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# Calculation of Digital Control Circuits using Scilab Environment

*The paper presents the computing of digital control circuits using Scilab environment. It exemplifies the influence of the sampling time in case of a transfer system described by a PT3 element composed of one PT1 and one PT2 element. For a continuous control system, the transfer function in z is computed and replaced with a digital control system. The presented calculation, done in Scilab, highlights the test responses of the process evidencing the systems performances.*

***Keywords***: *transfer function in z, sampling time, Scilab environment*

## 1. Introduction

Digital control systems are using the polynomial form of the transform function in z:

$$H(z)=\frac{b_m \cdot z^m + b_{m-1} \cdot z^{m-1} + ... + b_1 \cdot z + b_0}{a_n \cdot z^n + a_{n-1} \cdot z^{n-1} + ... + a_1 \cdot z + a_0} = \frac{num(z)}{den(z)}; n \geq m \quad (1)$$

The functions numerator and denominator can be introduced in Scilab environment in different ways [1], [2], i.e. .as line vectors, where the vectors elements are the polynomial coefficients of the *z* operator: num = [$b_m$, $b_{m-1}$,..., $b_1$, $b_0$] and den = [$a_n$, $a_{n-1}$,..., $a_1$, $a_0$].

## 2. Transfer function in z, sampling time, step and impulse response

Often digital controllers are designed for continuous adjustment path. The transfer function in *z* is defined through the retaining element and the sampling period, figure 1.

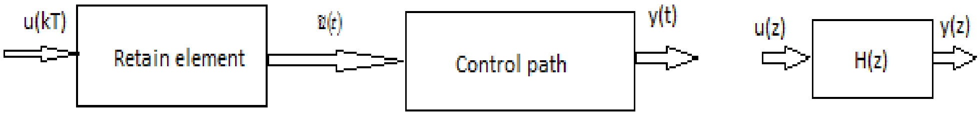


**Figure 1**. Defining the transfer function in z

The unilateral Z-transform of a digital sequence $x_n$, is given by

$$Z(x_n) = X(z) = \sum_{n=0}^{\infty} x_n z^{-n} , \tag{2}$$

where $z$ is a complex number in the so-called $z$-plane, mapping direct a discrete sequence $x_n$ from the sample domain $n$ into the complex plane $z$.

The input size of the retainer element is the string value *u(kT)*, determined by the control algorithm. The Digital Analog Converter produces a continuous size with discrete values, $\tilde{u}(t)$ as the input of the control path, figure 1. In simplest and most frequently encountered case, a retaining element of zero order is used for a sampling constant T. The Laplace transform of the control path $H_p(s)$, with zero order retaining element, the $Z$ transform is obtained as follows:

$$H_P(z) = \frac{z-1}{z} Z\left\{\frac{H_P(s)}{s}\right\} . \tag{3}$$

The discretization of the control path can be done with Tustin's relation (trapezoidal form):

$$s = \frac{2}{T}\frac{z-1}{z+1} . \tag{4}$$

The discretization is implemented with *dscr* function [6]. Starting from the Laplace transform of the control path, the sampling time T and the discretization method for computing the $Z$-transform function is chosen and if not otherwise stated, the retaining element is considered zero – order.

We follow the selection of the discretization time of a transfer system described through a PT3 type transfer function:

$$H_{PT3}(s) = \frac{1}{1+sT_1} \cdot \frac{\omega_0^2}{s^2 + 2 \cdot D \cdot \omega_0 s + \omega_0^2} , \tag{5}$$

composed of a PT1 element with pole $s_{p1} = -1/T_1 = -0.25$ and a PT2 element, with the poles $s_{p2,3} = -D\omega_0 \pm \omega_0\sqrt{D^2 - 1} = -2.5 \pm j \cdot 10$ and realize the discrete transfer function (computing algorithm).

The effect of the discretization time is analyzed for different values of the sampling time [3], figure 2, obtaining, based on the discretization criteria, $T$ = 0.3s. Also, a retaining element of zero order is considered. It can be seen that in case of increasing the discretization time, the specific oscillations for a PT2 element are not observed, those disappearing almost completely for $T$ = 0.4 sec.

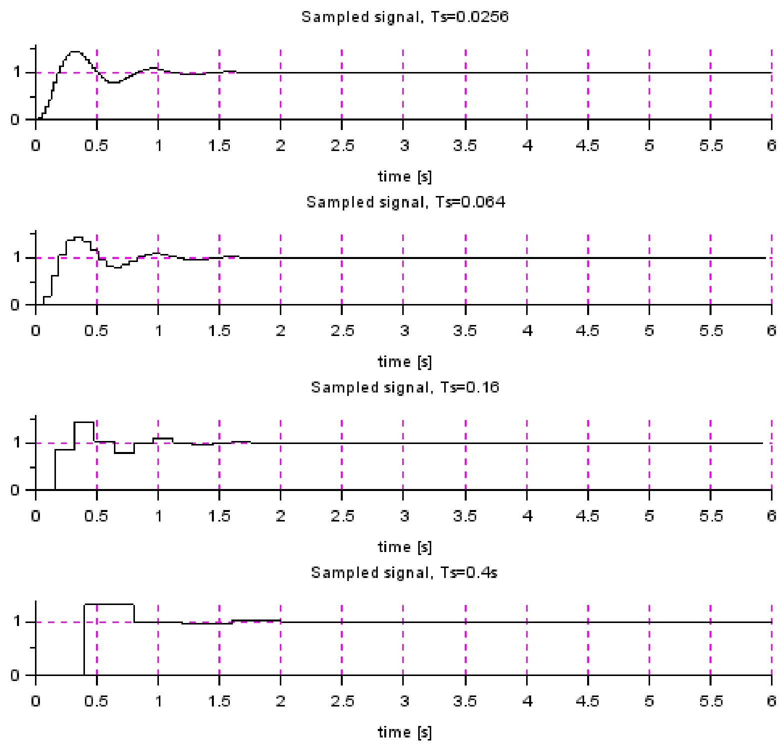


**Figure 2.** Step answer for different discretization constants

Figure 3 presents a control scheme in continuous time that will be replaced with a digital one, figure 4.

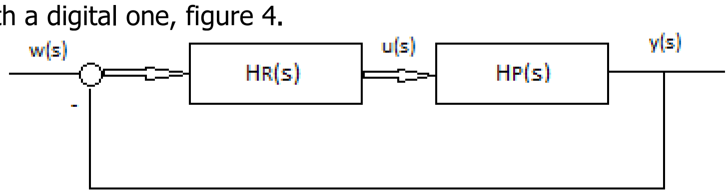


**Figure 3.** Block scheme of control system in continuous time

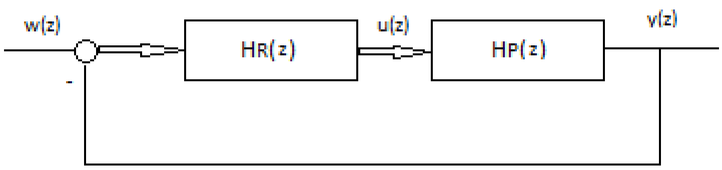


**Figure 4.** Block scheme of digital control system

We consider the system from figure 3 made up of *I* controller and the control path of PT1 type, with *K* = 3 and *T* = 2s, having the transfer functions [4]:

$$H_R(s)=\frac{K_R}{s},\ H_P(s)=\frac{K_S}{1+sT_S}, \tag{6}$$

resulting the global transfer function:

$$H(s)=\frac{3K_R}{s+2s^2}. \tag{7}$$

Thus for this PT2 type transfer function, is found that it can be reduced to two equal PT1 elements; the sampling will be done with a time constant of *T* = 1s.

The control system from figure 4, obtained by the z-transform, allows the simulation of the impulse and step response [5], using Scilab environment. The impulse response of the control path is determined by the function:

$$y(kT)=Z^{-1}\{H(z)\cdot w(z)\},\text{ with } w(z)=Z\{\delta(kT)\}=1. \tag{8}$$

Consequently figure 5, presents the impulse response of the command size *y(kT)* and of the control path.

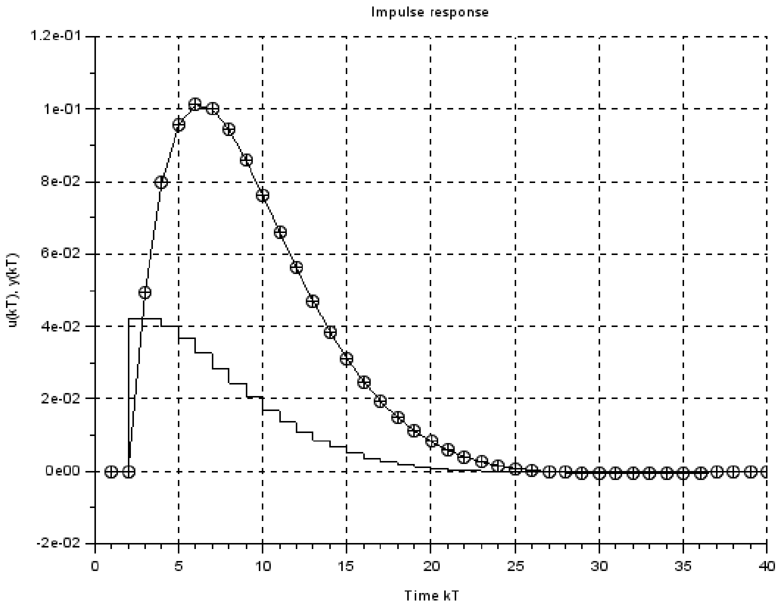


**Figure 5.** Impulse response of the control path and control command

The step response for the analyzed system is computed using the functions:

$$y(kT)=Z^{-1}\{H(z)\cdot w(z)\},\text{ with } w(z)=Z\{\sigma(kT)\}. \tag{9}$$

Figure 6 presents the step response of the command size and control path.

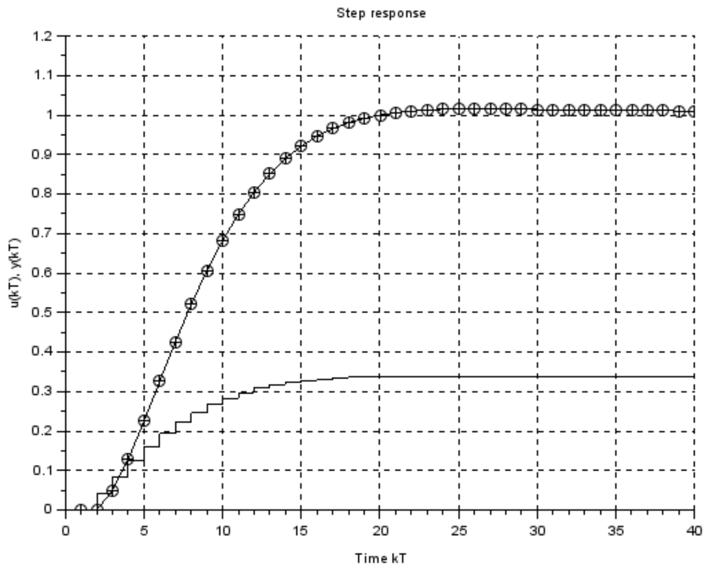


**Figure 6.** Step response of the control path and control command

### 3. Conclusion

The paper has highlighted the important role of the right sampling time in case of a PT3 type element, formed by a PT1 and PT2 element, through analyzing the step response, obtained in the Scilab environment, for four values of the sampling time. Then, a continuous control system is replaced with a digital control system, with the considered transfer function in *z* made by a I controller and an PT1 control path, for which, the step and impulse response was highlighted. For this reason, the Scilab / Xcos modeling and simulation programming language offers an effective practice instrument to compute the control circuit, being used on large scale.

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