (LM) algorithm is the most widely used optimization algorithm. It outperforms simple gradient descent and other conjugate gradient methods in a wide variety of problems. The LM algorithm is first shown to be a blend of vanilla gradient descent and Gauss-Newton iteration. Subsequently, another perspective on the algorithm is provided by considering it as a trust-region method. The problem for which the LM algorithm provides a solution is called Nonlinear Least Square Minimization. This implies that the function to be minimized is of the following special form:

$$f(x) = \frac{1}{2}\sum_{m}^{j=1} r_j^2(x) \quad (16)$$

Where $x = (x_1, x_2, \dots, x_n)$ is a vector, and each r_j is a function from R^n to R. The r_j are referred to as a residuals and it is assumed that $m \geq n$.

To make matters easier, f is represented as a residual vector r: R^n to R^m defined by:

$$r(x) = (r_1(x), r_2(x), \dots, r_m(x)) \quad (17)$$

Now, f can be rewritten as $f(x) = \frac{1}{2}\|r(x)\|^2$. The derivatives of f can be written using the Jacobian matrix J of r w.r.t x defined as:

$$J(x) = \frac{\partial r_j}{\partial x_i}, 1 \leq j \leq m, 1 \leq i \leq n \quad (18)$$

At the first, it considers the linear case where every r_i function is linear. Here, the Jacobian is constant and we can represent r as a hyperplane through space, so that f is given by the quadratic equation [17-18]:

$$f(x) = \frac{1}{2}\|J_x + r(0)\|^2 \quad (19)$$

4.2. Artificial neuron model

The most important part of ANNs is neural network training. Basically, there are two ways to train a network which are controllable training and uncontrollable training. Choosing the best and fastest training algorithm for solving problem is very important and critical step. ANNs are training through changing the weights of intermediate layers and using from these changed values in later iteration. Back propagation learning algorithm is the most usual kind of training.

To check the correctness of the result, some of statistic methods were being used like root mean square error (RMS), mean squared error (MSE), total variance fraction (R^2) and coefficient of variation in percent (cov). During the learning process, error was detected by RMS as follows:

$$RMS = \left[\frac{1}{n}\sum_{j=1}^{n}|t_j - o_j|^2\right]^{1/2} \tag{20}$$

Also, the performance of the network can be evaluated by the MSE, which is defined as:

$$MSE = \frac{1}{n}\sum_{j=1}^{n}|t_j - o_j|^2 \tag{21}$$

Total variance fraction and coefficient of variation in percent are defined as:

$$R^2 = 1 - \left(\sum_{j=1}^{n}|t_j - o_j|^2 \Big/ \sum_{j=1}^{n} {o_j}^2\right) \tag{22}$$

$$cov = \frac{RMS}{o_{mean}} \times 100 \tag{23}$$

where t is the target value, o is output value and o_{mean} is the mean value of all output data. Also, n shows the number of outputs of neural networks.

In order to increase network's precision, the amount of outputs and inputs of ANN were normalized in the range of (-1, 1). Input and output matrixes which was used for testing and training ANNs are describes as bellow:

$$Input = \begin{bmatrix} Re_1 & Ra_1 & \gamma_1 & \emptyset_1 \\ \vdots & \vdots & \vdots & \vdots \\ Re_n & Ra_n & \gamma_n & \emptyset_n \end{bmatrix} \tag{24}$$

$$output = \begin{bmatrix} Nu_1 & Nu_2 & Nu_3 \\ \vdots & \vdots & \vdots \\ Nu_{n1} & Nu_{n2} & Nu_{n3} \end{bmatrix} \tag{25}$$

The inputs and outputs which were used for training and testing ANNs had obtained from the FVM results.

4.3. Formulation with ANN

The aim of this study is detecting analytical relations by ANNs for Nusselt number of copper water Nanofluid in an inclined channel with three heat source.
Also reliable quantities were used in their validity range to test and train artificial network. The Reynolds number, channel inclination, Rayleigh number and solid volume fraction were taken as inputs of neural networks and Nusselt number was regarded as an output of neural network.

For ANNs training, the results of FVM are used. The feed forward back propagation learning algorithm was used by single hidden layer network. Neurons in input layer have no transfer function. Tangent sigmoid transfer function was used as the activation function for the hidden layer which presented by Eq.25

$$tansig(E_i) = 2/(1 + exp(-2 \times E_i)) - 1 \quad (26)$$

Pola-Ribiere conjugate gradient (CGP) and Levenberg-Marquardt (LM) are the variant algorithms were used in this study. The coefficient of multiple determinations (R^2) is 0.99866 when LM algorithm was used as training algorithm. The decrease in values of the MSE at the time of neural network is training show the neural networks convergence. Training period can be continued until the favorable convergence.

5. Results and discussion

In this paper, the Nanofluid mixed convection in an inclined channel with three discrete heat sources on the bottom wall has been studied. The results have been presented for the varying Reynolds number, Rayleigh number, inclination angle and solid volume fraction. Fig. 2 presents the isotherms and streamlines at $Ra=10^5$, $0 \leq Re \leq 50$ and $\gamma=0^\circ$, 90° for both pure fluid and nanofluid with Φ =5%. As it can be seen, in low Reynolds number when the channel is horizontal ($\gamma=0^\circ$), the isotherms near to heat sources are concentrated. Increase in Reynolds number will decrease this concentration so that at Re=50, there are no sign of this concentration. It is mentionable that the inclination angle has an important role in temperature distribution. As it shown in Fig. 2, increase in inclination angle leads to decrease in isotherm lines concentration. Solid volume fraction has a negligible effect in isotherm lines but because of increase in the effective thermal conductivity of Nanofluid respect to pure fluid, increase in solid volume fraction leads to increase in Nusselt number. At low Reynolds numbers (Re=1) for all of inclination angles, there are the rotating vortexes around the heat sources. By increase in Reynolds number the rotating vortexes will disappear, because in high Reynolds numbers the forced convection is dominated and it leads to disappearing the rotating vortexes. Also in high Reynolds numbers the stream lines distribution for Nanofluid are similar to pure fluid. Fig. 3 displays the variation of Nusselt number for three heat sources and various solid volume fraction at Reynolds numbers equal to 1, 50, 500, $Ra=10^3$, 10^5 and inclination angle $\gamma=0^\circ$. In general, for a constant Rayleigh number, increase in Reynolds number leads to increase in Nusselt number. Indeed, at low Reynolds numbers, by increasing Rayleigh number the Nusselt number increases remarkably. As it shown in Fig. 3, by increasing the Reynolds number the average Nusselt number for the first heat source increases more than the other ones. Also, the solid volume fraction has an important and remarkable effect on the Nusselt number. It is mentionable that at high Reynolds numbers, the inclination angle has a negligible effect on the Nusselt number.

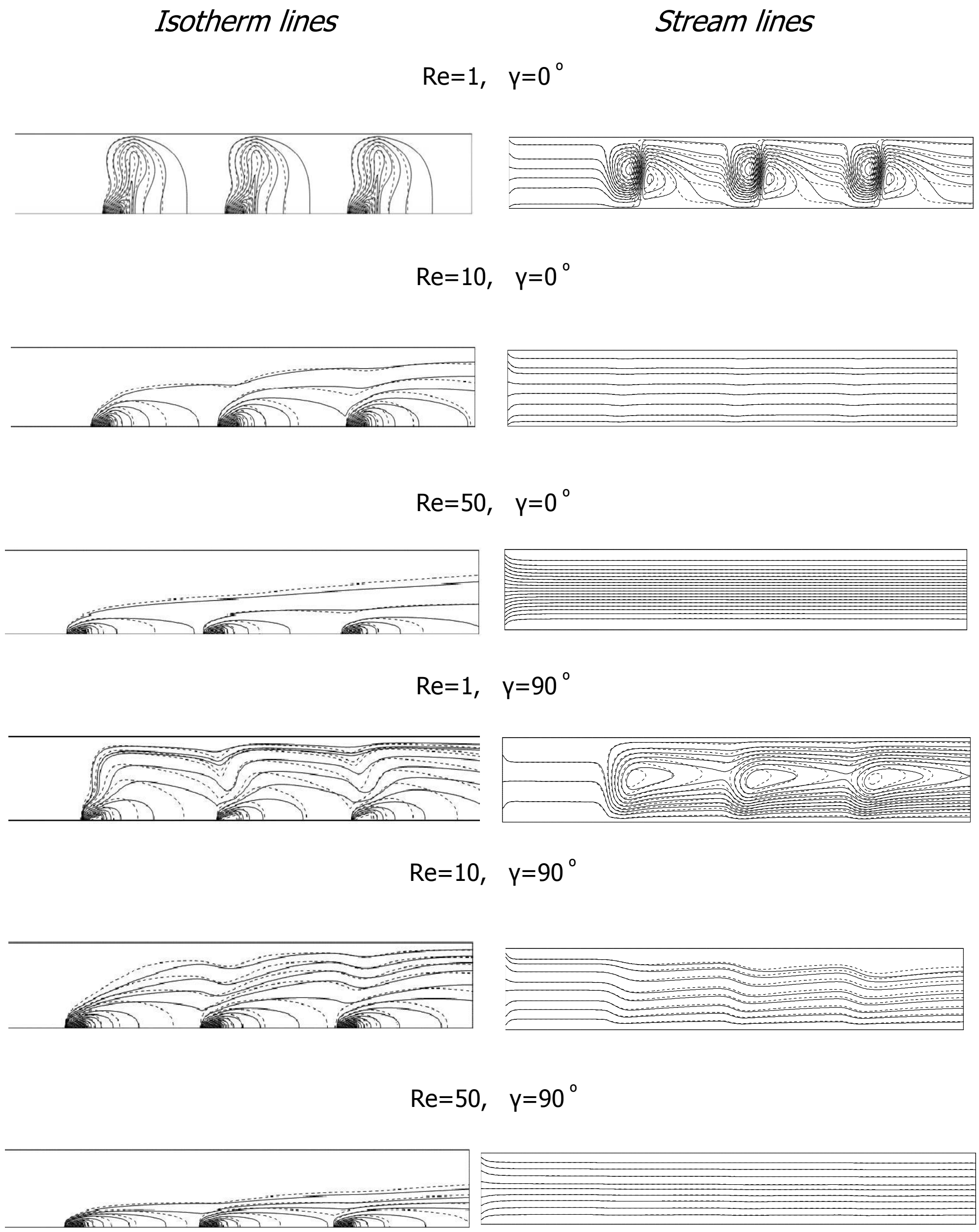

Figure 2. Isotherm lines and stream lines at $Ra=10^5$ and Re=1, 10, 50 and $\gamma=0^{\circ}$ and 90° (the bold lines are related to pure fluid and dashed lines are related to Nanofluid with $\Phi=0.05$)

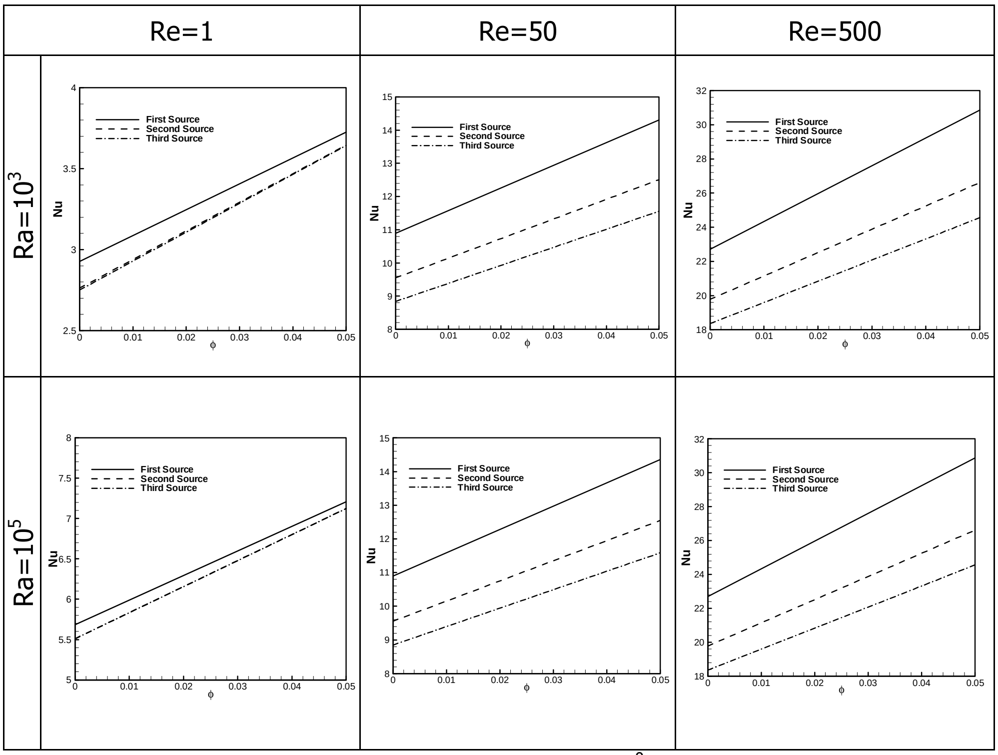

Figure 3. Variation of Nusselt number at $\gamma=0^{\circ}$ on three heat sources

Table 2 shows the relative percent of variation of Nusselt number on three heat sources for nanofluid with solid volume fraction equal to 5% respect to pure fluid at Reynolds numbers Re=1, 50 and 500 and Rayleigh numbers $Ra=10^3$, 10^5. At Re=1 and Ra= 10^3, the average Nusselt number of first heat source with solid volume fraction Φ =0.05 at inclination angle $\gamma=0^{\circ}$ has increased respect to pure fluid about 27%. On the other hand at Re=50 and Ra= 10^3, the percent of increasing is about 31% and finally, the percent of increasing at Re=500 for same Rayleigh number is about 36%. Hence, by increasing the Reynolds number the effect of solid volume fraction becomes more effective on increasing of Nusselt number. By keeping Ra constant and increasing Re, the difference of Nusselt number between the first heat source with second and third heat sources becomes more noticeable. It is due to undesirable effect of the first heat source temperature field on heat transfer coefficient of second heat source and by similar way, the second heat source temperature field on heat transfer coefficient of third heat source. Hence, at low Reynolds numbers the difference of Nusselt number between heat sources is small but by increasing Re this difference becomes more and more.

Table 2. Relative variation of average Nu (in percent) for three heat sources for Nanofluid with Φ =0.05 at γ=0.

Re	Heat Sources	$Ra=10^3$	$Ra=10^5$
1	1st	0.268	0.273
	2nd	0.292	0.320
	3rd	0.293	0.324
50	1st	0.317	0.314
	2nd	0.313	0.309
	3rd	0.309	0.307
500	1st	0.359	0.359
	2nd	0.345	0.345
	3rd	0.338	0.338

By using ANNs, a relation between Nusselt number and input parameters were obtained. Two kinds of training algorithms were used to train ANNs. Levenberg-Marquardt algorithm was appeared to have more accuracy and convergence rather than one. For training and testing the ANNs, 330 different kinds of data were used which 320 groups were used to train networks and the rest were used to test the networks. Also, the results of training process are available in Table 3. In order to detect the best structure for the neural networks, the neurons number of hidden layer changed from 3 to 7. Finally, the structures with the lowest MSE were chosen as the best structures which have 6 cells in their hidden layers as shown in Table 3.

Table 3. Comparison between various neurons in hidden layer

Neurons in hidden layer	MSE	RMS	R^2	Cov
3	1.66e-3	0.217253	0.99803	1.888015
4	1.30e-3	0.241538	0.99847	2.129649
5	1.24e-3	0.255033	0.99854	2.23345
6	1.13e-3	0.127439	0.99866	1.112492
7	1.14e-3	0.135861	0.99866	1.177925

ANNs were trained by real and normalized quantities separately. Values of the training and test data were normalized to a range of (−1, 1). Results indicate that networks trained by normalized quantities are more accurate.

The mathematical relations obtained from neural networks are represented. In these relations, the input quantity factors are used to derivate the sigma functions of neurons (E_i) and activation functions (F_i). Neurons in input layer have no transfer function. Tangent sigmoid transfer function has been used.

$$f(Ei) = 2/(1+\exp(-2\times Ei))-1 \qquad (27)$$

where E_i is the weighted sum of the input and should be taken from Table 4.

Table 4. Weight values for sigma function (E_i) with 4 neurons in input layer obtained by LM training algorithm

Neurons in hidden layer	Ei = C1i × Re + C2i × Ra+ C3i × Φ + C4i× γ + C5i				
i	C1i	C2i	C3i	C4i	C5i
1	1.0682	0.0239	-0.0922	-0.0071	1.8453
2	1.4752	0.6689	-0.0308	-1.1746	2.1431
3	19.4944	1.2855	-0.0601	0.0058	22.8742
4	0.0824	0.0204	0.4035	-0.0001	-0.1001
5	-1.0126	-0.1898	0.0287	0.2457	-3.0316
6	14.2745	-15.7068	0.6037	-2.8096	-2.4499

Finally, the outputs of the ANN, normalized quantities of Nusselt number, are as follows:

Table 5. Weight values for Nusselt number with 6 neurons for hidden layer obtained by LM training algorithm

Weight Factor	NuJ						
	B1i	B2i	B3i	B4i	B5i	B6i	B7i
J=1	4.5045	0.1866	8.8234	0.7657	10.5383	4.5045	-2.2407
J=2	4.4192	0.1376	7.7542	0.7652	8.5126	4.4192	-3.0577
J=3	4.3739	0.0977	6.5860	0.7677	6.8221	4.3739	-3.4909

$NuJ = B1i \times F_1 + B2i \times F_2 + B3i \times F_3 + B4i \times F_4 + B5i \times F_5 + B6i \times F_6 + F_7$

In the above equations, E_i is obtained from multiplying normalized input quantities in their related weights and then sum by related bias of hidden layer. All E_1 to E_6 and F_1 to F_6 sequentially indicate sigma functions and activation functions of the cells in hidden layer. Obtained relations for Nusselt number are based on normalized inputs, so network's outputs are normalized. It can realize high accuracy of neural network's results by comparisons between FVM data and those obtained by neural network for Nusselt number on the heat source. The normalized outputs obtained from neural network can be changed to real quantity by using the following relation:

$$X = Min(X_n) + (X_n + 1)(Max(X_n) - X_n)/2 \quad (28)$$

where X is real value and X_n is normalized value.

6. Conclusion

In this study, the Nanofluid mixed convection was examined through an inclined channel with three heat sources. Also, finite volume method (FVM) for solving the governing equations has been used and finally, a relation between Nusselt number and input parameters were obtained by using ANNs method. The heat transfer analysis showed, in general using of Nanofluid leads to increase in Nusselt number compared to pure fluid and this increasing is more noticeable at high Reynolds numbers (about 36%). A Mathematical relation between the Reynolds number, channel inclination, Rayleigh numbers and solid volume fraction as independent variable and the Nusselt number as a dependent variable was achieved by utilizing Artificial Neural Network. It can be seen that the L-M algorithm has the faster convergent rate than the SCG algorithm with the same epoch. The most suitable algorithm and neuron number in the hidden layer are found as Levenberg-Marquardt algorithm with tang sigmoid as activation function with 6 neurons in the hidden layer. It was found that the coefficient of multiple determinations (R^2) between the FVM and ANN values is equal to about 0.99866, maximum relative error is less than 5.9128%, mean of relative error is about 0.3127% and mean square error is about 1.13×10^{-3}.

References

[1] Guimarães P.M., Menon G.J., *Combined free and forced convection in an inclined channel with discrete heat sources*, Int. Communications in Heat and Mass Transfer, vol. 35, pp. 1267–1274, 2008.

[2] Rao G.M., Narasimham G.S.V.L., *Laminar conjugate mixed convec-tion in a vertical channel with heat generating components*, International Journal of Heat and Mass Transfer, vol. 50, pp. 3561–3574, 2007.

[3] Dogan A., Sivrioglu M., Baskaya S., Experimental investigation of mixed convection heat transfer in a rectangular channel with discrete heat sources at the top and at the bottom, International Communications in Heat and Mass Transfer, vol. 32, pp. 1244–1252, 2005.

[4] Lee S., Choi S.U.S., Li S., Eastman J.A., *Measuring thermal conductivity of fluids containing oxide nanoparticles*, Transaction. ASME Journal of Heat Transfer, vol. 121, pp. 280–289, 1999.

[5] Eastman J.A., Choi S.U.S., Li S., Yu W., Thompson, L.J., Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles, Applied. Physics. Letters, vol. 78, pp 718-720, 2001.

[6] Kalogirou S.A., Panteliou S., Dentsoras A., *Artificial neural-networks used for the performance prediction of thermosiphon solar water heater*, Renewable energy, vol. 18(1), pp. 87-99, 1999.
[7] Pacheco-Vega A., Sen M., Yang K.T., McClain R.L., *Neural network analysis of a fin-tube refrigerating heat-exchanger with limited experimental data*, International Journal of Heat and Mass Transfer, vol. 44(4), pp. 763-770, 2001.
[8] Palau A., Velo E., Puigjaner L., *Use of neural networks and expert systems to control a gas/solid sorption chilling machine*, International Journal of Refrigration, vol. 22 (1), pp. 59-66, 1999.
[9] Madadi R.R., Balaji C., *Optimization of the location of multiple discrete heat sources in a ventilated cavity using artificial neural networks and micro genetic algorithm*, International Journal of Heat and Mass Transfer, vol. 51, pp. 2299–2312, 2008.
[10] Santra A. K., Chakraborty N., Sen S., *Prediction of heat transfer due to presence of copper–water nanofluid using resilient-propagation neural network*, International Journal of Thermal Sciences, vol.48, pp.1311–1318, 2009.
[11] Patel H.E., Pradeep T., Sundararajan T., Dasgupta A., Dasgupta N., Das S.K., *A microconvection model for thermal conductivity of nanofluid*, Pramana- journal of physics, vol. 65, pp. 863–869, 2005
[12] Brinkman H.C., *The viscosity of concentrated suspensions and solutions*, Journal of Chemical Physics, vol. 20, pp. 571-581, 1952.
[13] Patankar S.V., *Numerical heat transfer and fluid flow, hemisphere*, New York, 1980.
[14] Santra A.K., Sen S., Chakraborty N., *Study of heat transfer due to laminar flow of copper–water nanofluid through two is thermally heated parallel plates*, International Journal of Thermal Science, vol. 48, pp. 391–400, 2009.
[15] Haykin S., *Neural Networks, A Comprehensive Foundation,* Prentice Hall, 1999.
[16] Hagan M.T., Menhaj M., *Training feed forward networks with the marquardt algorithm*, IEEE Transactions on Neural Network, vol. 5 (6), pp. 989-993, 1994.
[17] Levenberg K., *A method for the solution of certain problems in least squares*, Quart. Appl. Math.,Vol. 2, pp. 164–168, 1944
[18] Marquardt D., *An algorithm for least-squares estimation of nonlinear parameters*, SIAM J. Appl. Math., Vol. 11, pp. 431–441, 1963

Addresses:

- Hamid Teimouri, Department of Mechanical Engineering, University of Sistan and Baluchestan, P.O. Box 98164-161 Zahedan, Iran Hamidtmr@gmail.com
- Navid Bozorgan, Mechanical Engineering Department, Abadan Branch, Islamic Azad University, Abadan, Iran, n.bozorgan@gmail.com