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An Extension of the Burridge-Knopoff Model for Friction

The paper presents an extension of the Burridge-Knopoff (BK) model with an additional kinetic equation for the friction force in order to reproduce the both the velocity weakening friction between the tire and the road and the increase of static friction with time when the car is not moving. The BK was initially proposed to investigate statistical properties of earthquakes. In this model the sliding force decreases monotonously from a reference value, and the static friction can have negative values to prevent back sliding. The stability of the system is affected and the sliding regime at small sliding velocities and large stiffness cannot be reproduced. The extended model BK assures the stability of the diagram sliding-stationary sliding, and correctly reproduces the stability diagram for sliding friction under various loading conditions.

Keywords: *friction, Burridge-Knopoff, model, stability*

1. Introduction

The tires on the road can perform stick-slip events when the brakes are pushed so hard that they lock up the wheels. In this case a pure stick in the brake-wheels system is put into evidence. The tires slide on the road instead of rolling. In other words, we have sticking to the road. In this case, the stick state corresponds to tires normally rolling, and the slip state corresponds to a sudden slip on the road, which can induce wear of the tires, i.e. loss of material and irreversible deformations. The behavior which defines stick-slip is intermittent being done to an intermittent braking. The figures seen on the roads, i.e. the regularly spaced skid marks are not directly related to stick-slip, but rather are the consequence of the use of the break.

The stick-slip phenomenon occurs on different length scales. Even when the motion of the center of mass seems smooth, it is possible local stick-slips to occur at the interface between the sliding solid and its substrate. These local events may be understood by studying the elastic waves emitted from the sliding interface [1-5]. From the microscopically point of view, it is possible to have contacts in a sticking state and some in a sliding state. Modern theories of friction regard sliding as a sequence of slips between sticking asperity zones, which reverberate across the contact surface as shear stresses build up and dissipate locally. The way this happens is said to be similar to the sliding of geological fault lines, which is manifest as a discrete sequence of earthquakes.

The Burridge-Knopoff (BK) model, or spring-block model, is a description of the friction at the mesoscopic level between two plates A and B elastically coupled together. This model was advanced in the context of seismology by Burridge and Knopoff in 1967. The tectonic plate B is divided in virtual blocks connected each to other by elastic interactions. The load is performed via elastic interactions with a plate A which moves with velocity V_0 (Figure 1). The motion equation is given by

$$m\ddot{x}_i = k_0(V_0 t - x_i) + k_1(x_{i+1} - 2x_i + x_{i-1}) - \varphi_i \,, \tag{1}$$

where x_i is the departure of block i from its equilibrium position, k_0 the stiffness of the connection with the driving plate, k_1 the stiffness of the interactions between blocks and m the mass of each block, and φ_i is the local friction force acting on the block i. In the BK model, the sliding friction force is a lubricated creep-slip friction law with viscous properties at both the low and high velocity limits (see the dashed line in Figure 2).

The friction threshold is not equal for all blocks. The system behaves differently from one block to another. The BK friction law shows a threshold (red vertical line) where the plate starts to slip.

The BK model can be associated to the stick-slip law of Carlson and Langer [6] (see solid line in Figure 2)

$$\varphi = \frac{\varphi_0 \operatorname{sgn}(V_0)}{(1 + |V_0|)} \,. \tag{2}$$

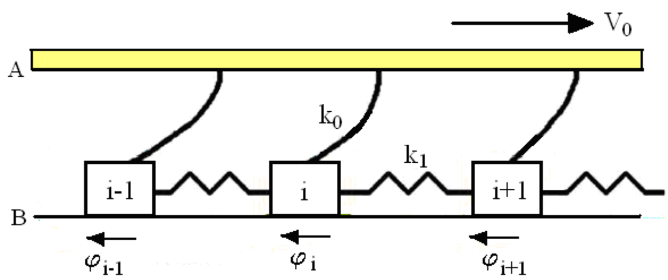

Figure 1. Burridge-Knopoff model

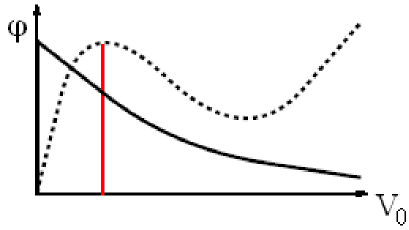

Figure 2. The stick-slip friction law [6]

In this paper, starting from BK model, we build and study an extension of this model with an additional kinetic equation for the friction force in order to reproduce the both the velocity weakening friction between the tire and the road and the increase of static friction with time when the car is not moving. The frictional events introduce nonlinearities in the stiffness and damping characteristics of contact interfaces.

The friction plays a dual role by transmitting energy from one surface to the other and by dissipating energy of relative motion [7-9]. Experiments show that in most cases of dry friction the sliding stabilizes for either sufficiently large velocities or sufficiently large stiffness of the system.

These properties are explained in details in the books [10, 11]. The BK model is related to the nonlinear Klein-Gordon equation which admits soliton-like solutions [12]. Cartwright et al. [13] studied the BK model of earthquake faults with viscous friction and show that the model admits the van der Pol- FitzHugh- Nagumo equation for excitable media with elastic coupling. In both situations, the solutions of the BK problem are composed by a number of propagating pulses with a proper choice of the initial conditions.

2. Modified BK model

Equation (1) can be written in a continuum language [13]

$$\ddot{\chi} = c^2\chi_{,xx} - (\chi - vt) - \gamma\phi\,, \qquad (3)$$

where $\chi(x,t)$ is the local time-dependent longitudinal deformation of the surface A with respect to the static reference of the plate B, and γ is the magnitude of the friction, c is the longitudinal speed of sound and v is the velocity of the plate A or slip rate.

The friction force ϕ is given by the BK lubricated creep-slip friction law

$$\phi = \frac{1}{3}\dot{\chi}^3 - \dot{\chi}\,. \qquad (4)$$

From (3) we can obtain the local velocity $\psi = \dot{\chi}$ of the interface between the plates A and B. So, (3) can be written as two differential equations of first-order

$$\dot{\psi} = \gamma(\eta - \phi), \quad \dot{\eta} = -\frac{1}{\gamma}(\psi - v - c^2\psi_{,xx}). \tag{5}$$

The BK model (1) and (2) is a simplification of real properties of static and kinetic friction. This equation does not reproduce the correct stability diagram for sliding-stationary sliding. It is important to say that the system is always unstable [14]. This is why we introduce here a modified version of BK model including the state dependent friction term. The new model describes a velocity weakening of friction between moving car and an increase of static friction during stick periods. It provides a stable diagram for the transition from smooth sliding to stick-slip behavior as observed experimentally.

This new law includes a viscous term $\varsigma\dot{x}_i$ and it is represented as

$$m\ddot{x}_i = k_0(V_0 t - x_i) + k_1(x_{i+1} - 2x_i + x_{i-1}) - \varsigma\dot{x}_i - \varphi_i\tau. \tag{6}$$

In a continuum language, (6) becomes

$$\ddot{\chi} = c^2\chi_{,xx} - (\chi - vt) - \delta\dot{\chi} - \gamma\phi, \tag{7}$$

where δ is the is the magnitude of the friction rate.

Equation (7) can be written as two differential equations of first-order

$$\dot{\psi} = \gamma(\eta - \phi), \quad \dot{\eta} = -\frac{1}{\gamma}(\psi - v - c^2\psi_{,xx} - \delta\gamma\eta + \delta\gamma\phi). \tag{8}$$

The friction force ϕ in (7) and (8) is given by the BK lubricated creep-slip friction law (4). Different initial conditions can be attached in the study of stability of the problem (8) and (4).

3. Stability of the model

But the problem is not so simple. The large full band-gap depends not only on the number of scatterers but on the material the scatterers are made and on their geometry. The geometry can be diverse: spheres, hollow spheres, cylindrical shells, rods. The stability of solutions for (8) and (4) for initial and boundary conditions $\psi(0) = 0.1$, $\eta(0) = 0.1$ and $\psi|_{x=0} = -1.5$, is investigated with respect to v. The system has uniformly propagating solutions for v approaching the singular limit $v = c$. In this limit the pulses become discontinuous, so that (8) becomes ill-defined. Equation (8) has bounded solutions only if $v^2 < c^2$. In this case the solutions depend slightly on v and the period of oscillations decreases with the increase of *v*. Since our estimates predict that v becomes smaller than *c* and approaches zero as the number of pulses in the system is increased, there should be a maximum number of pulses allowed in the solutions. A similar result was obtained by Cartwright et al. [13].

The parameters have the values $\gamma = 3$, $\delta = 0.2$ and $c = 1$. Also, $k_0 = k_1 = 0.2$. In Figure 3a, a two pulse solution propagating to the right with the velocity 4.88 is presented. Figure 3b presents the phase portrait of this solution. The red line is the nulcline line, and the blue point shows the position of the unstable fixed point. The nullcline lines are sometimes called zero-growth isoclines. The fixed points of the system are located where all of the nullcline lines intersect.

Figure 4a shows a four pulse solution propagating to the left with the velocity 2.48 and thus closer to the sound velocity *c*, as expected from the estimates.

Figure 4b presents the phase portrait of this solution. Figure 5a shows an eleven pulse solution propagating to the left with the velocity 1.08 very closer to *c*. Figure 5b presents the phase portrait of this solution.

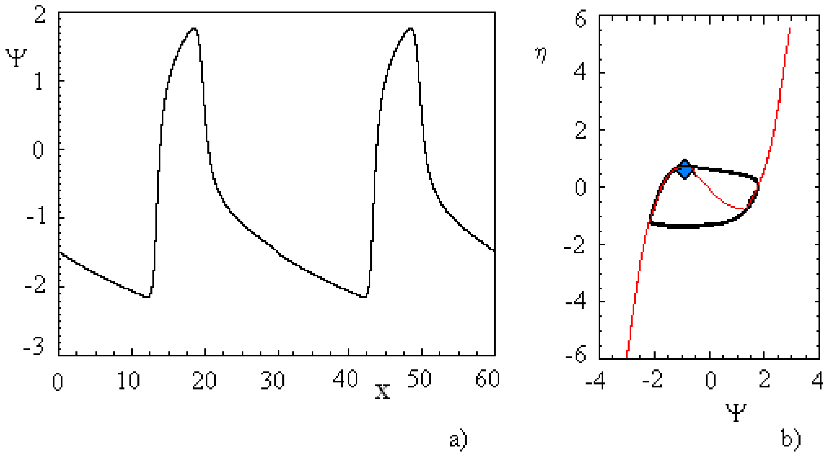

Figure 3. a) Two pulse solution; b) Phase portrait

Equation (6) contains a viscous damping term $\varsigma \dot{x}_i$. This term avoids the local oscillations in the system, being a benefit to the stability. When applying viscous damping it is necessary to avoid over-damping, which occurs when the root of the characteristic equation has two real solutions. That gives $\varsigma \le \sqrt{mk_1}$. In our study we use $\varsigma = \sqrt{0.1mk_1}$.

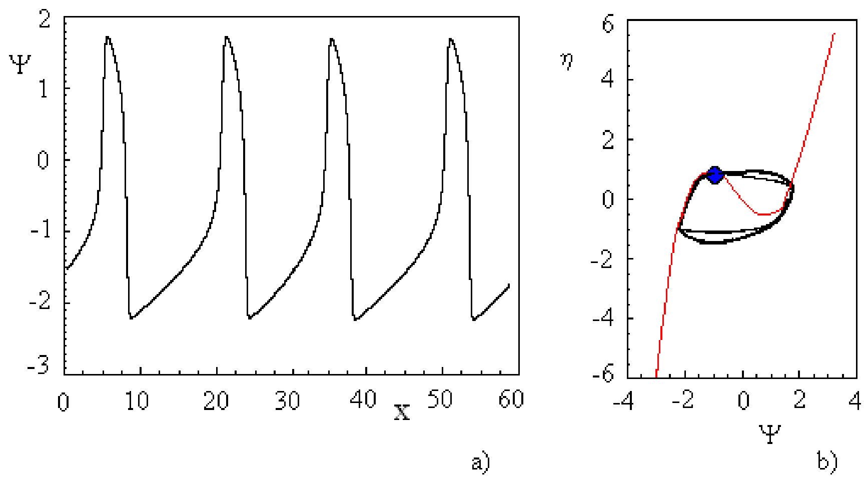

Figure 4. a) Four pulse solution; b) Phase portrait.

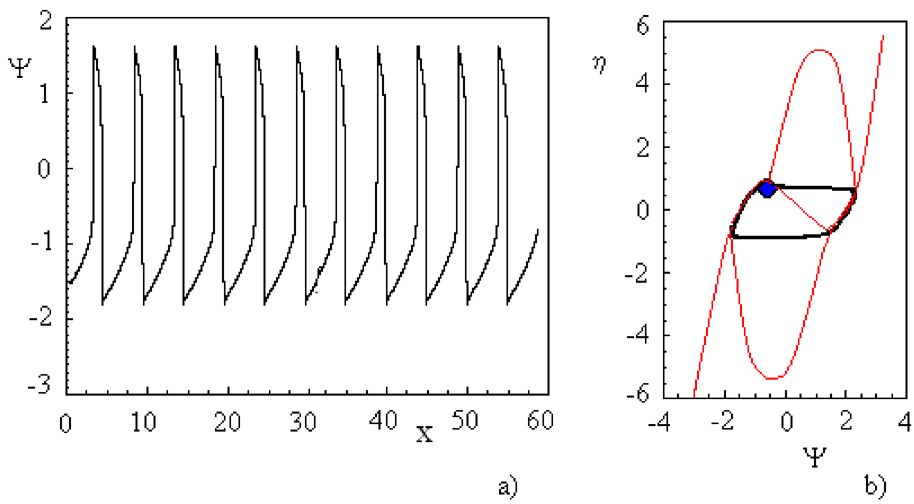

Figure 5. a) Eleven pulse solution; b) Phase portrait.

The stability diagram in the plane ($\log k_1$ - $\log v$) is presented in Figure 6. The line separates the regions of sliding and stick-slip motion.

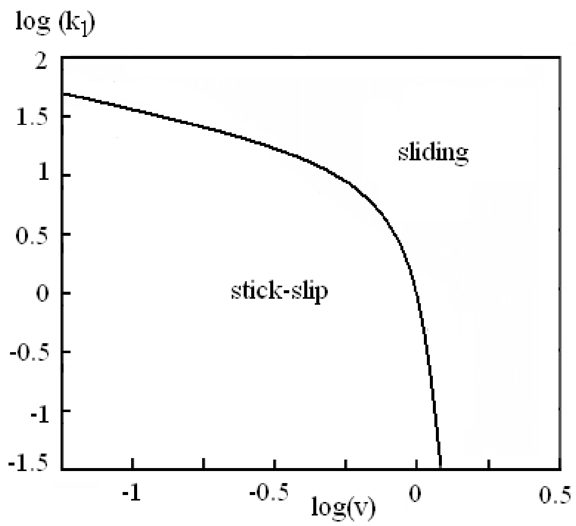

Figure 6. Stability diagram

4. Conclusion

The paper discusses the way of controlling the mechanical properties of sonic composites. These properties are referring to the full band-gaps generation, where the sound is not allowed to propagate due to complete reflections. The Bloch's theorem which is used to build the composite in the repetitive way represents the main concept for the architecture of the scatterers in order to obtain special features in their response to external waves**.** The band-gaps occur at different frequencies inverse proportional to the central distance between two scaterers. The complete reflection on the boundaries of scatterers is due to the full band-gap property itself, independent of the incident angle.

The technique for transforming the conventional foams into auxetic foams is discussed in this paper by exploiting the property of the governing equations to be written in a covariant form such that the metric is only involved in the material parameters. The geometric transformations lead to material properties that are, if not impossible to obtain, at least challenging for manufacture of new materials

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