ANALELE UNIVERSITĂȚII
"EFTIMIE MURGU" REȘIȚA
ANUL XXII, NR. 2, 2015, ISSN 1453 - 7397

Iulian Girip, Ruxandra Diana Ilie

Acoustical Wave Propagation in Sonic Composites

The goal of this paper is to discuss the technique of controlling the mechanical properties of sonic composites. The idea is to architecture the scatterers and material from which they are made, their number and geometry in order to obtain special features in their response to external waves. We refer to perfectly reflecting of acoustical waves over a desired range of frequencies or to prohibit their propagation in certain directions, or confining the waves within specified volumes. The internal structure of the material has to be chosen in such a way that to avoid the scattering of acoustical waves inside the material. This is possible if certain band-gaps of frequencies can be generated for which the waves are forbidden to propagate in certain directions. These band-gaps can be extended to cover all possible directions of propagation by resulting a full band-gap. If the band-gaps are not wide enough, their frequency ranges do not overlap. These band-gaps can overlap due to reflections on the surface of thick scatterers, as well as due to wave propagation inside them. growth.

Keywords: *sonic composite, full band-gap, control*

1. Introduction

By definition, a sonic composite is a periodic array of scatterers embedded in the epoxy material. The sonic composite exhibits a large sound attenuation band (full band-gap) obtained through superposition of multiple reflected waves within the array according to Bragg's theory. Bragg reflections occur at different frequencies inverse proportional to the central distance between two scaterers. The local band-gaps can overlap to generate the full band-gap independent of the incident angle [1-7].

The rationale for the band-gap generation is the lack of purely real wave vector for certain modes of waves at certain frequencies and in consequence, the evanescent waves distributed across the boundary of the waveguide [8-11]. The wave amplitude may decay exponentially sustaining an evanescent mode, or can increase exponentially and a defect can terminate this exponential growth to sustain also an evanescent mode [12].

Complete sound atenuation for a certain frequency range can be obtained tyhrough controlling the "classical wave spectral gap", originally introduced for electromagnetic waves (photonic band gap). The idea is related to a strong periodic modulation in sound velocity in order to create spectral gaps that forbid wabve propagation [13]. We refer to perfectly reflecting of acoustical waves over a desired range of frequencies or to prohibit their propagation in certain directions, or confining the waves within specified volumes.

2. Bloch's theorem

The Bloch's theorem states that the response of a sonic composite with periodical scatterers is characterized by the response of the unit cell generator (a unit scatterer) which is used to build the composite in the repetitive way in order to obtain periodicity. The wave propagation in the composite can be described by the wave propagation in the unit scatterer [14].

Consider that the periodic 2D sonic structure is consisting of an array of rectangular acoustic scatterers embedded in an epoxy matrix. This structure is built as a repetition of a unit rectangular scatterer of length a and b in the directions d_1 and d_2 (Figure 1 left). Each point P in the structure has a corresponding point Q in the unit scatterer. The structure is generated by translating a number of n_1 scatterers along d_1 and a number of scatterers n_2 along d_2

$$r_P = r_Q + n_1 d_1 + n_2 d_2 \,. \tag{1}$$

Block theorem: *The response $u(r_P, \omega)$ of a 2D periodic structure is expressed in terms of the response of the unit scatterer and an exponential term which defines the amplitude and the phase change of the wave propagating from one scatterer to the next*

$$u(r_P, \omega) = u_0(r_Q, \omega) \exp k(n_1 d_1 + n_2 d_2) \,. \tag{2}$$

where $u_0(r_Q, \omega)$ is the reference response of the unit scatterer, ω is the angular frequency and k is the wave vector.

The propagation vector $\mu(\mu_1, \mu_2)$

$$\mu = kd \,, \tag{3}$$

defines the complex phase shifts in the directions d_1 and d_2.

With (3), Eq. (2) becomes

$$u(r_P, \omega) = u_0(r_Q, \omega)\exp \mu n\,. \tag{4}$$

where n is a vector which indicated the number of scatterers translated in both directions.

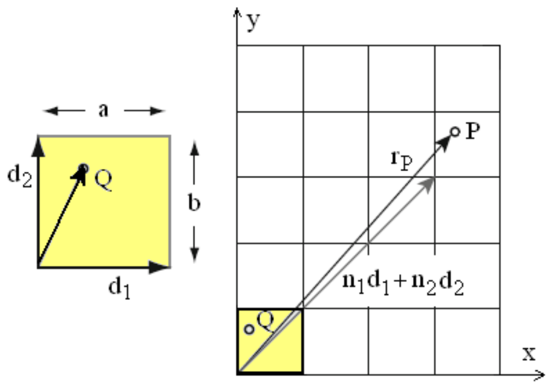

Figure 1. The unit scatterer of the structure (left) and the periodic structure with many cells (right).

3. Modeling of unit scatterer

The finite element method is suitable to be used to describe the behavior of the unit scaterrer. The steady-state harmonic response of the unit scatterer is written in the matrix form

$$(K - \omega^2 M)q = f\,, \tag{5}$$

where K is the stiffness matrix, M is the mass matrix, q the generalised displacements vector and f is the generalised forces.

Displacement vector q can be reduces in a simplified form q_{red}, when taking into account the boundary conditions on the unit scatterer. In this case, Eq. (5) can be written in the simplified form, if no external forces are applied on the internal nodes of the structure

$$(K_{red} - \omega^2 M_{red})q_{red} = 0\,, \tag{6}$$

where index *red* refers to the reduced form of the matrices.

The key of the band-gap generation is the lack of purely real wave vector for certain modes of waves at certain frequencies. The wave amplitude may decay exponentially sustaining an evanescent mode [12].

Therefore, by aassuming free wave propagation for which propagation vector is strictly imaginary $\mu = i\varepsilon$, Eq. (6) is solved with respect to the angular frequency ω as a function of $i\varepsilon$.

Eq. (6) is solved for every possible combination $\mu_1 = i\varepsilon_1$ and $\mu_2 = i\varepsilon_2$. The angular frequencies form a surface in the region $(\varepsilon_1, \varepsilon_2)$, i.e. so-called the dispersion surface $\omega = f(\varepsilon_1, \varepsilon_2)$. This dispersion surface is periodic and hence not the entire dispersion surface must be analysed. Only a periodic zone in this surface, so-called the Brillouin zone, or the first Brillouin zone, must be investigated [13-17].

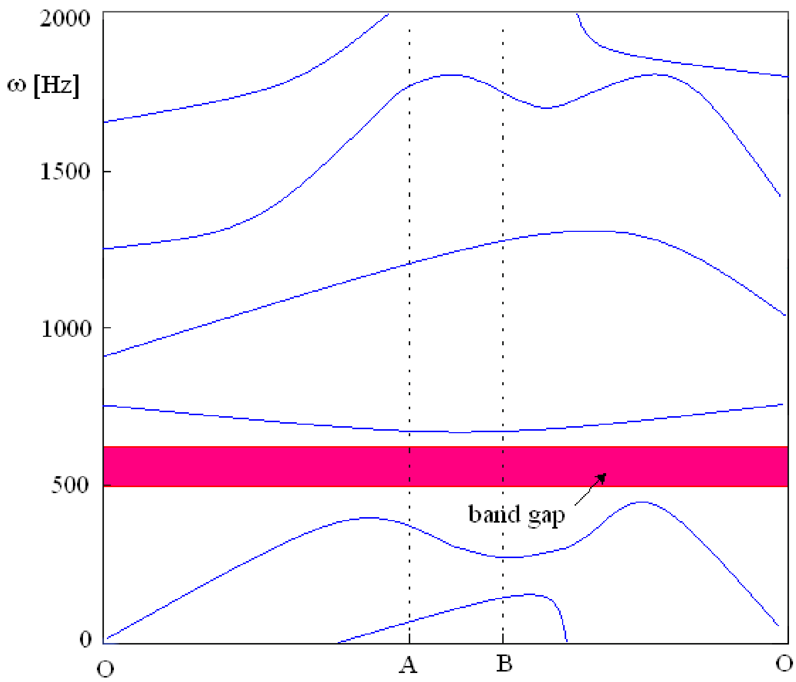

Figure 2. Dispersion curves for the unit scatterer

This surface is very important because it yields the solutions in frequencies for which the waves are propagating without attenuation. The frequencies which are not solutions of (6) are frequencies for which the waves with attenuation are possible to propagate. The last frequencies belong to band-gaps of the unit scaterrer. We can say that first Brillouin zone is the smallest zone which contains all information about the wave propagation, not only in the unit scatterer but in entire periodic structure. The first Brillouin zone contains sufficient information for determining the band-gaps. We call these band-gaps within the unit scatterer the local band-gaps.

In the 2D case, the surface $\omega = f(\varepsilon_1, \varepsilon_2)$ has corner points as O(0,0), A$(4/3\pi, 2/3\pi)$, B(π, π) and O(0,0) [18]. Figure 2 shows the dispersion curves for unit scatterer, in which we see a band-gap between 500Hz and 625Hz. The dispersion curves are calculated along contour of the first Brillouin zone.

4. Modeling of entire structure

Due to the periodic arrangement of scatterers, sonic structure has the property of selective sound attenuation in specific range of frequencies. This range of frequencies is known as the band gap, and it is found that sound propagation is significantly reduced in this band gap region. The reason for such sound attenuation is due to the destructive interference of waves in the band of frequencies. It is also shown that the propagating wave has an evanescent behavior (decaying amplitude) which causes the sound attenuation to take place in the band gap region.

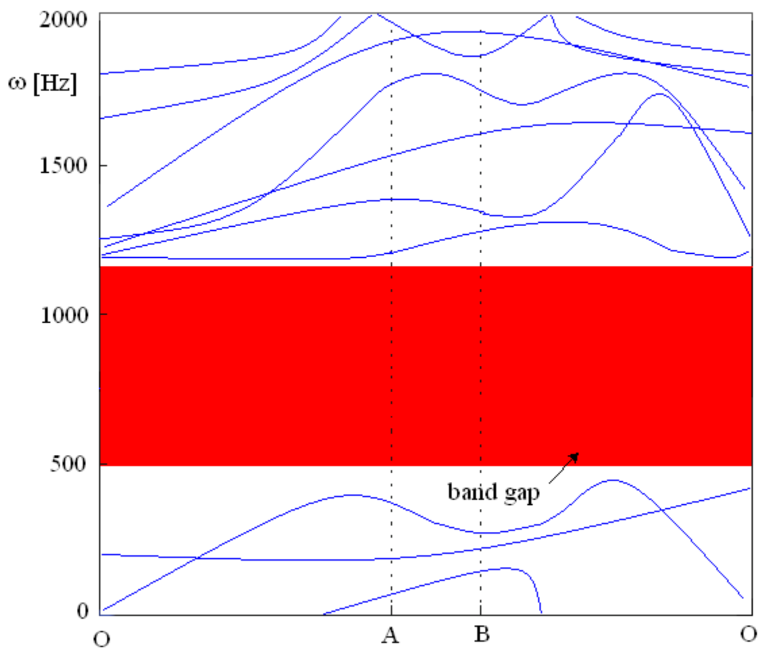

Figure 3. Dispersion curves for the structure with 16 scatterers.

For entire structure, the steady-state harmonic response is described in a similar way with (6)

$$(K-\omega^2 M)q=0\,, \tag{7}$$

where K is the stiffness matrix, M is the mass matrix, q the generalised displacements vector for entire structure.

Figure 3 shows the dispersion curves for a structure with 16 scatterers, in which we see a band-gap between 500Hz and 1130Hz. The band-gap from Figure 3 is called the full band-gap because it is obtained by superposition of all band-gaps associated to each scatterer.

From here we draw an important conclusion: If we want to have a large full band-gap, we must have many local band-gaps in the same frequency region. It depends of course on the number of scatterers. If this frequency region is small, the local band-gaps are narrow and do not overlap. As this region increases, the local band-gaps widen and begin to overlap in the same frequency region leading to a full band-gap independent of the polarization.

But the problem is not so simple. The large full band-gap depends not only on the number of scatterers but on the material the scatterers are made and on their geometry. The geometry can be diverse: spheres, hollow spheres, cylindrical shells, rods.

5. On the compression viewed as a geometric transformation

A modeling of the compression by using the property of Helmholtz equation to be invariant under geometric transformations is an alternative way to control the properties of a sonic composite [19, 20]. The compression can be theoretically controlled by the geometric transformations. As an example, new architectures for auxetic materials [21-23] can be built up by applying the geometric transformations.

Let us consider the geometric transformation from the coordinate system (x',y',z') of the compressed space to the original coordinate system (x,y,z), given by $x(x',y',z')$, $y(x',y',z')$ and $z(x',y',z')$. The change of coordinates is characterized by the transformation of the differentials through the Jacobian $J_{xx'}$ of this transformation, i.e.

$$\begin{pmatrix}\mathrm{d}x\\ \mathrm{d}y\\ \mathrm{d}z\end{pmatrix}=J_{xx'}\begin{pmatrix}\mathrm{d}x'\\ \mathrm{d}y'\\ \mathrm{d}z'\end{pmatrix},\quad J_{xx'}=\frac{\partial(x,y,z)}{\partial(x',y',z')}\,. \tag{8}$$

From the geometrical point of view, the change of coordinates implies that, in the transformed region, one can work with an associated metric tensor

$$T = \frac{J_{xx'}^{\mathrm{T}} J_{xx'}}{\det(J_{xx'})}. \tag{9}$$

We consider for example, the Helmholtz equation for the pressure waves propagating in a bounded fluid region $\Omega \subset \mathrm{R}^3$

$$\nabla \cdot (\underline{\underline{\rho}}^{-1} \nabla p) + \frac{\omega^2}{\kappa} p = 0, \tag{10}$$

where p is the pressure, $\underline{\underline{\rho}}$ is the rank-2 tensor of the fluid density, κ is the compression modulus of the fluid, and ω is the wave frequency. The equation (10) is invariant under geometric transformation (8).

In terms of the material parameters, one can replace the material from the original domain (homogeneous and isotropic) by an equivalent compressed one that is inhomogeneous (its characteristics depend on the spherical (r', θ', ϕ') coordinates) and anisotropic (described by a tensor), and whose properties, in terms of $J_{x'x}$, are given by

$$\underline{\underline{\rho}}' = J_{x'x}^{-\mathrm{T}} \cdot \rho \cdot J_{x'x}^{-1} \cdot \det(J_{x'x}), \quad \kappa' = \kappa \det(J_{x'x}). \tag{11}$$

Here, $\underline{\underline{\rho}}'$ is a second order tensor. This theory can be applied for theoretical compression of the conventional foams. The scope is to transform conventional foam which occupies a disk $r \le R_2$ into auxetic material which fills the annulus $R_1 \le r \le R_2$. The new material is inhomogeneous and anisotropic.

A linear geometric transformation (8) which maps the disk $r \le R_2$ into an annulus $R_1 \le r \le R_2$ is given by

$$r' = R_1 + r\frac{R_2 - R_1}{R_2}, \quad 0 \le r \le R_2, \ \theta' = \theta, \ 0 \le \theta \le 2\pi, \quad x_3' = x_3, \quad x_3 \in \mathrm{R}, \tag{12}$$

where r', θ', x_3' are radially contracted coordinates r, θ, x_3. The Cartesian basis (x_1, x_2, x_3) is defined as $x_1 = r\cos\theta$, $x_2 = r\sin\theta$. The original domain is a disk of radius R_2 filled with conventional non-auxetic cellular foam. The spatial compression is obtained by applying the geometric transformation (12). The transformed domain is an annulus of radii R_1 and R_2.

The most important physical parameter to dominate the negative Poisson's ratio transformation is the compression ratio $\vartheta = \frac{(R_2'^2 - R_1'^2)}{R_2^2}$, where prime denotes the final parameters. The initial domain with $R_2 = 15$mm. The dependence of the properties of the auxetic foam on the Poisson's ratio ν and the coordinates is illustrates next. The variation of the reduced moduli on ν is displayed in Figure 4. For a given ν it is possible to determine a set of permissible material constants.

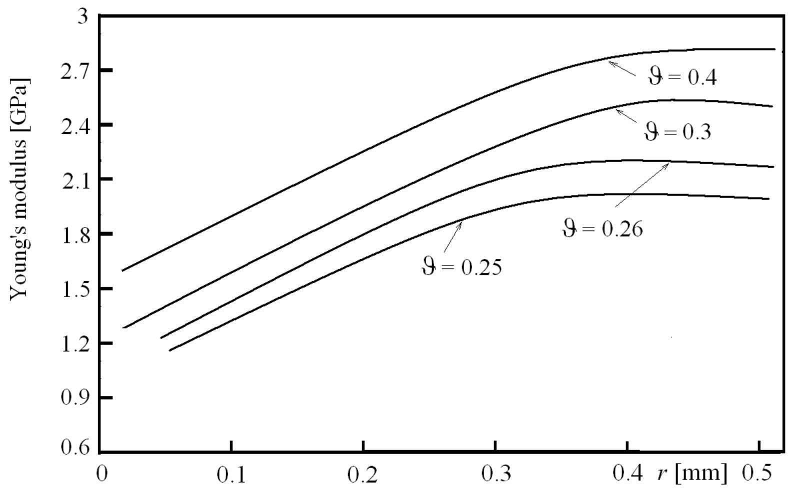

Figure 4. Variation of the Young's modulus with respect to radial coordinate

6. Conclusion

The paper discusses the way of controlling the mechanical properties of sonic composites. These properties are referring to the full band-gaps generation, where the sound is not allowed to propagate due to complete reflections. The Bloch's theorem which is used to build the composite in the repetitive way represents the main concept for the architecture of the scatterers in order to obtain special features in their response to external waves**.**

The band-gaps occur at different frequencies inverse proportional to the central distance between two scaterers. The complete reflection on the boundaries of scatterers is due to the full band-gap property itself, independent of the incident angle.

The technique for transforming the conventional foams into auxetic foams is discussed in this paper by exploiting the property of the governing equations to be written in a covariant form such that the metric is only involved in the material parameters. The geometric transformations lead to material properties that are, if not impossible to obtain, at least challenging for manufacture of new materials

Acknowledgement

The authors gratefully acknowledge the financial support of the National Authority for Scientific Research ANCS/UEFISCDI through the through the project PN-II-ID-PCE-2012-4-0023, Contract nr.3/2013.

References

[1] Girip I., Ioan R., *On the contact interfaces in the sonic composites*, PAMM-Proceedings in Applied Mathematics and Mechanics, December 2014, 357-358, work presented to 85th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM2014), March 10-14, 2014 Friedrich-Alexander Universität Erlangen-Nürnberg.

[2] Ilie R., Ioan R., Brişan C., *On the sonic composites subjected to severe acoustic loads*, PAMM- Proceedings in Applied Mathematics and Mechanics, December 2014, 687-688, work presented to 85th Annual Meeting of the International Association of Applied Mathematics and Mechanics (GAMM2014), March 10-14, 2014 Friedrich-Alexander Universität Erlangen-Nürnberg

[3] Munteanu, L., Chiroiu, V., Donescu, St., Brişan, C., *A new class of sonic composites*, Journal of Applied Physics, 115, 104904, 2014.

[4] Munteanu L., Chiroiu V., Serban V., *From geometric transformations to auxetic materials*, CMC: Computers, Materials & Continua, 42(3), 175-203, 2014.

[5] Miyashita T., Taniguchi R., Sakamoto H., *Experimental full band-gap of a sonic-crystal slab structure of a 2D lattice of aluminum rods in air*, Proc. 5th World Congress on Ultrasonics TO-PM04.02, 2003.

[6] Miyashita T., *Full band gaps of sonic crystals made of acrylic cylinders in air-numerical and experimental investigations*, Jpn. J. Appl. Phys. 41, 3170-1-3175, 2002.

[7] Miyashita T., *Sonic crystals and sonic wave-guides. A review*, Measurement Science and Technology, 16, R47-R63, 2005.

[8] Hirsekorn M., Delsanto P.P., Batra N.K., Matic P., *Modelling and simulation of acoustic wave propagation in locally resonant sonic materials*, Ultrasonics, 42, 231–235, 2004.

[9] Chiroiu V., Brişan C., Popescu M.A., Girip I., Munteanu L., *On the sonic composites without/with defects,* Journal of Applied Physics, vol. 114 (16), pp. 164909-1-10, 2013.

[10] Munteanu L., Chiroiu V., *On the dynamics of locally resonant sonic composites*, European Journal of Mechanics-A/Solids, 29(5), 871–878, 2010.

[11] Romero-García V., Sánchez-Pérez J.V., Garcia-Raffi L.M., *Evanescent modes in sonic crystals: Complex relation dispersion and supercell approximation*, Journal of Applied Physics, 108(4), 108-113, 2010.

[12] Joannopoulos J.D., Johnson S.G., Winn J.N., Meade R.D., *Photonic Crystals*, Princeton University Press, second edition, 2008.

[13] Liu Z., Zhang X., Mao Y., Zhu Y.Y., Yang Z., Chan C.T., Sheng P., *Locally resonant sonic materials*, Science, 289, 1734–1736, 2000.

[14] Claeys C.C., Vivolo M., Sas P., Desmet W., *Numerical and experimental study of local cell resonators to obtain low-frequency vibrational stopbands in periodic lightweight structures*, chapter 4 in: Topics in Modal Analysis II, vol.6, Conference Proceedings of the Society for Experimental Mechanics Series, the 30th IMAC, A Conference on Structural Dynamics, Jacksonville, Florida, Jan. 30-Feb. 2, 2012, Springer 2012.

[15] Brillouin L., *Wave propagation in periodic structures*, McGraw-Hill Book Company, New York, 1946.

[16] Cox, S.J., Dobson, D.C., *Maximizing band gaps in two-dimensional photonic crystals*, SIAM J. Appl. Math., 59(6), 2108–2120, 1999.

[17] Kittel C., *Introduction to solid state physics*, 8th edn. Wiley, New York, 2005.

[18] Srikantha P.A., Woodhouse J., Fleck N.A., *Wave propagation in two-dimensional periodic lattices*, J. Acoust. Soc. Am., 119, 1995–2005, 2006.

[19] Munteanu L., Chiroiu V., Donescu St., Brişan C., *A new class of sonic composites*, Journal of Applied Physics, 115, 104904, 2014.

[20] Munteanu L., Chiroiu V., Serban V., *From geometric transformations to auxetic materials*, CMC: Computers, Materials & Continua, 42(3), 175-203, 2014.

[21] Munteanu L., Chiroiu V., *On the three-dimensional spherical acoustic cloaking*, New Journal of Physics, 13(8), 083031, 2011.

[22] Munteanu L., Dumitriu D., Donescu St., Chiroiu V., *On the complexity of the auxetic systems*, Proc.of the European Computing Conference, Lecture Notes in Electrical Engineering, 28(2), 1543–1549, 2009.

[23] Munteanu L., Donescu St., Delsanto P.P., Dumitriu D., Mosnegutu V., *On the characterization of auxetic materials*, *Research Trends in Mechanics* vol.2, pp. 205-234, Editura Academiei (eds. D.Popa, V.Chiroiu, I.Toma), 2008.

Addresses:

- Dr. Iulian Girip, Institute of Solid Mechanics, Romanian Academy, Ctin Mille 15, Bucharest 010141, iuliangirip@gmail.com
- Drd. Ruxandra-Diana Ilie, Technical University of Civil Engineering, Dept. of Mathematics and Informatics, Bucharest, rux_i@yahoo.com