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Vibration-Based Structural Health Monitoring: Theoretical Foundations and Experimental Validation on Reinforced Concrete Beams

Quick identification of damages in structures is of great importance to engineers. Among the various techniques available for the evaluation of reinforced concrete structural integrity, non-destructive tests method remain a viable one as its use can lead to speedy decisions that bring savings on repairs or replacement of damaged reinforced concrete structures. This research uses modal parameter-based non- destructive tests to assess damages in reinforced concrete beams under static load. Four-point static loadings were applied to the 3 RC beams to induce three damage scenarios. After each static loading, a dynamic test was performed to access the degree of stiffness degradation. Modal frequencies and mode shapes obtained depicts clearly the stiffness degradations of the beams as the severity of damages on the beams became more pronounced. Results obtained showed that the research procedure adopted is a smart approach for damage assessment in reinforced concrete elements.

Keywords: *Damage Assessment, Dynamic Test, Non-Destructive Tests, Reinforced Concrete Beams*

1. Introduction

Vibration-based structural damage assessment was conceived in analogy to the use of vibration as a machine condition indicator. Machines hardly break down without warning. The signs of impending failure are usually present long before breakdown makes the machine unusable. Machine troubles are almost always characterised by an increase in vibration level which can be measured on some external surface of the machine and thus act as an indicator. With the frequency analysis of vibration signals, it is possible to locate the source of many of the frequency components present. The frequency spectrum of a machine in a normal running condition can therefore be used as a reference "signature" for that machine. Subsequent analyses can be compared to this reference so that not only the need for action is indicated but also the source of the fault is often diagnosed.

Based on this reasoning, vibration tests are adopted widely for structural health monitoring/damage assessment since the frequency spectrum of a structure in an un-damaged/normal condition can be used as a reference signature for that structure in subsequent moments or working conditions.

2. The evolution of structural health monitoring

The past 50 years have witnessed major developments in the theory and application of linear dynamic systems, work that were heavily influenced by Kalman's results in the early 1960s. Much of these works developed as system identification, originated from systems and control engineering and are applicable in many different engineering fields, in economics and in medicine etc. System identification basically means modelling of the dynamic systems from experimental data. Structural health monitoring or damage detection is an important application of system identification. This consist in conducting non-destructive test or inspection of a structure to determine the existence, location and extent of damages, and, in some cases make a prediction on the future life of the system. This practice that started in the mechanical-based industries and have taken an increased importance in aerospace sector have finally crossed over to the civil applications within the last few decades. This has greatly increased the use of aircrafts structures far beyond their original life expectancy. Additionally, as the civil infrastructures ages, the determination of their integrity for continued safe usage becomes critical. Costs associated with inspection, maintenance, and system downtime also provides motivation for improved inspection and damage identification practices. For these reasons, new methods of structural health monitoring are being explored to better determine the functional safety of structures ([1], [2], [3]). In particular, system identification has turned out to be very useful in damage assessment of complex civil engineering structures, such as towers, dams, bridges, offshore structures.

This vibration based health monitoring algorithm that have its root in structural system identification utilizes changes in response functions or modal parameters such as natural frequencies, mode shapes or their derivatives, to identify damage locations and levels of damages. Analysis of changes seen in parameters between sequential tests over time is used to determine damage characteristics. Excellent reviews of model-based health monitoring methods can be found in ([4], [5], [6], [7]).

Model-based health monitoring methodologies are excellent tools in the model updating process. For this purpose, one set of data from the analysis model is compared to a set of data from the physical structure. The presence of damage is detected when the features of the model do not match the experiment. Localized methods are much better suited for this purpose, as it is better to update individual elemental or sub-structural properties rather than global properties across the whole design. Analytical sensitivities of response parameters to changes in physical properties are used to update modelling assumptions.

2.1 Fundamentals of Structural Dynamics

Vibration is a common phenomenon verified in our surroundings whenever dynamic forces excite structures. It is often a destructive and annoying side effect of a useful process, but is sometimes generated intentionally to perform a task. It can originate from natural phenomenon or man-made actions. The *effects* of vibrations often cause discomfort, fatigue, health hazards and in extreme cases destructions. Within the recent times, the technological advancement has brought about the invention of more complex structures which are accompanied by increased dynamic problems. In this era in which a lot of emphasis is being placed on competitive and sustainable growth, there is an ever increasing need for a reliable dynamic analysis. A comprehensive understanding of structural dynamics [see e.g. [8], [9], 10]) is essential for the design and development of new structures, and to solving noise and vibration problems on existing structures. One of the reliable tools for vibration analysis is *Modal Analysis*.

Modal analysis covers a variety of applications on the analysis of modal parameters. These parameters describe specific dynamic characteristics of the structure. The application of modal analysis implies knowledge of a broad range of physical laws and mathematical concepts. The basic assumptions for linear modal analysis consist of schematizing the structure as a linear system whose dynamic behaviour can be described by a set of differential equations, the structure obeying Maxwell's reciprocity theorem and the structure being time invariant.

2.1.2. Single Degree of Freedom Systems (SDOF)

The simplest linear system adopted is Single Degree of Freedom System (SDOF). The dynamic behaviour of the linear *SDOF* system can be regarded as the basis for much of the analysis and interpretation of results for more complicated vibratory systems. In particular, the response of multi-degree-of -freedom (*MDOF*) linear systems is a superposition of modal responses of *SDOF* systems. All dynamic properties of mechanical systems are distributed in space. However, in linear vibration analysis, the basic properties are considered as separated into simple discrete elements which often represent the dynamic properties of the system to sufficient accuracy. The discretization of a single degree of freedom (SDOF) oscillator with its properties are represented by the elements of an analytical model in figure 1. It is an abstract system consisting of a point mass *(m)* supported by a massless linear spring of constant stiffness *(k)* and connected to a linear viscous damper *(c).* The mass is constrained so that it can move in only one direction (x).

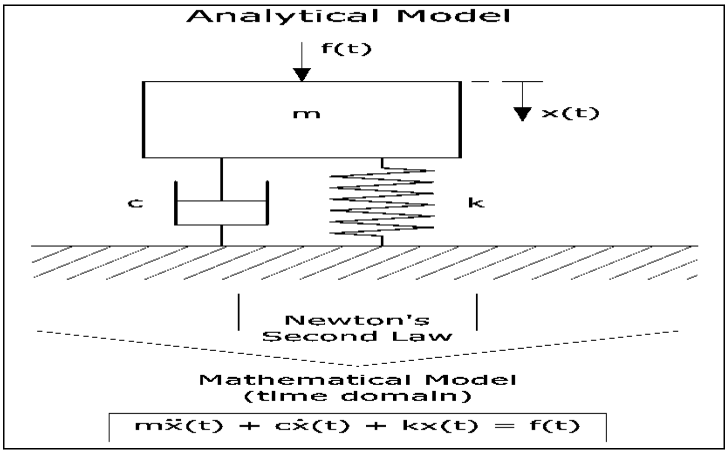

Figure 1. A single Degree of Freedom System.

A mathematical model in the time domain can be derived by applying Newton's Second Law to the analytical model. By equating the internal forces (inertia, damping and elasticity) with the external (excitation) force, we obtain the model which is a second-order differential equation

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \quad (1)$$

where $f(t)$ and $x(t)$ are respectively the time dependent excitation force applied to the system and the corresponding displacement. The initial displacement and velocity conditions ($t = 0$) are $x(0)$ and $\dot{x}(0)$.

The solution of equation (1) is the sum of the solution of the corresponding homogeneous equation of the free vibration with a particular integral of the non-homogeneous equation of the forced Vibration.

While the un-damped solution (($\xi = 0$) corresponds to a harmonic motion, of frequency ω_n (known as the un-damped natural frequency) and with constant amplitude, the under-damped solution ($0 < \xi < 1$) has an oscillating motion of frequency $\omega_d = \omega_n\sqrt{1-\xi^2}$ (damped modal frequencies) which tends exponentially to zero.

This solution which is closer to what is obtained in real structures has the following solution:

$$x(t) = e^{-\xi\omega_n t}(C_1 e^{i\omega_D t} + C_2 e^{-i\omega_D t}) + \frac{F}{\sqrt{(k-\omega^2 m)^2 + (\omega c)^2}} e^{(i\omega t + \phi)} \quad (2)$$

Steady state solution is the part relative to the forced vibration. Adopting $\beta = \omega/\omega_n$ as a dimensionless parameter representing the ratio of the forcing frequency to the un-damped natural frequency of the system, we can consider the steady state solution of the forced vibration in terms of the following quantity:

$$\frac{X}{X_s} = Q = \frac{1}{\sqrt{(1-\beta^2)^2+(2\xi\beta)^2}} \quad (3)$$

where $X_s = F/k$ is the static deformation of the system if loaded by a constant force F and Q is amplification factor. When $\xi = 0$, and $\beta = 1$ $(\omega = \omega_n)$ the steady state vibration has infinite amplitude $X^{\dagger}$ and this particular situation is called resonance. Avoiding resonance is of great importance for engineering design since it can lead to ruin of any structure.

Since real dynamic system always have some energy dissipating mechanisms, the amplitude at resonance never arrive at infinity. For low damping, it can assume very large values. The maximum value of the amplitude of the steady state vibration occurs for $\omega = \omega_n\sqrt{1-2\xi^2}$. At resonance,

$$Q = 1/2\xi \quad (4)$$

The frequency domain solution for the forced vibration is contained in the mathematical expression relating the output to the input:

$$\frac{\bar{X}}{F} = H(\omega) = \frac{1}{(k-\omega^2 m)+i(\omega c} \quad (5)$$

and is referred to as the system's Frequency Response Function (FRF).

2.1.3. Multiple Degree of Freedom Systems (MDOF)

The SDOF model served to describe a dynamic system in its simplest term. Real structures are continuous and non-homogeneous elastic systems with infinite number of degrees of freedom. Therefore, their analysis entails the adoption of a finite number of degrees of freedom (DOFs), which, permits an approximation that ensures accuracy. Choosing the DOFs must be one of the beginning points of any analysis. The DOFs are the number of independent coordinate necessary to completely describe the motion of the system.

Typically, the equilibrium conditions for linear time-invariant continuum mechanics are discretized through spatial displacement interpolation to a finite number of variables (e.g. finite element methods), resulting in *n*-dimensional set of second-order linear differential equations having **M, C** and **K** as *n x n* mass, damping and stiffness symmetric matrixes respectively of the system. $\ddot{q}$ $\dot{q}$ and q are *nx1* vectors of time-varying acceleration, velocity and displacement responses, respectively. This is then accompanied with by a sensor output vector and output influence matrices for displacement, velocity and acceleration, respectively.

2.1.4. Natural Frequencies and Mode Shapes of Un-damped MDOF system. Proportionally damped systems

The natural frequencies and mode shapes of an un-damped MDOF system is obtained by solving for a non-trivial solution of equation

$$[\mathbf{K}\text{-}\omega^2\mathbf{M}]\mathbf{X} = 0 \qquad (6)$$

or a characteristic equation in the form of:

$$[\mathbf{K}\text{-}\omega^2\mathbf{M}]\mathbf{X} = 0 \qquad (7)$$

This equation yields *n* possible positive real solutions$\omega_1^2, \omega_2^2, \dots, \omega_n^2$ known as the eigenvalues of equation (6), while $\omega_1, \omega_2, \dots, \omega_n$ are the un-damped natural frequencies of the system. Substituting each natural frequency value in equation (6) and solving for **X**, we obtain *n* possible vector solutions $\{\varphi_r\}, r = 1, 2, \dots, n$ known as the mode shapes, modes of vibration or eigenvectors of the system under analysis. The *n* elements of these eigenvectors $\{\varphi_r\}$ are real quantities and can be graphed to give a clear view of how the system moves at that particular mode.

The mass normalized mode shapes of the modal matrix are obtained from the un-damped modal mode shapes as:

$$\{\phi_r\} = \frac{1}{\sqrt{m_r}} \{\varphi_r\} \qquad (8)$$

2.2. Execution of modal test and extraction modal parameters

A typical modal test can be executed by setting up the modal test, taking measurements and estimating the parameters. You start by specifying the Degrees of Freedom (DOFs) of interest. The number of DOFs needed depends on the purpose of the test, on the complexity of structural geometry, and must be chosen to capture the total dynamics of the structure. Hammer excitation or electrodynamic vibration exciter can be adopted, based on the frequency range of interest. The exciter is best positioned at a point where both the symmetric and asymmetric modes will exhibit maximum motion.

The simplest set of instrumentations is composed of a dual-channel signal analyzer, with hammer excitation, and an accelerometer to measure the response signal. For the first few bending modes, few DOFs aligned in the vertical direction will be sufficient. After the setting up of the instrumentation, preliminary adjustments are often necessary as to guarantee an improve the final results

The measurement phase is the most critical and important part of the whole operation. The quality of results to be obtained depends primarily on the accuracy obtained at the measurement. Here, a set of FRF measurements between the excitation DOF and all the other defined DOFs is taken and stored. A dual-channel FFT analyzer is used to measure the FRF. For the measurements, the analog input signals are filtered, sampled, and digitized to give a series of digital records. Over a finite time these records represent the *time history* of the signals. The sampling rate and the record lengths determine the frequency range, and the resolution, of the analysis. Each record from a continuous sequence may be weighted by a window function, which tapers the data at both the beginning and end of each record to make the data more suitable for block analysis. The weighted sequence is transformed to the frequency domain as a complex spectrum, by the use of a Discrete Fourier Transformation. The measurements are usually made in terms of acceleration and the results arranged as elements of mobility matrix. For *n* defined DOFs, the number of possible input/output combinations is n x n. The individual FRF measurements can be arranged as the elements of a mobility matrix **H.** Each element $\mathbf{H}_{ij}(\omega)$ is a particular FRF measurement. Each row of the matrix contains FRFs with a common response DOF while in each column they have a common excitation DOF. The diagonal of [H] contains a class of FRFs for which the response and excitation DOFs are the same. These are the *driving point FRFs.* The off-diagonal elements are *transfer FRFs.* Because reciprocity helps in mobility matrix, the number of measurements needed is equal to the number of specified DOFs.

Preferably, a mobility measurement should simply involve exciting the structure with a measurable force, measuring the response, and then calculating the ratio between the force and response spectra. In practice however, we are faced with the problems of noise and limited analysis resolution. To minimize these problems, we have to apply some statistical methods and averaging process to estimate the FRFs from our measurements. The Autospectra of the force and the response, together with the Cross Spectrum between the force and response are then needed for the FRF estimates.

From any measured FRF we can determine the modal frequencies and dampings, and thus obtain the pole locations. A structure with lightly coupled modes behaves as a single-degree-of freedom system around its modal frequencies and it can be assumed that all the response is due to that particular mode. The modal frequencies are determined simply by observing the maximum magnitudes on the FRF.

The modal dampings are not so simple to determine, and will often be the parameters measured with the greatest degree of uncertainty. One technique that can be used to measure the damping is to find the -3dB bandwidths. On a lightly damped structure the resonances are sharp and the peaks are too narrow for accurate measurements of the bandwidths. This problem can often be overcome by making a zoom analysis to obtain sufficient frequency resolution for the measurements.

The *mode shapes* can be determined if we fix a response, or an excitation DOF as a reference and then make a set of measurements. The imaginary parts of the measured FRFs can be "picked" at the modal frequencies at which they represent the modal displacement for that specific DOF. If the measurements were made with calibrated instrumentation, the mode shapes could then be scaled.

3. Experimental Validation on Reinforced Concrete (RC) Beams

This experimental research aims to verify the efficiency of using dynamic tests procedures presented in section 2 of this paper to assess damages in reinforced concrete beams under static load. The test specimens, material properties and test methods used are hereby presented. Three RC beams of 15 x 20 cm cross-section and a length of 220cm, reinforced with four ribbed longitudinal steel bars were adopted for the experimental program. The RC beam is shown in figure 2 while table 1 shows the material properties of the beams.

Table 1. Reinforced concrete material properties

PROPERTIES	VALUES	
	CONCRETE	STEEL
Young's Modulus	28400 [MPa]	200000 [MPa]
Poisson's ratio	0.18	0.3
Compressive Strength	60 [MPa]	500 [MPa]

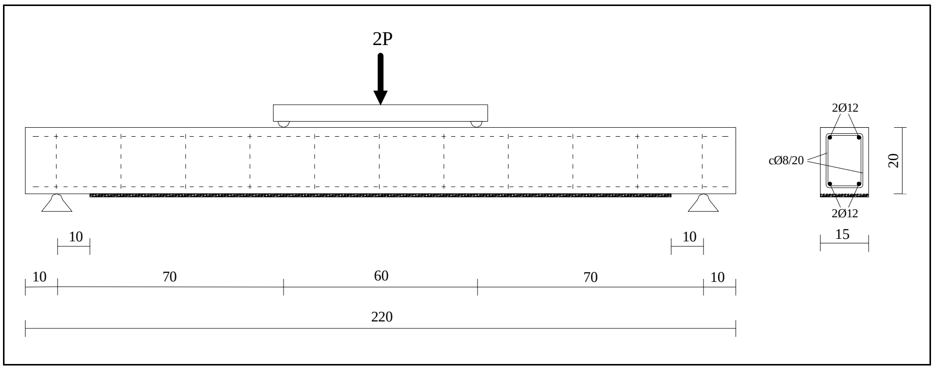

Figure 2. Geometry (cm) and the four point bending loading of the beams

To generate damages on the RC beams, four- point static loadings were applied to the beams to induce three damage scenarios. After each static loading, a dynamic test was performed to access the degree of stiffness degradation. Three PCB Piezometrics' accelerometers were adopted for the dynamic test and were placed at 50cm, 65cm and 100cm, respectively from the beams support. The procedure was similar to that adopted in [11]. The position of the accelerometers are shown in figures 3, while the excitation points are shown in figures 4.

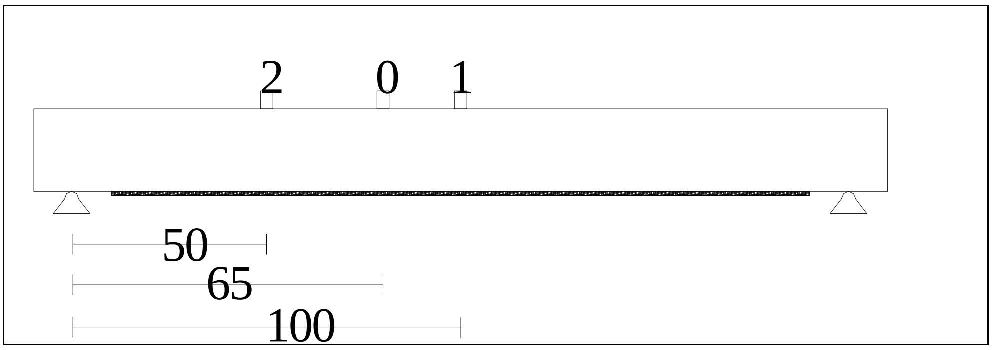

Figure 3. Accelerometers locations

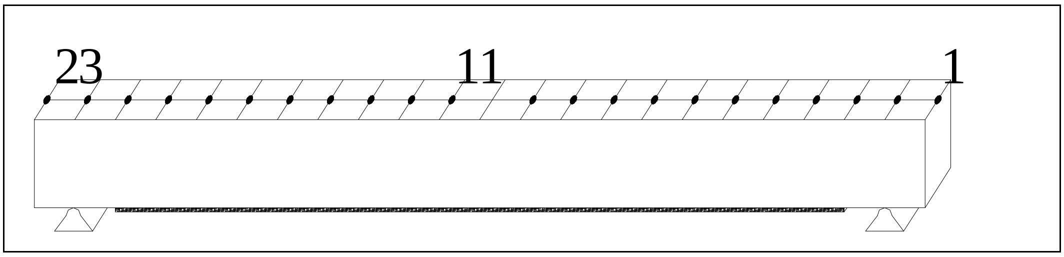

Figure 4. Excitation points along the beams

For all the beams, a dynamic test was performed at the beginning to obtain the reference dynamic properties. After every static a dynamic test was performed to verify the corresponding dynamic parameters at that stage. Starting from an edge of the beam and within a 10 cm spacing, 23 impact hammer excitations were performed to induce vibration of the beams for each damage scenario. The time history free responses were filtered with the Fast Fourier Transform (FFT) and the Frequency Response Functions (FRF) were obtained.

4. Experimental Results

Results of the experimental programs aimed at verifying the efficiency of using the dynamic tests procedure presented in section 3 of this paper to assess damages in RC beams under static load is hereby presented. Figures 5 and 6 show the sampled 1st and 2nd mode shapes for the RC beam. The mode shapes' null points did not coincide exactly with the position of the beam's supports, showing the imperfections that could arise from the assumptions of the fixity. Figure 7 shows the diminishing trend of the frequencies to forecast the degradation of stiffness as the severity of damages increased.

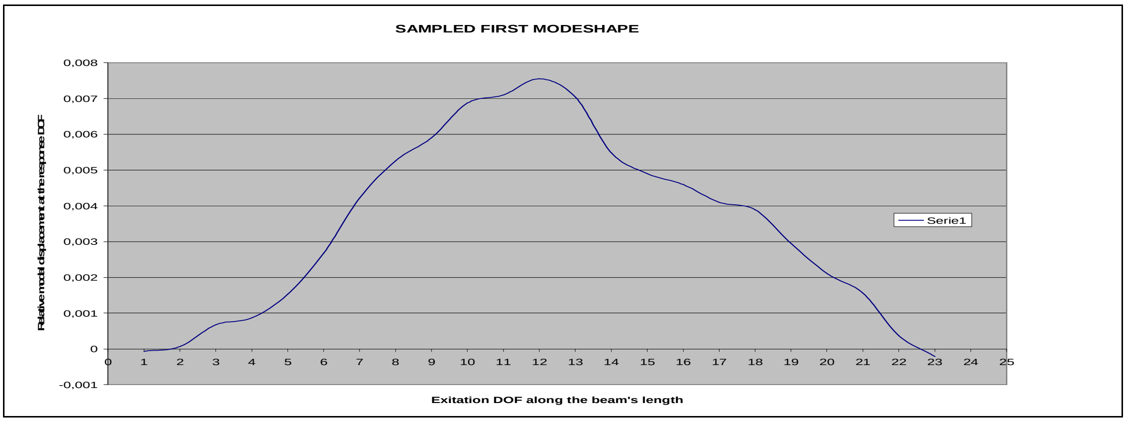

Figure 5. Sampled 1st mode shape of a RC beam

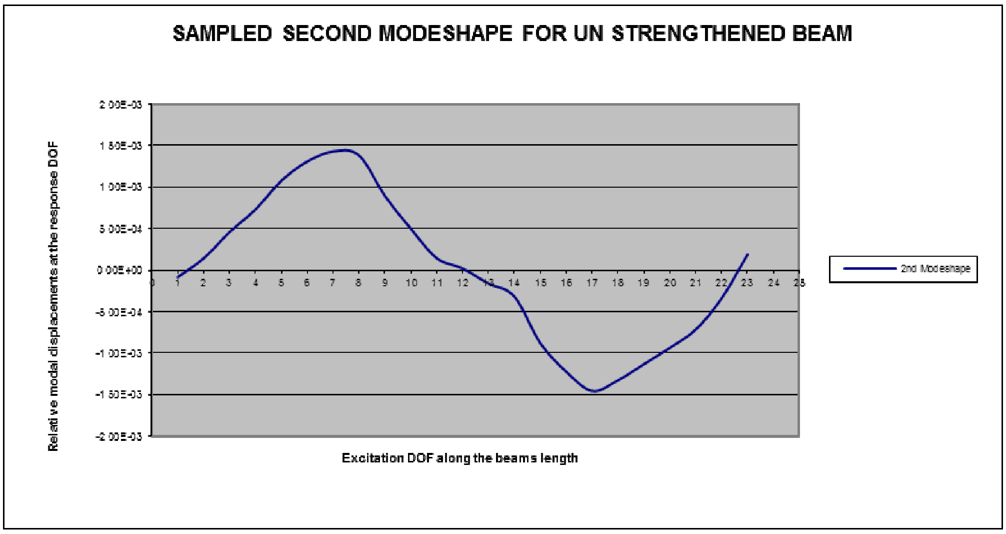

Figure 6. Sampled 2nd mode shape of a RC beam

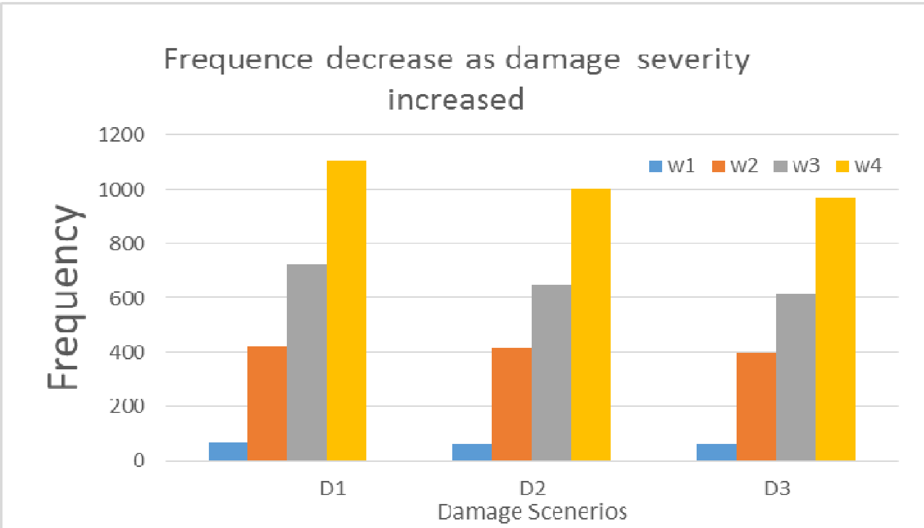

Figure 7. shows the diminishing trend of the frequencies as the stiffness degraded

5. Conclusion

The results showed that dynamic-based damage assessment method adopted in this research is good in monitoring damage evolution in RC beams under static loads. Narrow miss of the fixity points by the modal shape curves highlights the uncertainty that can accompany the realization of structural supports. The results obtained confirms the assessment procedures used in this research as a quick and reliable diagnostic approach to verify the degradation of stiffness of structural elements if the initial dynamic signature of the structure is known.

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