ANALELE UNIVERSITĂȚII
“EFTIMIE MURGU” REȘIȚA
ANUL XXII, NR. 2, 2015, ISSN 1453 - 7397

Cornel Hatiegan, Marian-Dumitru Nedeloni, Cristinel Popescu, Marius Tufoi, Ioan Pădureanu, Cristian Rudolf

Comparative Study through Modal Analysis of Thin Trapeze Shape Plates Clamped on Contour without and with Damages

This paper presents a comparative study of four types of plates in the form of trapeze, clamped on the contour, regarding modal analysis. After this analysis, there have been obtained the Eigen frequencies for 30 vibration modes between the limits 183.37 ÷ 1206.3 [Hz]. For these results there has been computed the absolute variation, respectively the relative frequency shifts of the analyzed plates.

Keywords: *Comparative study, thin trapeze shape plates, clamped on contour, damages*

1. Introduction

In practical engineering applications, different plates, concerning the geometric form, leaning type, section forms as well as their physical and mechanical characteristics are used [1].

Many studies in the specialty literature have focused on the studies of plates in the static and dynamic behavior, being established methods and models of modal analysis of the plates for different leaning ways, as well as for the deformed shape of the plates [2] - [5].

Also, about the behavior of plates, with or without damages, there have been done studies regarding [6] - [12]: the dynamic analysis/dynamic behavior of them; the modal analysis (simulating through the finite element method) of the Eigen frequencies and the vibration mode shapes and the identification of the damages in plates by analytical and experimental methods.

In the present paper, a comparative study is desired, by modal analysis, of four types of plates (4 cases), clamped on the contour, regarding the Eigen frequencies and vibration modes, as well as the differences that appear after the results obtained.

In this sense, for the modal analysis, on has chosen a trapeze shape plate, without an imposed damage, with the following dimensions expressed in [mm]: long axis 1500, short axis 500, height 250 and thickness 2.

The area of this plate, with the dimensions described above is of 250000 mm^2, being computed with the relation (1):

$$A = \frac{(B+b)\cdot h}{2}, \qquad (1)$$

Where:

- B, is the long axis of the trapeze;
- b, is the short axis of the trapeze;
- h, is the height of the trapeze.

For simulating the other 3 analyzed cases, has considered the same plate but with 3 damages of the same dimension 10 x 5 [mm], in 3 different places, namely: a lateral side of the plate, respectively at the long axis and at the short axis.

2. The modal analysis of plates

The modal analysis has been done by the finite element method with the SolidWorks software [13], regarding the 3D designing and simulation of plates.

From the SolidWorks software, there has been used the module of static analysis for thin plates under the action of their own weight at a chosen temperature of 20 ^{o}C, in which there have been determined 30 vibration mode shapes.

Also, the chosen material was 1023 Carbon Steel (the properties being described in table 1), and for the mesh, one has used the option High for Mesh Quality. Regarding the mesh information concerning the number of nodes and elements, these are described in table 2.

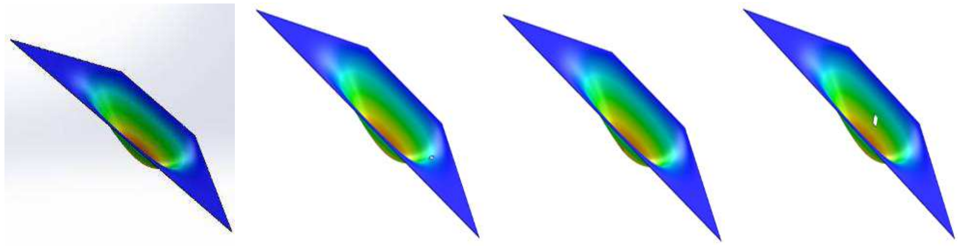
Table 1. Properties of the 1023 Carbon Steel

Elastic Modulus	Shear Modulus	Mass Density	Poisson's Ratio
[N/mm^2]	[N/mm^2]	[kg/m^3]	[-]
205000	80000	7858	0.29

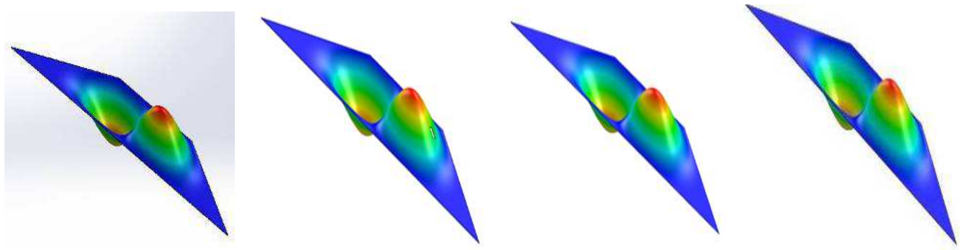
Table 2. Mesh information of the analyzed plate types

Number of nodes/elements	Type of plate			
	Without damage	Lateral damage	Long axis damage	Short axis damage
Nodes	61243	61691	61133	60682
Elements	30112	30449	30033	29684

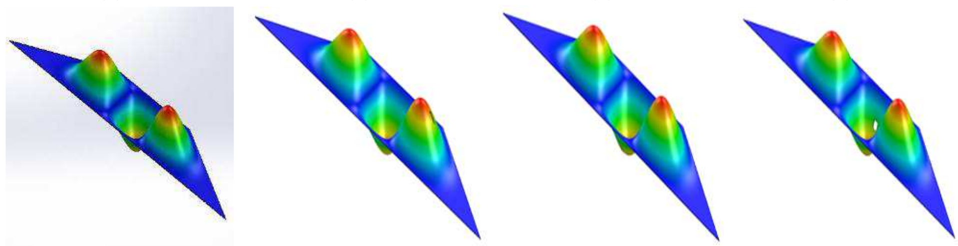
In figure 1, there are comparatively presented the 30 vibration modes, in the following order: plate without damage, plate with lateral damage, plate with damage at the long axis, respectively plate with damage at the short axis.



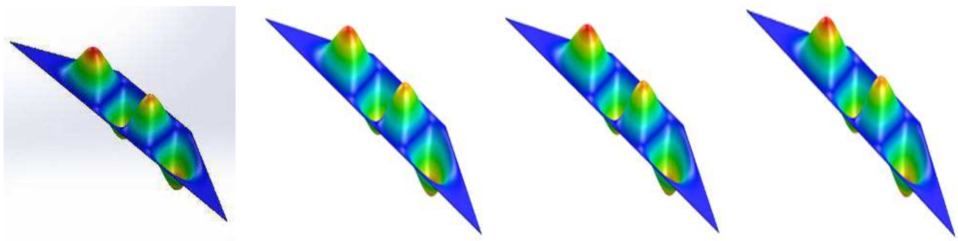
Mode 1 Mode 1 Mode 1 Mode 1



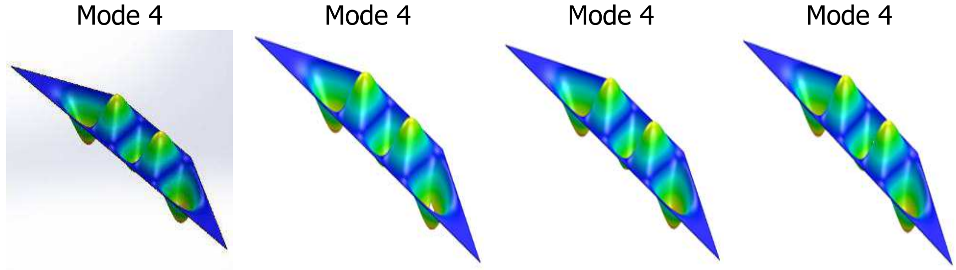
Mode 2 Mode 2 Mode 2 Mode 2



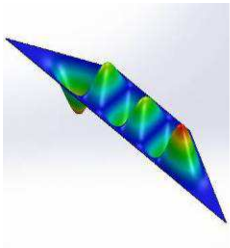
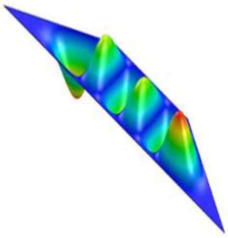
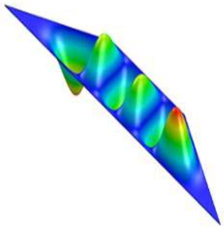
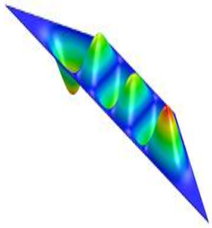
Mode 3 Mode 3 Mode 3 Mode 3



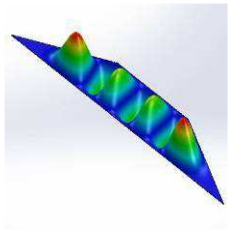
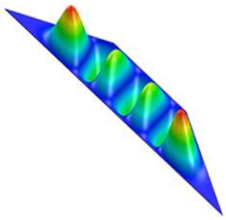
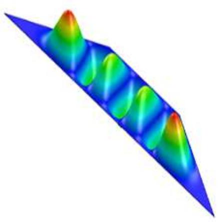
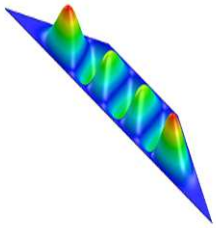
Mode 4 Mode 4 Mode 4 Mode 4



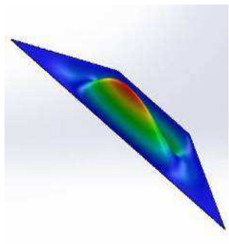
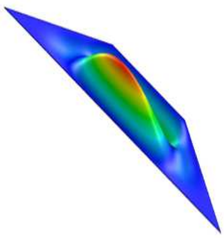
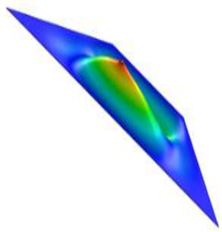
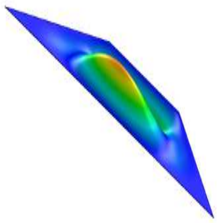
Mode 5 Mode 5 Mode 5 Mode 5

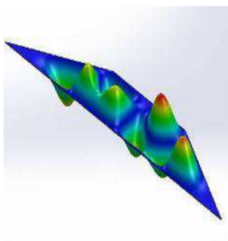
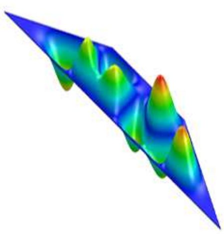
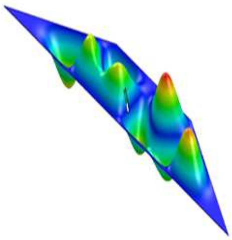
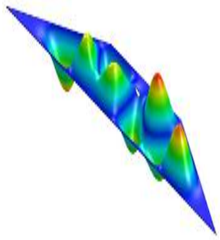
Mode 6 Mode 6 Mode 6 Mode 6

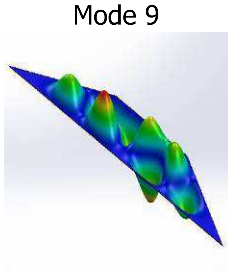
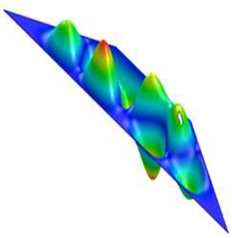
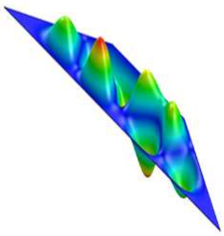
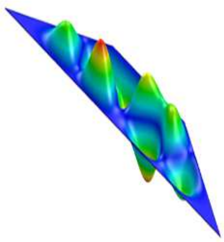
Mode 7 Mode 7 Mode 7 Mode 7

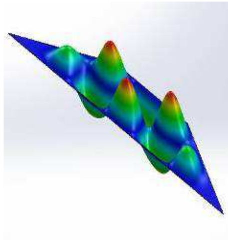
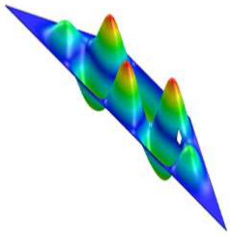
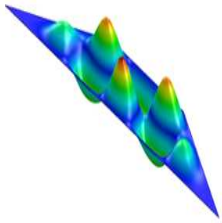
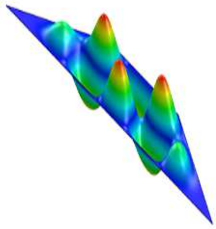
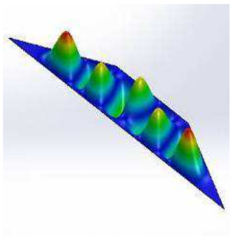
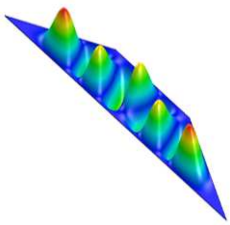
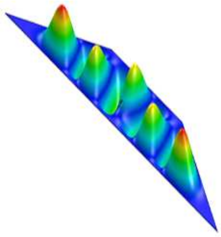
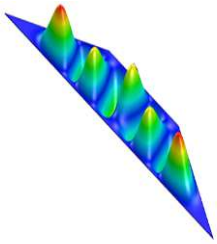
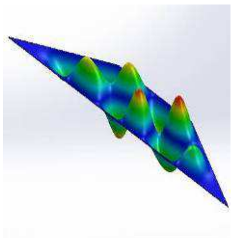
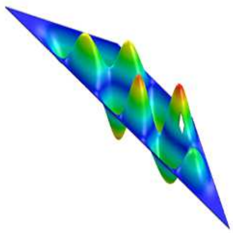
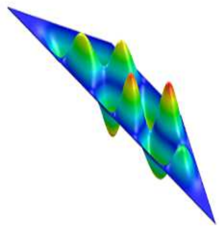
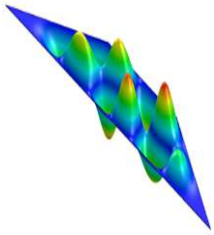
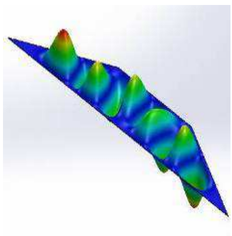
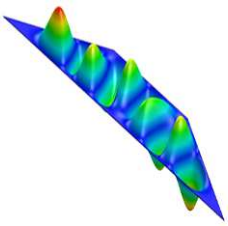
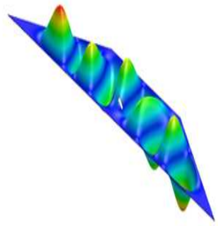
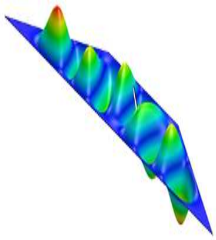
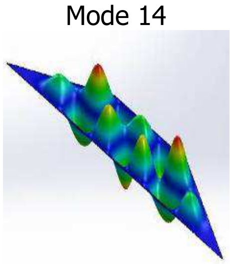
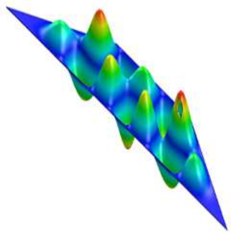
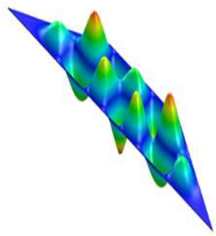
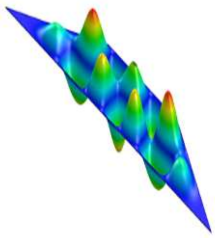
Mode 8 Mode 8 Mode 8 Mode 8

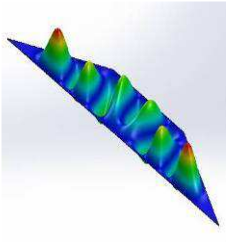
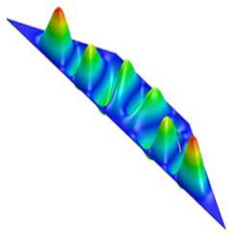
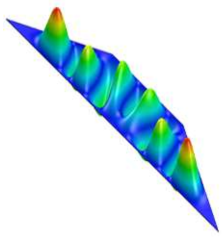
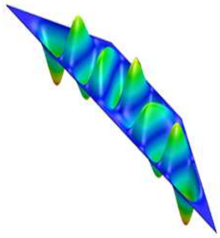
   

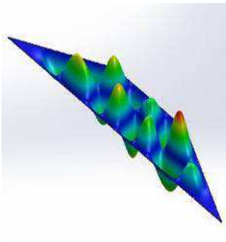
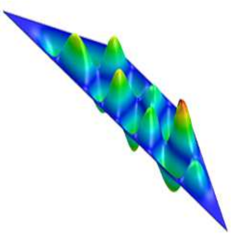
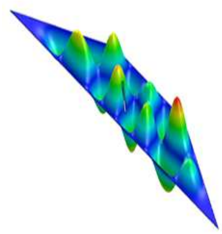
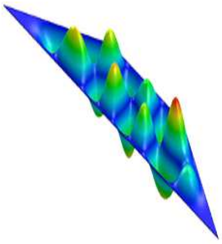
Mode 9 Mode 9 Mode 9 Mode 9

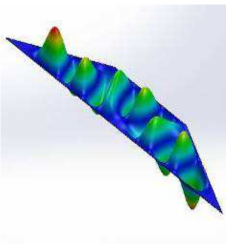
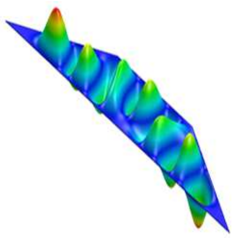
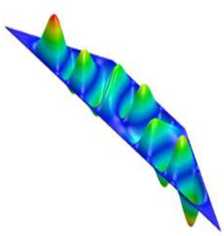
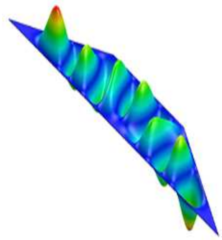
   

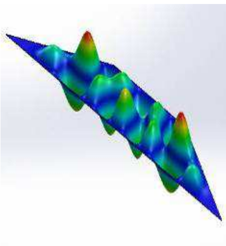
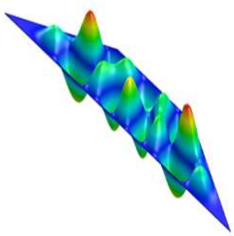
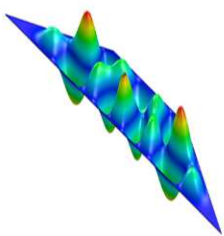
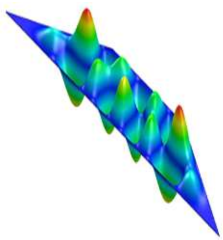
Mode 10 Mode 10 Mode 10 Mode 10

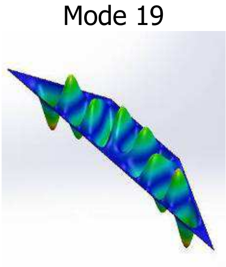
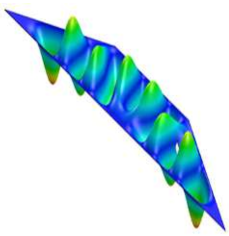
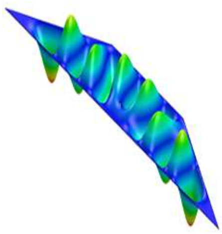
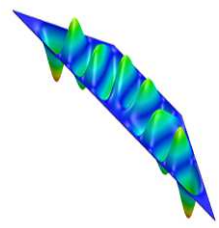

Mode 11

Mode 11

Mode 11

Mode 11

Mode 12

Mode 12

Mode 12

Mode 12

Mode 13

Mode 13

Mode 13

Mode 13

Mode 14

Mode 14

Mode 14

Mode 14

Mode 15

Mode 15

Mode 15

Mode 15

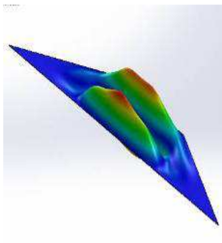
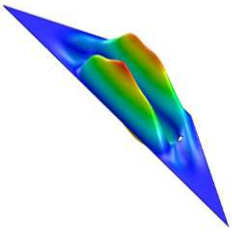
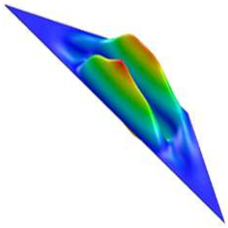
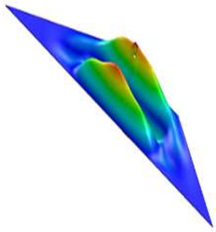

Mode 16

Mode 16

Mode 16

Mode 16

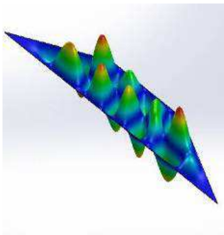
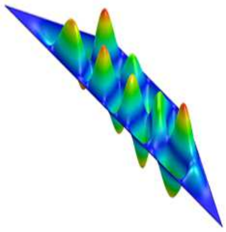
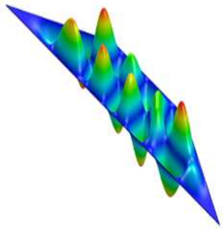
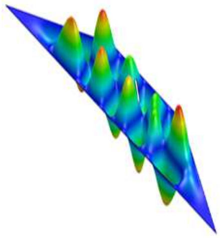

Mode 17

Mode 17

Mode 17

Mode 17

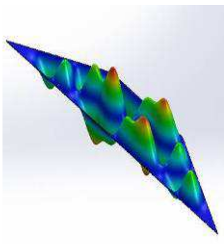
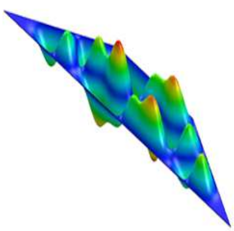
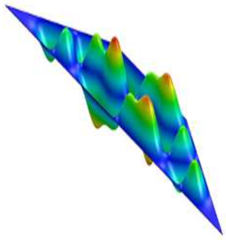
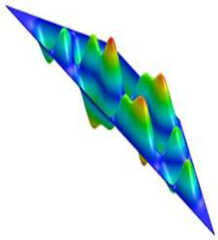

Mode 18

Mode 18

Mode 18

Mode 18

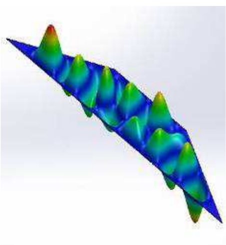
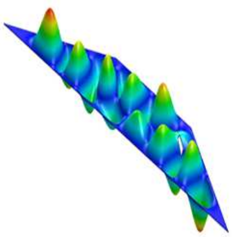
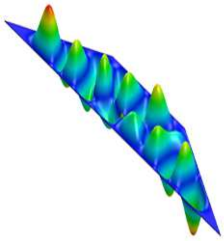
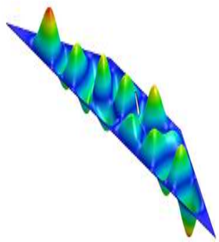

Mode 19

Mode 19

Mode 19

Mode 19

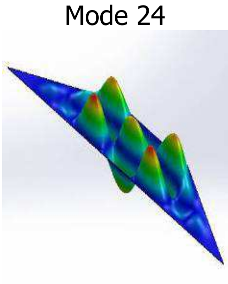
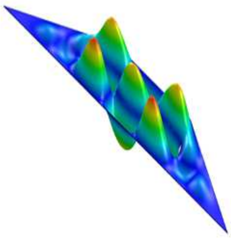
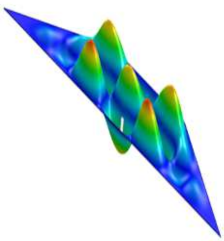
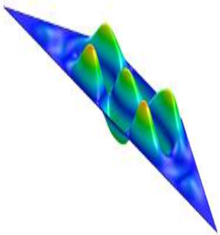

Mode 20

Mode 20

Mode 20

Mode 20


Mode 21

Mode 21

Mode 21

Mode 21


Mode 22

Mode 22

Mode 22

Mode 22


Mode 23

Mode 23

Mode 23

Mode 23


Mode 24

Mode 24

Mode 24

Mode 24


Mode 25

Mode 25

Mode 25

Mode 25

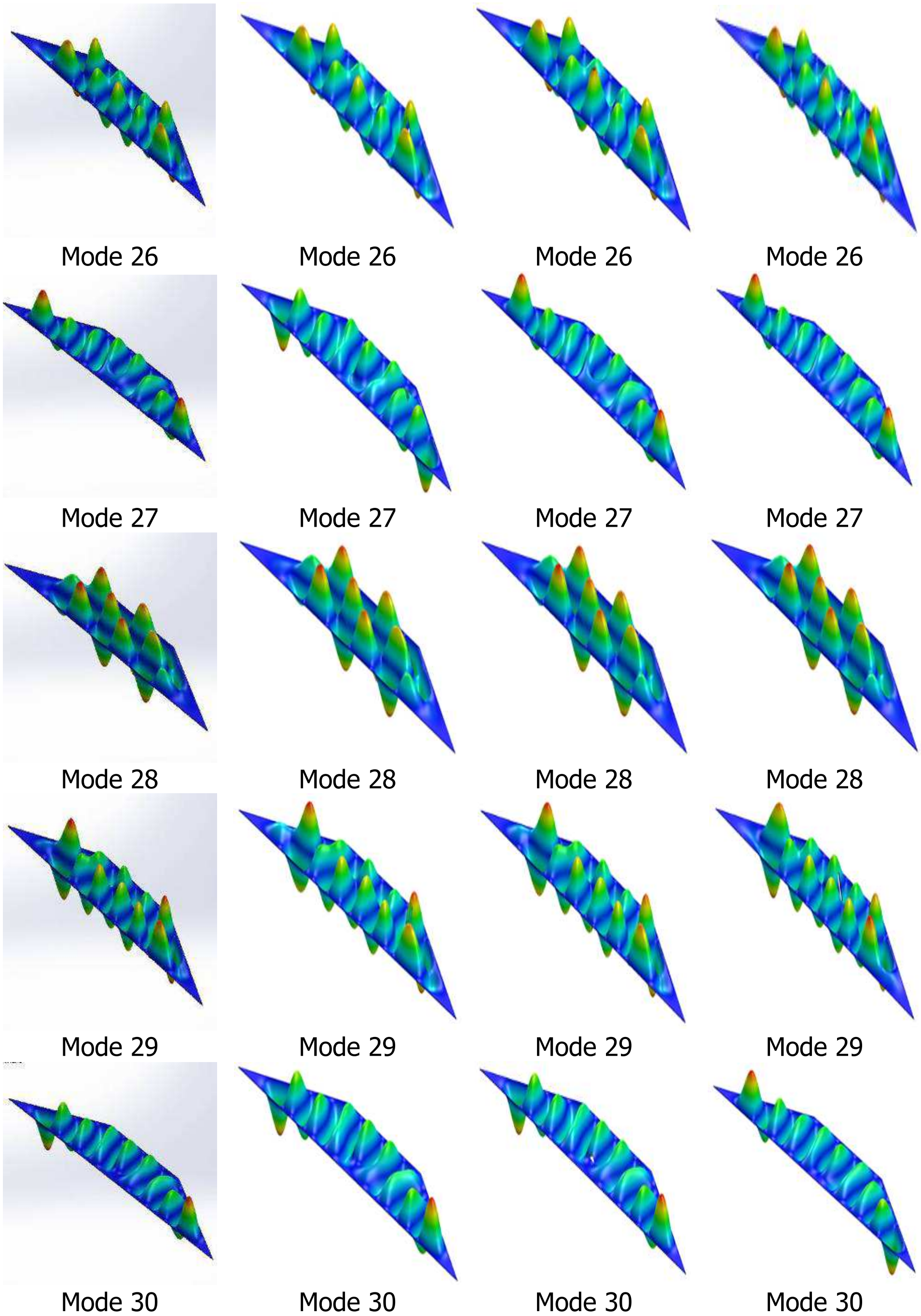


Figure 1. The vibration modes for the 4 cases.

3. Simulating results

The obtained results by simulation for the Eigen frequencies of the first 30 vibration modes described in figure 1 are numerically presented in table 3.

Table 3. The Eigen frequencies for the 4 cases [Hz]

Vibration mode	Plate without damage	Plate with lateral damage	Plate with damage at the long axis	Plate with damage at the short axis
M 1	183.39	183.37	183.5	183.53
M 2	204.87	204.85	204.73	204.39
M 3	241.48	241.45	241.48	240.81
M 4	289.74	289.6	289.44	288.39
M 5	343.25	343.17	343.07	342.03
M 6	399.04	399	398.78	397.5
M 7	459.96	459.9	459.63	458.37
M 8	497.16	497.08	495.18	495.79
M 9	524.81	524.68	524.61	523.49
M 10	529.85	529.46	529.37	527.77
M 11	575.22	575.05	573.52	571.7
M 12	600.5	600.06	599.98	597.91
M 13	641.95	641.82	641.29	637.44
M 14	676	675.83	675.56	672.84
M 15	722.31	721.86	720.97	716.26
M 16	754.98	754.75	754.35	751.25
M 17	806.62	805.81	806.07	800.94
M 18	837.6	837.46	837.1	833.31
M 19	891.59	891.1	890.82	885.44
M 20	923.97	924.04	923.48	919.24
M 21	968.84	968.78	967.28	965.47
M 22	982.65	982.75	982.3	975.58
M 23	1006.1	1006.1	1005.4	1003.2
M 24	1015.2	1015.5	1014.7	1010.3
M 25	1066.9	1067.6	1066.4	1058
M 26	1087	1087.9	1086.3	1078.9
M 27	1108	1108	1107.4	1102.9
M 28	1149.2	1149.9	1147.4	1138.4
M 29	1194.9	1195.9	1194.8	1184.9
M 30	1206.2	1206.3	1205.8	1201.1

For these Eigen frequencies, according to the relation (2) there are computed the absolute variation (table 4) that appear between the plate with lateral damage, the plate with damage at the long axis, respectively the plate with damage at the short axis and the plate without damage (Δf_1, Δf_2 and Δf_3).

$$\Delta f = \left| f_a - f_b \right|, \tag{2}$$

Where:

Δf - Represents the absolute variation for the values of the Eigen frequencies of the plates with damages towards the plate without damage;

f_a - The value of the Eigen frequency for the plate without damage;

f_b - The value of the Eigen frequency for the plate with lateral damage, the plate with damage at the long axis and the plate with damage at the short axis.

Table 4. The absolute variation of Eigen frequencies of the plates [Hz]

Vibration mode	Δf_1	Δf_2	Δf_3
M 1	0.02	0.11	0.14
M 2	0.02	0.14	0.48
M 3	0.03	0	0.67
M 4	0.14	0.3	1.35
M 5	0.08	0.18	1.22
M 6	0.04	0.26	1.54
M 7	0.06	0.33	1.59
M 8	0.08	1.98	1.37
M 9	0.13	0.2	1.32
M 10	0.39	0.48	2.08
M 11	0.17	1.7	3.52
M 12	0.44	0.52	2.59
M 13	0.13	0.66	4.51
M 14	0.17	0.44	3.16
M 15	0.45	1.34	6.05
M 16	0.23	0.63	3.73
M 17	0.81	0.55	5.68
M 18	0.14	0.5	4.29
M 19	0.49	0.77	6.15
M 20	0.07	0.49	4.73
M 21	0.06	1.56	3.37
M 22	0.1	0.35	7.07
M 23	0	0.7	2.9
M 24	0.3	0.5	4.9
M 25	0.7	0.5	8.9
M 26	0.9	0.7	8.1
M 27	0	0.6	5.1
M 28	0.7	1.8	10.8
M 29	1	0.1	10
M 30	0.1	0.4	5.1

Due to the close values of the Eigen frequencies (table 3), as well as the absolute variation highlighted (which is very small - table 4), one desires the computing of the relative frequency shifts for the analyzed plates with the relation [15]:

$$\Delta f^* = \frac{\Delta f}{f_a} = \frac{|f_a - f_b|}{f_a} \cdot 100 \ [\%], \qquad (3)$$

Where:

Δf^* - Represents the relative frequency shifts for the values of the Eigen frequencies with damage towards the one without damage. The other notations of this mathematic relation have been described in relation (2).

The values for the relative frequency shifts are presented in table 5, where following notations have been used:

Δf_1^* - The relative frequency shifts between the plate with lateral damage and the plate without damage;

Δf_2^* - The relative frequency shifts between the plate with damage at the long axis and the plate without damage;

Δf_3^* - The relative frequency shifts between the plate with damage at the short axis and the plate without damage.

Table 5. The relative frequency shifts for the analyzed plates [%]

Vibration mode	Δf_1^*	Δf_2^*	Δf_3^*	Vibration mode	Δf_1^*	Δf_2^*	Δf_3^*
M 1	0.011	0.060	0.076	M 16	0.030	0.083	0.494
M 2	0.010	0.068	0.234	M 17	0.100	0.068	0.704
M 3	0.012	0.000	0.277	M 18	0.017	0.060	0.512
M 4	0.048	0.104	0.466	M 19	0.055	0.086	0.690
M 5	0.023	0.052	0.355	M 20	0.008	0.053	0.512
M 6	0.010	0.065	0.386	M 21	0.006	0.161	0.348
M 7	0.013	0.072	0.346	M 22	0.010	0.036	0.719
M 8	0.016	0.398	0.276	M 23	0.000	0.070	0.288
M 9	0.025	0.038	0.252	M 24	0.030	0.049	0.483
M 10	0.074	0.091	0.393	M 25	0.066	0.047	0.834
M 11	0.030	0.296	0.612	M 26	0.083	0.064	0.745
M 12	0.073	0.087	0.431	M 27	0.000	0.054	0.460
M 13	0.020	0.103	0.703	M 28	0.061	0.157	0.940
M 14	0.025	0.065	0.467	M 29	0.084	0.008	0.837
M 15	0.062	0.186	0.838	M 30	0.008	0.033	0.423

For the values obtained/computed in tables 3, 4, and 5 we continue with their graphic representation in figures 2, 3, respectively 4.

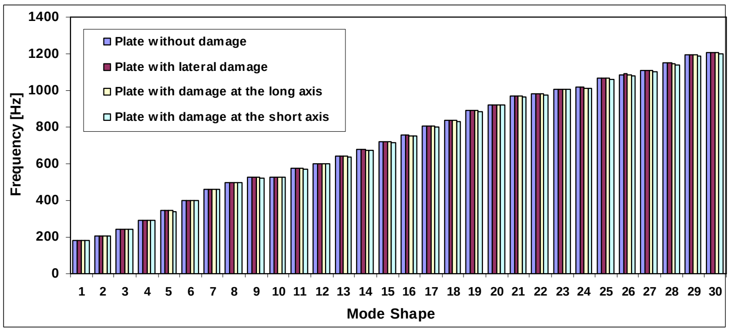


Figure 2. The Eigen frequencies for the 4 type of plates.

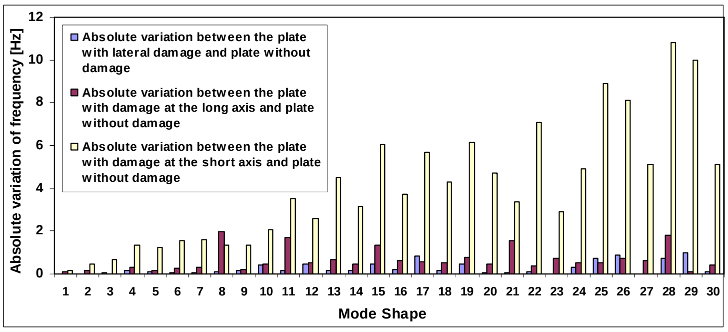


Figure 3. The absolute variations of Eigen frequencies for the analyzed plates.

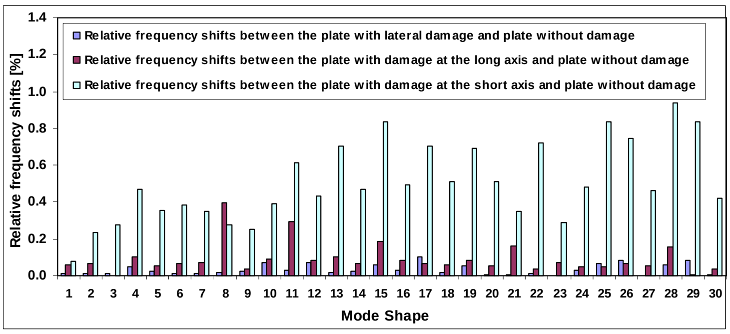


Figure 4. The relative frequency shifts of the analyzed plates.

4. Conclusion

From the results obtained by modal analysis and analytical results that highlight the comparative study of the four types of plates analyzed, the following conclusions can be made, with regard to:

- plates with damage at the short axis, which had significantly different values compared to other plates with damages for the number of nodes and elements, for Eigen frequencies or absolute variation and relative frequency shifts;

- plates with lateral damages and the plates with damages at the long axis, which had values very appropriated to the plates without damage sometimes on absolute variation for plate with lateral damage there were no differences, that value was zero;

- frequency values obtained from simulation, between the limits 183.37 ÷ 1206.3 [Hz], which had a rate of increase in 30 vibration modes chosen.

For another comparison study by modal analysis of the plates we can simulate other geometric shapes, we may choose other positions and damages, and for better accuracy of these results there can be done experimental researches on specialized stands regarding the recording of eventual deviations.

Acknowledgments

The work has been funded by the Sectorial Operational Program Human Resources Development 2007-2013 of the Ministry of European Funds through the Financial Agreement POSDRU/159/1.5/S/132395.

References

[1] Vrabie M., *Calculul plăcilor (Plates computing)*, Editura Societăţii Academice Matei-Teiu-Botez, Iaşi, 2011.
[2] Choi S.H., *Development of Nondestructive Damage Detection Algorithms for Plate Structures*, Strength of Materials, vol. 6, no. 4, 2002, 495-501.
[3] Bratu P., *Analiza structurilor elastice. Comportarea la acţiuni statice şi dinamice (Elastic Structure Analyses. Static and Dynamic Actions Behavior)*, Editura Impuls, Bucureşti, 2011.
[4] Radeş M., *Vibraţii mecanice (Mechanical vibration)*, Editura Printech, Bucureşti, 2008.
[5] Haţiegan C., *Identification of Damages in Thin Elastic Plates by Means of Modal Analysis*, PhD Thesis, University "Eftimie Murgu" of Resita, 2013.
[6] Tufoi M., Hatiegan C., Gillich G.R., Protocsil C., Negru I., *Frequency Changes in Thin Rectangular Plates due to Geometrical Discontinuities. Part I: Finite Element Analysis*, Analele Universitatii "Eftimie Murgu" Resita, year XX, no. 3, 2013, 221-232.
[7] Tufoi M., Hatiegan C., Vasile O., Gillich G. R., *Dynamic Analysis of Thin Plates with Damages by Experimental and FEM Methods*, Romanian Journal of Acoustics and Vibration, year 10, no. 2, 2013, 83-88.
[8] Tufoi M., Gillich G. R., Praisach Z.I., Ntakpe J.L., Hatiegan C., *An Analysis of the Dynamic Behavior of Circular Plates from a Damage De-*

tection Perspective, Romanian Journal of Acoustics and Vibration, year 11, no. 1, 2014, 41-46.

[9] Nedelcu D., Ianici D., Nedeloni M.D., Daia D., Pop F.M., Avasiloaie R.C, *The Aerodynamic Force Calculus for a Plate Immersed in a Uniform Air Stream Using SolidWorks Flow Simulation Module*, Proceedings of the 4th WSEAS International Conference on Finite Differences-Finite Elements-Finite Volumes- Boundary Elements, 2011, 98-103.

[10] Nedeloni M.D., Nedelcu D., Ion I., Ciobotariu R., *Calibration of a Sonotrode from a Stand Component for Test Cavitation Erosion through Direct Method.*, Constanta Maritime University Annals, year 13, issue 17, 2012, 125-130.

[11] Nedeloni M.D., Nedelcu D., *Variants of Sonotrode for a Vibratory Apparatus for Test Cavitation Erosion by the Indirect Method*, Constanta Maritime University Annals, year 13, issue 17, 2012, 131-136.

[12] Elshafey A.A., Marzouk H., Haddara M.R., *Experimental Damage Identification Using Modified Mode Shape Difference*, J. Marine Sci. Appl., vol. 10, 2011, 150-155.

[13] Nedelcu D., *Proiectare şi simulare numerică cu SOLIDWORKS (Digital Prototyping and Numerical Simulation with Solid-Works)*, Editura Eurostampa, Timişoara, 2011.

[14] Haţiegan C., Suciu L., *Fizică tehnologică. Teorie şi aplicaţii (Technological Physics. Theory and Applications)*, Editura Eftimie Murgu, Reşiţa, 2010.

[15] Gillich G.R., Praisach Z.I., *Robust method to Robust method to identify damages in beams based on frequency shift analysis*, SPIE Smart Structures and Materials & Nondestructive Evaluation and Health Monitoring, San Diego, USA, 2012, 83481D-12.

Addresses:

- Lect. PhD. Phys. Cornel Haţiegan, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, c.hatiegan@uem.ro
- Assist. PhD. Eng. Marian-Dumitru Nedeloni, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, m.nedeloni@uem.ro
- Assoc. Prof. PhD. Eng. Cristinel Popescu, "Constantin Brâncuși" University of Târgu Jiu, Calea Eroilor, No. 30, Târgu Jiu, cristi67pop@yahoo.com
- PhD. Eng. Marius Tufoi, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, m.tufoi@uem.ro
- Assist. PhD. student Eng. Ioan Pădureanu, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, i.padureanu@uem.ro
- Lect. PhD. Cristian Rudolf, "Eftimie Murgu" University of Reşiţa, Piaţa Traian Vuia, nr. 1-4, 320085, Reşiţa, c.rudolf@uem.ro