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On the Evanescent Waves in Sonic Band-gaps

The band-gaps or the Bragg reflections occur at different frequencies inverse proportional to the central distance between two scaterers. The evanescent waves may converge or diverge as distance goes to infinity. If the waves converge, they decay exponentially within the band-gaps and sustain an evanescent mode. If the waves diverge, a defect can terminate this exponential growth.

Keywords: *sonic composite, full band-gap, evanescent waves*

1. Introduction

The existence of full band-gaps in sonic composites is reported not only by experimental works, but also by theoretical works [1-5].

The key of the band-gap generation is the lack of purely real wave vector for certain modes of waves at certain frequencies. The wave amplitude may decay exponentially sustaining an evanescent mode, or can increase exponentially and a defect can terminate this exponential growth to sustain also an evanescent mode [6, 7]. The primary goal of this paper is to study a sonic system periodic/noperiodic in order to discern some of the most important features of the sonic composites, such as the full band-gaps and evanescent modes that are localized around defects.

The periodic sonic system is consisting of an array of acoustic scatterers embedded in an epoxy matrix. The acoustic scatterers are hollow spheres made from a nonlinear isotropic piezoelectric ceramic, while the matrix is made from a nonlinear isotropic epoxy resin [8-15]. Acoustic scatterers are composed by piezoceramic hollow spheres of functionally graded materials - the Reddy graded hollow spheres. The simplest possible case for study the properties of a sonic composite is multilayer films consisted of alternating layers of material with different properties. This arrangement is not a new idea. Lord Rayleigh published in 1877 one of the first analysis of the optical properties of multilayer films. He shown that this type of photonic crystal can act as a mirror (a Bragg mirror) for light with a frequency within a specified range, and it can localize light propagation if there are any defects in its structure. This concept is used in dielectric mirrors and optical filters [16, 17].

The non-periodic sonic system is a multilayer sonic film consists of alternating layers of material with different mechanical properties, following a triadic Cantor sequence [15]. Study of these sonic composites is enables to obtain the dispersion relation for defect modes, and the prediction of the evanescent nature of the modes inside the band-gaps.

2. Periodic structures and band-gaps

Let us consider a composite thin plate consting of an array of acoustic scatterers embedded in an epoxy matrix [8, 11]. The acoustic scatterers are hollow spheres made from a nonlinear isotropic piezoelectric ceramic (PZ), while the matrix is made from a nonlinear isotropic epoxy resin (ER). The sonic plate consists of 72 local resonators of diameter a. A rectangular coordinate system $Ox_1x_2x_3$ is employed. The origin of the coordinate system $Ox_1x_2x_3$ is located at the left end, in the middle plane of the sample, with the axis Ox_1 in-plane and normal to the layers and the axis Ox_3 out-plane and normal to the plate. The length of the plate is l, its width is d, while the diameter of the hollow sphere is a and its thickness is $e > a$.

Consider now two piezoceramic hollow spheres with the ratio of the inner and outer radii ξ_0. Two laws represent the functionally graded property of the material. The first one is the Reddy law [18] given by

$$M = M_p\mu^{\lambda} + M_z(1-\mu^{\lambda}), \tag{1}$$

where $\mu = \dfrac{b-r}{b-a}$, λ is the inhomogeneoty parameter or gradient index, M_p and M_z are material constants of two materials, namely PZT-4 and ZnO. The case $\lambda = 0$ corresponds to a homogeneous PZT-4 hollow sphere and $\lambda \to \infty$, to a homogeneous ZnO hollow sphere.

Two and three strong attenuation bands, respectively, in the audible range are found at frequencies at 0.8 kHz and 8.5 kHz, and 0.8 kHz, 4.2kHz and 7.8 kHz, respectively, with a relative attenuation of 25dB.

The guided waves are accompanied by evanescent waves which extend to the periodic array of the scatterers surrounding the wave-guide. It is strongly expected that mode coupling waves arise between adjacent wave-guides. The output of the coupled modes is compared with the input waves, as shown in Figure 1, in the case of Reddy law.

A remarkable result is that the ratio of the coupled and input waves is −3 to −4 dB around the frequency of 8kHz to 8.8kHz in the band-gap of the sonic material.

Figure 2 plots the dispersion curve including the first partial band-gaps for the composite.

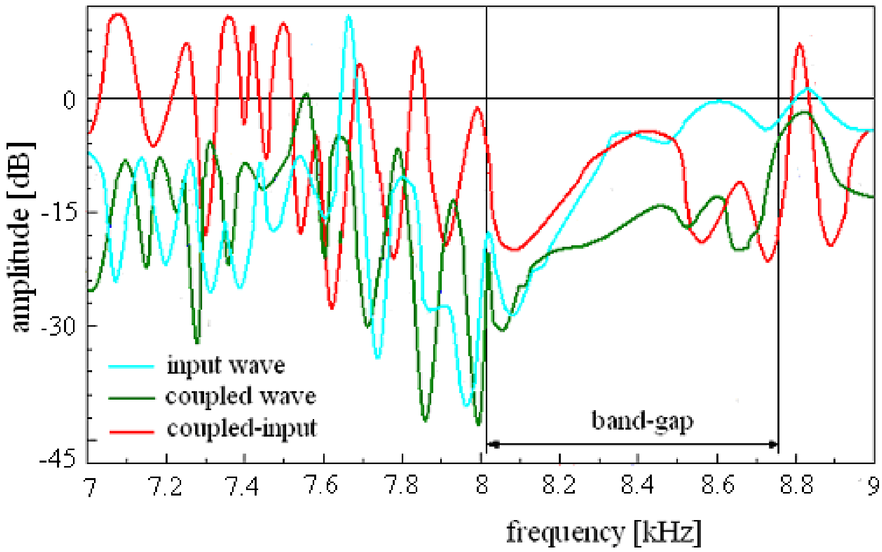

Figure 1. The input and coupled waves for the sonic composite

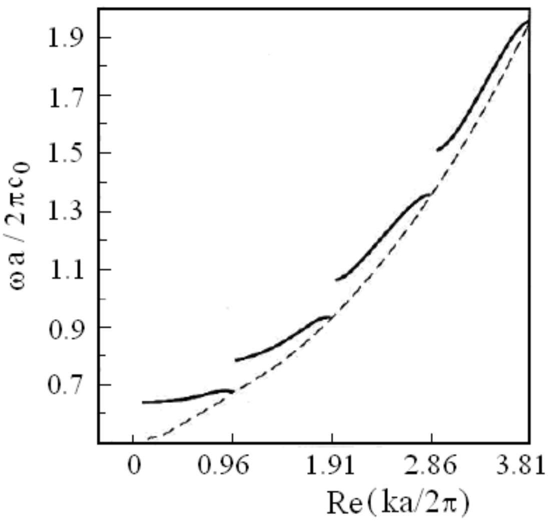

Figure 2. Linear dispersion

The reduced units for the frequency are $\omega a / 2\pi c_0$, with c_0 the speed of sound in air. We see that the point defects confine acoustic waves in localized modes and in consequence the band-gaps are larger than in the case of the complete composite. The guided waves are accompanied by evanescent waves which extend to the periodic array of the scatterers surrounding the wave-guide.

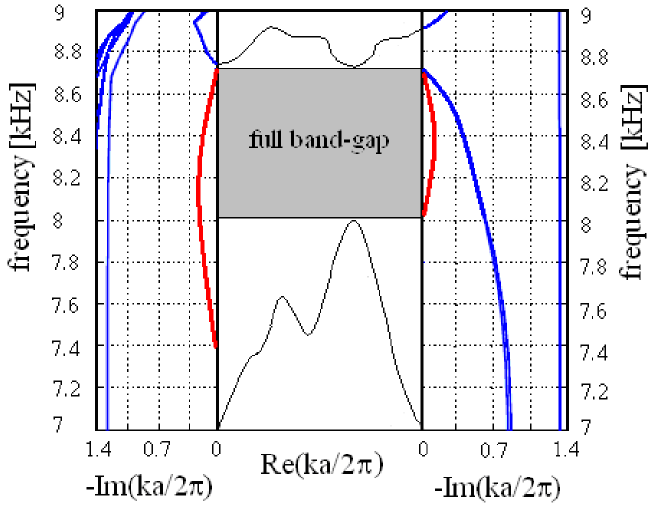

Figure 3. Band structure for the sonic composite

Using the Joannopoulus representation [7] for the bad-gap structure, Figure 3 presents the band structure with the evanescent modes with exponential decay for the sonic composite. The modes present purely imaginary wave vectors. The central grey region is the full band-gap ranged between 8.02 kHz and 8.72 kHz, given by the real part of the wave vector constrained in the first Brillouin zone for each frequency. The left region represents the imaginary part of the wave vector for longitudinal direction frequency (tension/compression), while the right region is the imaginary part of the wave vector for transverse direction frequency (shear). The red lines represent the imaginary part of the wave vector of the evanescent modes inside the bad-gap.

If we want to have a full band-gap, we must have structures with band-gaps for both longitudinal and transverse waves in the same frequency region.

The difference in the sound velocities between transverse and longitudinal modes causes partial gaps at different frequencies. If the mechanical contrast is small, these partial gaps are narrow and do not overlap.

As mechanical contrast increases, the partial gaps widen and begin to overlap in the same frequency region leading to the appearance of a full band-gap independent of the polarization. It is strongly expected that mode coupling waves arise between adjacent wave-guides.

3. Non-periodic structures and band-gaps

Let us consider a 2D multilayer film of alternating layers of piezoelectric ceramics (PZ) and the epoxy resin (ER), following a triadic Cantor sequence with 31 elements (Figure 4). The length of the plate is l, the width of the smallest layer is $l/81$ and the thickness of the plate is $2h$. The width of the plate is d [9].

We choose this kind of structure due to its property of generating the sub-harmonic waves which have a significant importance in the generation of the full band-gaps.

Alippi et al. [19-21] and Craciun et al. [22] show the experimental evidence of extremely low thresholds for subharmonic generation of ultrasonic waves in 1D artificial piezoelectric plates with Cantor-like structure, as compared to the corresponding homogeneous and periodical plates. An anharmonic coupling between the extended-vibration (phonon) and the localized-mode (fracton) regimes explained this phenomenon. They demonstrate that the large enhancement of non-linear interaction results from the more favorable frequency and spatial matching of coupled modes (fractons and phonons) in the Canto structure. The existence of multiple fracton and multiple phonon-mode regimes in the displacement field in such structure was analyzed by Scalerandi et al. (1999) and Chiroiu et al. (2001).

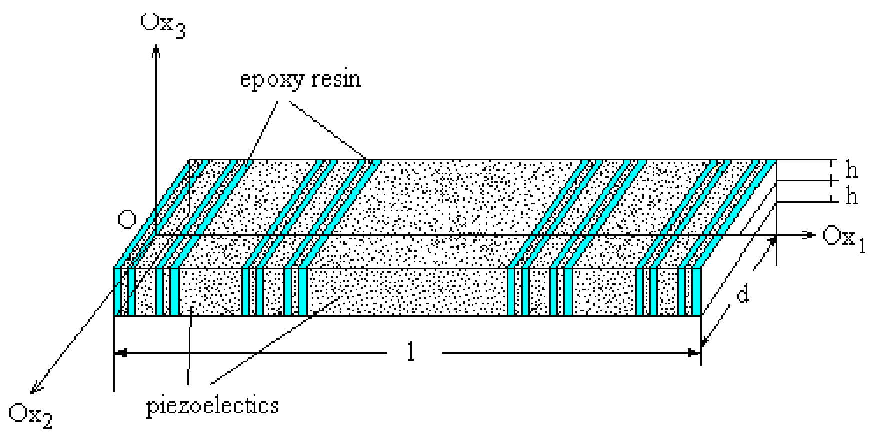

Figure 4. The non-periodic structure with Cantor-like structure

The calculus is carried out for $l =$ 67.5 mm and $2h = 0.3$ mm. We refer to the plates with 31 elements (for $a = l/81$). The resonant Lamb modes are excited by applying an external electric field $\overline{E}_1 = \overline{E}_3 = \overline{E}^0 \exp(\mathrm{i}\omega_0 t)$ on both sides of the plate. The surface Lamb waves are superposition of longitudinal (symmetric waves) and shear modes (anti-symmetric waves or SV waves), which dominate the radial in-plane and vertical motion of particles in the film.

The structure and size of the band-gap depend on $\bar{E}^0$. If $\bar{E}^0$ is increased above a threshold value $\bar{E}^0_{th} = 5.27\text{V}$ the $\omega/2$ subharmonic generation is observed. Note that Alippi obtains in the Cantor-like sample typical values of the lowest threshold voltages of 3-5 V. The amplitude of waves at the surface of the plate is function of $\bar{E}^0$.

Figure 5 shows the Lamb normal mode $\omega/2\pi$ = 334kHz, and the subharmonic mode $\omega/4\pi$ = 167kHz. On the abscissa is a sketch of the plate geometry (dashed, piezoelectric ceramic and blue, epoxy resin).

For the homogeneous plate the mismatch $\omega_n - \omega/2$ is due to the symmetry of fundamental modes with respect to x. Only symmetric odd n can induce a subharmonic, but never $\omega/2$ coincides with a plate vibration mode.

The fracton and phonon regimes are represented in Figures 6 and 7. The fracton mode is found mostly near the eigenfrequencies.

The band structure for the wave propagation in direction Ox_1 is displayed in Figure 8. The reduced unit of frequency is $\omega a/2\pi c_0$ with c_0 the speed of sound in air. For the frequency within this gap, there is no allowed waves mode in the material, regardless of k.

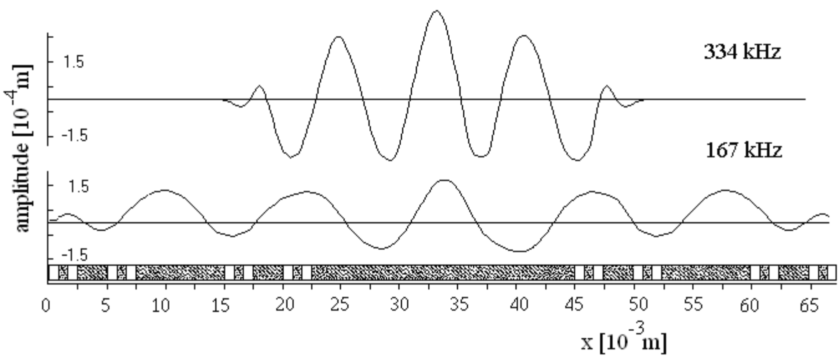

Figure 5. Lamb displacement of the normal and subharmonic modes

We saw that the sonic film of alternating layers of material following a triadic Cantor sequence can prohibit the propagation of surface waves in Ox_1 direction. The band-gaps are generated in the band structure of the film, meaning that the waves are forbidden to propagate with certain frequencies in Ox_1 direction. Alongside of Lamb waves there is another kind of motion of particles, namely in-plane but in a direction perpendicular to the direction of wave propagation. These are Love waves (SH waves).

We are interested to investigate if the band-gap can be extended to cover the Love waves that are propagating in the Ox_1 direction. By overlapping of all band-gaps, a full band-gap can be resulting.

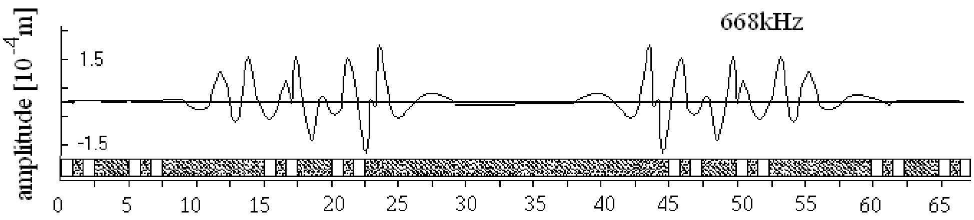

Figure 6. The fracton regime

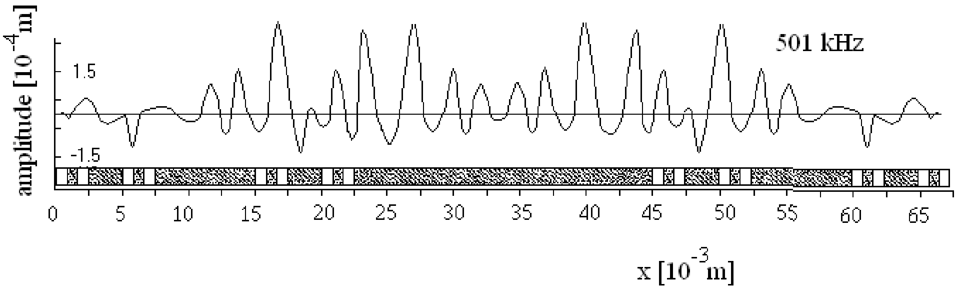

Figure 7. The phonon regime

The Love band structure is presented in Figure 9 to the left, and the full band-gap to the right. We see that the overlapping is not too successfully in that the full band-gap has irregular shape that tangles the applications. An important topic that benefits the full band-gap generation is the presence of defects into the material. The points and line defects can correct the shape of the full band-gap because they may permit the localized modes to exist, with frequencies inside the band-gaps. If a mode has a frequency in the gap, then it must exponentially decay once it enters the film. The defects can terminate the evanescent modes with exponential growth, to sustain also an evanescent mode.

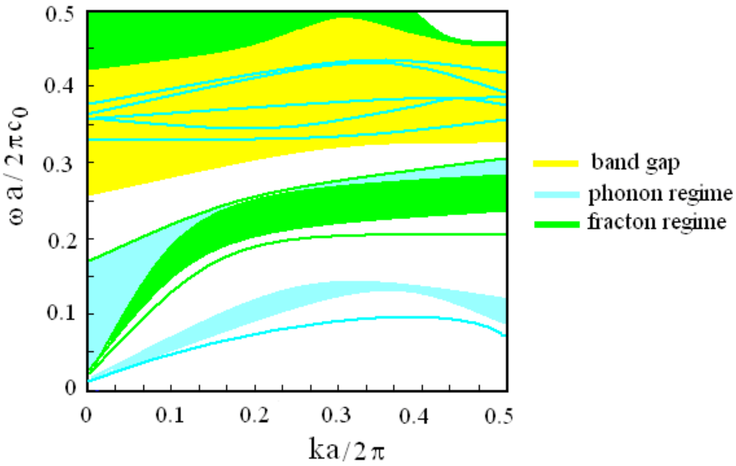

Figure 8. Lamb band structure

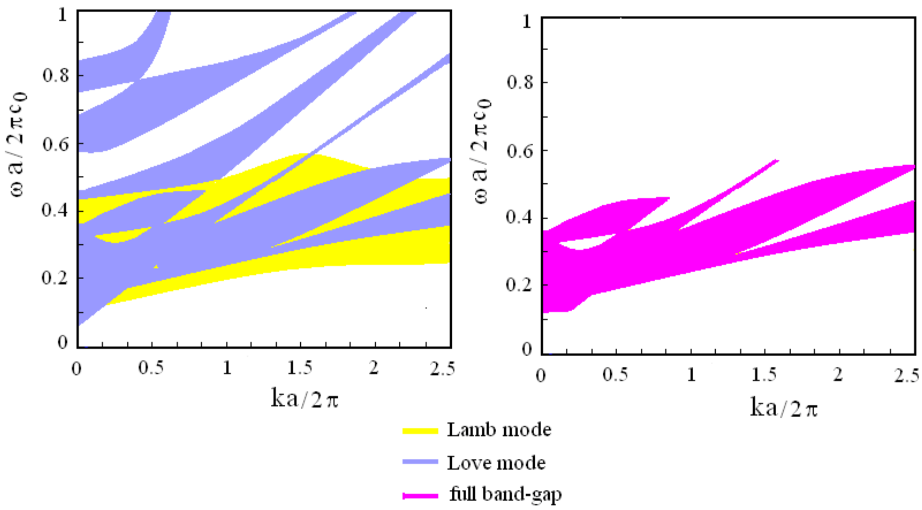

Figure 9. Love band structure (left) and the full band-gap (right)

4. Conclusion

The sonic composites exhibit important features such as full band-gaps where the waves (sound) is not allowed to propagate due to complete reflections, and modes that are localized around defects. The band-gaps or the Bragg reflections occur at different frequencies inverse proportional to the central distance between two scatterers. If the band-gaps are not wide enough, their frequency ranges do not overlap. Consequently, any wave is reflected completely from this periodic structure in the frequency range where all the band-gaps for the different periodical directions overlap. The key of the band-gap generation is the lack of purely real wave vector for certain modes of waves at certain frequencies. The wave amplitude may decay exponentially sustaining an evanescent mode, or can increase exponentially and a defect can terminate this exponential growth to sustain also an evanescent mode.

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