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Critical Buckling Force Variation for Beams with Discontinuities

This paper introduces a method to evaluate the critical buckling force for beams with geometrical discontinuities. First the shape of the healthy deformed beam due to axial forces is analyzed, in order to predict the distribution of the strain energy in the first buckling mode. Afterwards, the critical buckling forces for the similar beams with discontinuities positioned at different locations are determined by means of the finite element method (FEM), in order to find a similarity between the critical force drop and the damage position. It was finally demonstrated that it is a correlation between these two features; in fact, the force decrease is proportional with the energy locally stored in the beam.

Keywords: *Double-clamped beam, thermal buckling, damage*

1. Introduction

The phenomenon by which a beam with the length greater than the cross-sectional dimensions, subjected to compressive force directed along the axis, passes from its original stable equilibrium in a form of unstable equilibrium to overcome a certain amount of force called critical force, is named buckling.

Under a force greater than the critical buckling force the beam loses its stability, the average fiber deforms and in the beam bending moments appear that cause further tension. The stress caused by the bending moments occurring in compressed fibers adds up with the tensions caused by axial force.

Comparing the stress related to the critical buckling force with the elastic limit of the material of which the beam is made the buckling can be [2]:

- elastic buckling, if the stress corresponding to the critical force is less than the elastic limit of the material;

- plastic buckling, if the stress corresponding to the critical force is greater than the elastic limit of the material.

In this paper we want to study the variation of the critical force for the beams with discontinuities, when buckling occurs in the elastic range.

The literature [1]-[5] presents the buckling in the constant section beams and the buckling in the composed section beams, determining some mathematical relationships for calculating the critical buckling force in the elastic range for different supporting cases.

Next we analyze the case of the double clamped beam.

The critical buckling force [3] determined from the equation of the deformed average fiber and from boundary conditions is given by:

$$P_{cr} = \frac{(2n\pi)^2 EI_{min}}{l^2} \tag{1}$$

Where: n is a positive integer, E the modulus of elasticity, I_{min} the minimum moment of inertia of the cross section.

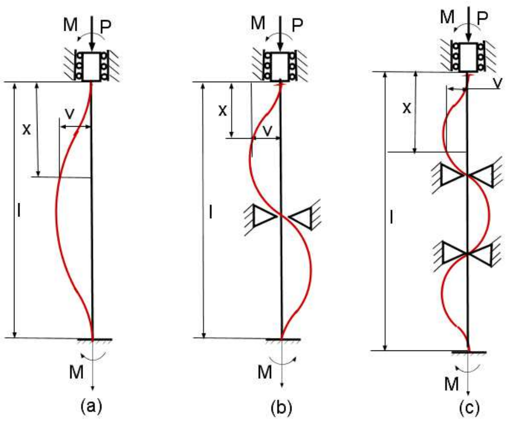

Figure 1. The first three buckling modes shapes

Formula (1) allows the calculation of the critical force for n buckling modes. The shape for the deformed average fiber of the first three buckling modes is shown in Figure 1. In reality the deformed average fiber of a double-clamped beam whose lateral movement is not prevented can only take the form shown in Figure 1.a., which is corresponding to n = 1, to the first solution of characteristic equation.

In this case the formula for calculating the critical force becomes:

$$P_{cr} = \frac{4\pi^2 EI_{min}}{l^2} \tag{2}$$

From the formula (1) and Figure 1.b and 1.c it is seen that the critical buckling force increases with n^2 if we introduce (n-1) restrictions for the lateral displacement of the beam.

Given that the subject of this work is a double-clamped beam, without intermediate supports only the first buckling mode will be studied, as seen in Figure 1.a.

2. Deformations of the undamaged elastic beam subjected to buckling

When the bending moments appear because of the buckling, the average fiber of the beam deforms as it can be seen in Figure 1.a.

From [1] we have the mathematical relationship that describes the shape of the deformed average fiber:

$$v=\frac{M_0}{P}\left[1-\cos\left(2\pi\frac{x}{l}\right)\right] \tag{3}$$

where: v is the transverse displacement , M_0 - embedding at the clamped end, P -axial load, x -distance from the clamped end to the point in which the transversal displacement is v and l -beam length. In Figure 2 the deformed average fiber is represented, according to the relation 3.

Table 1. The transverse displacement

x/l	0	0.25	0.5	0.75	1
v	0	M_0/P	$2M_0/P$	M_0/P	0

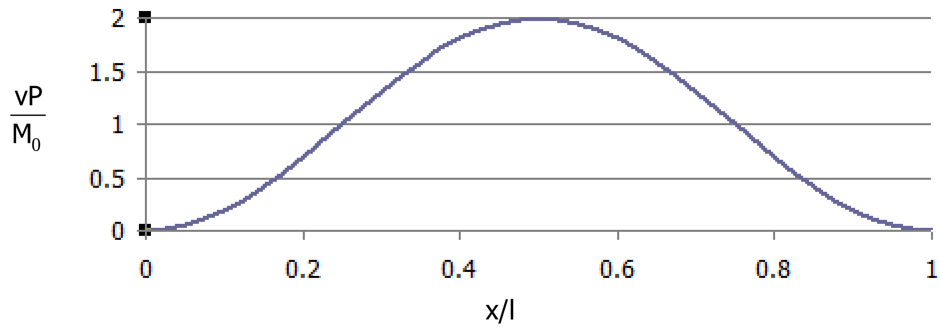

Figure 2. Shape of the deformed average fiber, for the first mode of buckling

From Table 1 and Figure 2 it is observed that the maximum bending occurs in the middle of the beam.

The variation of the bending moment along the length of the beam is given by the second derivative of displacement that has the following mathematical expression:

$$\frac{d^2v}{dx^2} = \frac{M_0}{EI_{min}} \cos\left(2\pi \frac{x}{l}\right) = -M \tag{4}$$

where M is the bending moment in a section placed at distance x.

Table 2. Values of the bending moment depending on the moment in the clamped end, elastic modulus and the minimum moment of inertia of the cross section, at the distance x / l from that end.

x/l	0	0.25	0.5	0.75	1
M	M_0 / EI_{min}	0	$-M_0 / EI_{min}$	0	M_0 / EI_{min}

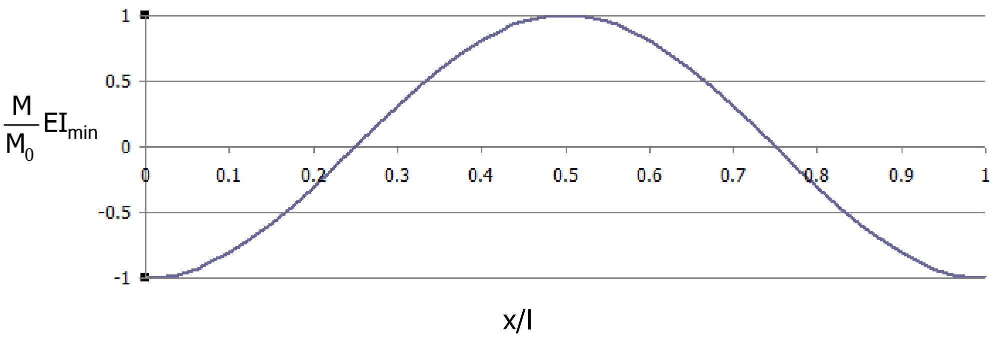

Figure 3. Dimensionless bending moment of the beam, subjected to buckling

From Table 2 and Figure 3 results that the bending moment at both ends of the buckled beam is equal, in absolute value, with the bending moment in the middle of the beam, but with opposite sign.

The beam deforms under the action of the bending moment and thus performs mechanical work.

Considering that the deformations which occur as a result of the bending moments are in the elastic domain, the mechanical work is fully stored as elastic potential energy and released when the beam is fully unloaded. From [3] we have U_d the elastic potential energy given by:

$$U_d = \int_0^l \frac{M^2}{2EI} dx \quad (4)$$

It follows that the elastic potential energy is directly proportional to the square of the bending moment, i.e. the locally stored energy.

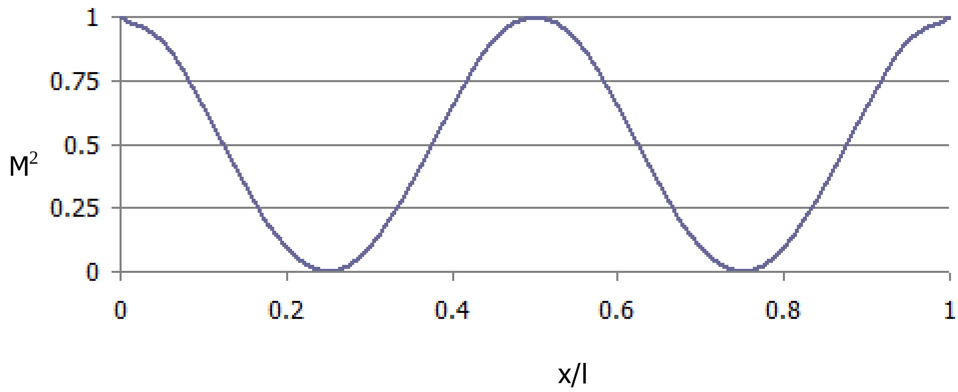

Figure 4. Quadratic bending moment on the length of the beam

3. Study of the critical buckling force variation for the damaged beams by numerical simulations

3.1. The influence of damage width over of the critical buckling force

A double-clamped beam with damages of 4 mm deep, widths between 0,0001mm and 1 mm and located at the 235 mm and 500 mm respectively at one clamped end, was analyzed by finite element method using Solid Works software.

An axial force of 30000 N is acting on the beam. The physical and mechanical characteristics of the beam with 1000x50x10 mm dimensions are the following:

- yield point: $5{,}3\times10^8\ \mathrm{N/m^2}$
- modulus of elasticity: $E=2\times10^{11}\mathrm{N/m^2}$
- thermal expansion coefficient: $1{,}15\times10^{-5}/^{\circ}\mathrm{C}$
- Poisson constant: 0.3
- density: $7850\ \mathrm{kg/m^3}$

The beam was meshed and we obtained 119 754 elements, 187 484 nodes, with a maximum size of an item being 3,18611 mm. A solid type mesh with curved base was used. The buckling coefficients presented in Table 3 and Table 4 were determined in simulations, as well as the buckling forces.

Table 3. Critical buckling forces for beams with damages situated at 235 mm to one clamped end

Width [mm]	0	1	0.1	0.01	0.001
Buckling coefficient c	1.08974	1.08925	1.08933	1.08937	1.08938
P_{cr} [N]	32692.2	32677.5	32679.9	32681.1	32681.4
ΔP_{cr} [N]	0	14.7	12.3	11.1	10.8
T_{cr} [° C]	53.29	53.28	53.28	53.28	53.28

Table 4. Critical buckling forces for beams with damages situated at 500 mm to one clamped end

Width (mm)	1	0.1	0.01	0.001	0.0001
Buckling coefficient c	1.05009	1.05695	1.05856	1.05858	1.05858
P_{cr} [N]	31502.7	31708.5	31756.8	31757.4	31757.4
ΔP_{cr} [N]	1189.5	983.7	935.4	934.8	934.8
T_{cr} [° C]	52.26	52.44	52.48	52.48	52.48

Analyzing the values presented in Tables 3 and 4 we observe that the critical buckling force variation through the damage width, in the studied interval, is less than 3% and increases with the widening of the discontinuity.

3.2. The influence of discontinuity depth on critical buckling force

Using the finite element method we analyze a beam with the same physical and mechanical characteristics, in the same supporting and meshing conditions as those set out in paragraph 3.1, considering that the beam has a discontinuity with δ a depth between 1 mm and 8 mm situated at a distance of 500 mm from the clamped end.

Table 5. Critical buckling forces determined through numerical simulations with discontinuities of depth between 1mm and 8mm

δ [mm]	1	2	3	4
Buckling coefficient	1.0877	1.0818	1.0715	1.0541
P_{cr} [N]	32632.2	32455.8	32145	31623.6
δ (mm)	5	6	7	8
Buckling coefficient	1.025	0.9781	0.8878	0.7136
P_{cr} [N]	30751.2	29343.21	26634.81	21409.59

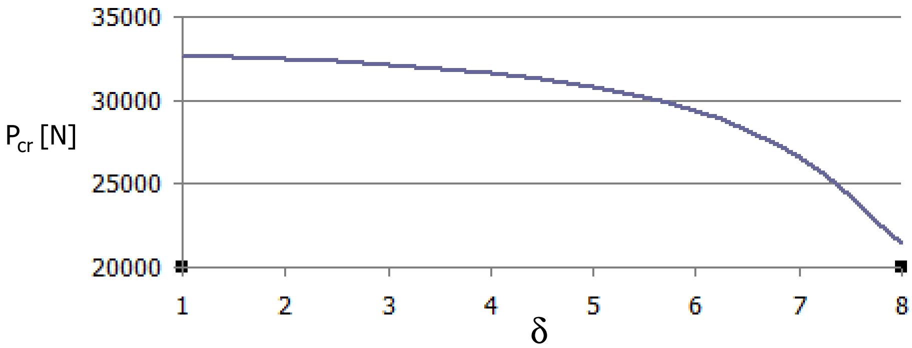

Figure 5. Variation of the critical buckling force determined through numerical simulations based on damage depth

Analyzing the values presented in Table 5 and plotted in Figure 6 we see that the critical buckling force decreases with the depth of the damage. The decline is accentuated when the depth of the damage is greater than half of the cross-section.

3.3. The influence of damage position over the critical buckling force

The numerical simulation using the Solid Works program were made for the beam with features, support system, loading and mesh shown in paragraph 3.1 with the amendment that a damage of 0.5 mm width and a depth of 4 mm was considered, located at a variable distance between 15 mm and 985 mm, from the clamped end.

Table 6. Critical buckling forces for the damage located at a distance between 15 and 985 mm

x_D [m]	0.015	0.025	0.03	0.05	0.078	0.1	0.156	0.235
P_{cr} [N]	31639.5	31647.9	31661.1	31721.4	31861.8	31989	32347.8	32679.9
x_D [m]	0.411	0.44	0.47	0.5	0.53	0.56	0.589	0.677
P_{cr} [N]	31910.1	31753.5	31652.43	31623.6	31652.43	31753.5	31910.1	32481
x_D [m]	0.765	0.844	0.9	0.922	0.95	0.97	0.975	0.985
P_{cr} [N]	32679.9	32347.8	31989	31861.8	31721.4	31661.1	31647.9	31639.5

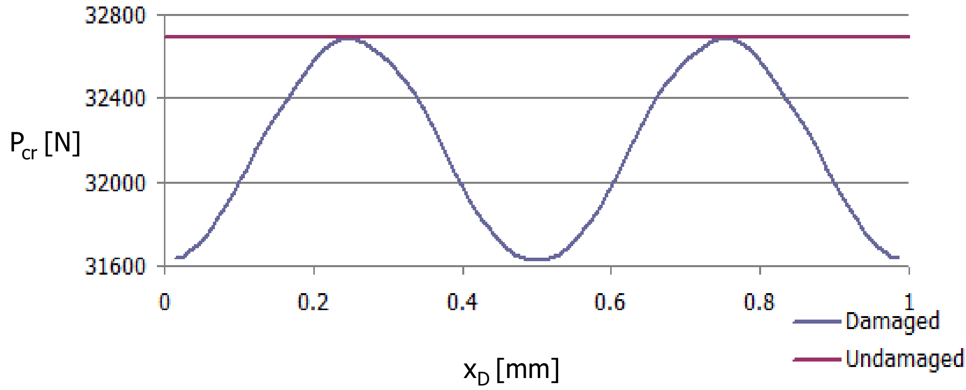

Figure 6. Critical buckling force variation with the damage position, for a damage of 0.5 mm width and a depth of 4 mm

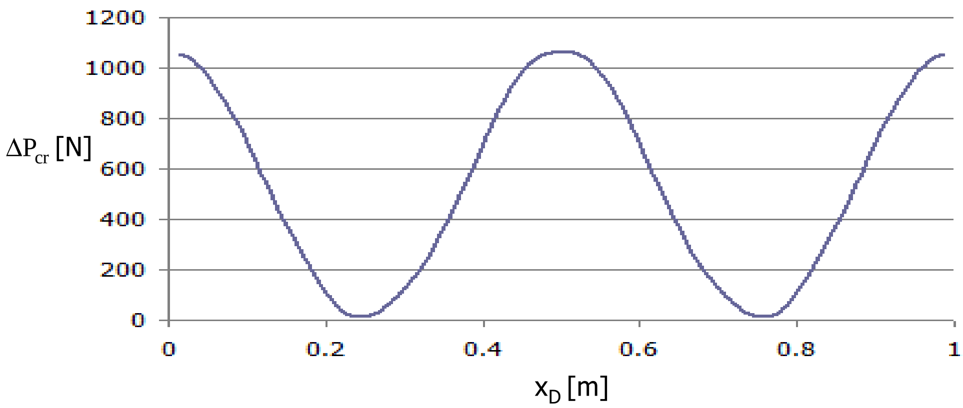

Figure 7. Variation of the difference between the critical buckling force of the undamaged beam and the critical force of the damaged beam, depending on the distance

The results are given in Table 6 and are plotted in Figure 7. Analyzing the graphs in Figures 7, 2 and 3 it is seen that the critical buckling force of the damaged beam has the maximum value close to the critical buckling force of the undamaged beam when the position of discontinuity corresponds to the position of the deformed medium fiber's inflection points i.e. corresponding to the position of the beam section where the bending moment is zero.

We denote by ΔP the difference between the critical force for the undamaged beam and the critical force for the damaged beam. Comparing the graph from figure 7 with that from figure 4 we see that the difference between the critical force for the undamaged beam and the critical force for the damaged beam varies along the beam, as well as the square of bending moment, having minimum and maximum points located on the same sections. Based on this observation and the expression of potential energy of deformation (4) we conclude that the difference between the two critical buckling forces, for the undamaged beam and for the damaged beam respectively, varies directly proportional to the potential energy of deformation, on the entire length of the beam.

4. Conclusion

This paper introduces a method to evaluate the critical buckling force for beams with geometrical discontinuities. The analysis was made on a prismatic beam with two clamped ends.

The results obtained by numerical simulations using Solid Works program highlight the following:

- the critical buckling force is significantly influenced by the depth and position of damage on the beam and is less influenced by variation of the damage width;
- the existence of a connection between the critical buckling force of a damaged beam, which is the square of the bending moment and consequently the potential energy stored in the beam in the affected region.

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References

[1] Bejan M., *Rezistenta Materialelor*, vol. 2, Editura AGIR, Bucuresti, 2006

[2] Javidinejad A., *Buckling of beams and columns under combined axial and horizontal loading with various axial loading application locations*, Journal of Theoretical and Applied Mechanics, Sofia, 2012, vol. 42, No. 4, pp. 19–30.

[3] Buzdugan G., *Rezistenta Materialelor*, Editura Tehnica, Bucuresti, 1974.

[4] Pastrav I., *Rezistenta Materialelor*, vol.1 si 2, Institutul Politehnic Cluj Napoca, 1979.

[5] Tripa P., Hlușcu M., *Rezistenta Materialelor. Notiuni Fundamentale si Aplicatii*, Editura MIRTON, Timisoara, 2006.

[6] Rades, M., *Rezistenta Materialelor*, vol.2, Editura PRINTECH, 2007.

[7] Tufoi M., Gillich G.R., Praisach Z.I., Iancu V., Furdui H., *About the Influence of Temperature Changes on the Natural Frequencies of*

Clamped-Clamped Euler-Bernoulli Beams, Romanian Journal of Acoustics and Vibration, vol. XI, issue 2, 2014.

[8] Tufoi M., Gillich G.R., Iancu V., Negru I., *Damage identification in rectangular plates using spectral strain energy distribution*, Proc. SPIE 9438, Health Monitoring of Structural and Biological Systems 2015, 943818 (March 23, 2015); doi:10.1117/12.2085009

[9] Muntean F., Gillich G.R., Praisach Z.I., Iancu V., Haţiegan C., *Studiul comportării dinamice a stâlpilor cu diferite nivele de încărcare*, Ştiinţă şi Inginerie, 2014.

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