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Numerical Methods Application for Reinforced Concrete Elements-Theoretical Approach for Direct Stiffness Matrix Method

A detailed theoretical and practical investigation of the reinforced concrete elements is due to recent techniques and method that are implemented in the construction market. More over a theoretical study is a demand for a better and faster approach nowadays due to rapid development of the calculus technique. The paper above will present a study for implementing in a static calculus the direct stiffness matrix method in order capable to address phenomena related to different stages of loading, rapid change of cross section area and physical properties. The method is a demand due to the fact that in our days the FEM (Finite Element Method) is the only alternative to such a calculus and FEM are considered as expensive methods from the time and calculus resources point of view. The main goal in such a method is to create the moment-curvature diagram in the cross section that is analyzed. The paper above will express some of the most important techniques and new ideas as well in order to create the moment curvature graphic in the cross sections considered.

Keywords: *manipulating of the stiffness matrix; direct stiffness matrix method; numerical methods; initial stress link between stiffness matrix and initial strain*

1. Introduction

After a deep analysis of incremental computational techniques and research carried out in linear structures analyses I oriented towards the direct method, considered by the author to provide more convenient on the potential for improvement and development. In fact, to characterize a finite element reinforced and/or prestressed these key points are sufficient. In fact, as stresses is shown figure 1, they delimit the working and stress/strain stages and reinforced concrete enough to be implemented in the calculations, according to Kwak and Kim [1]. Such an approach involves assessing the initial rigidity of the structural element (in this case it is proposed initial tangent stiffness) after revaluation in subsequent steps of secant stiffness with respect to load stage.

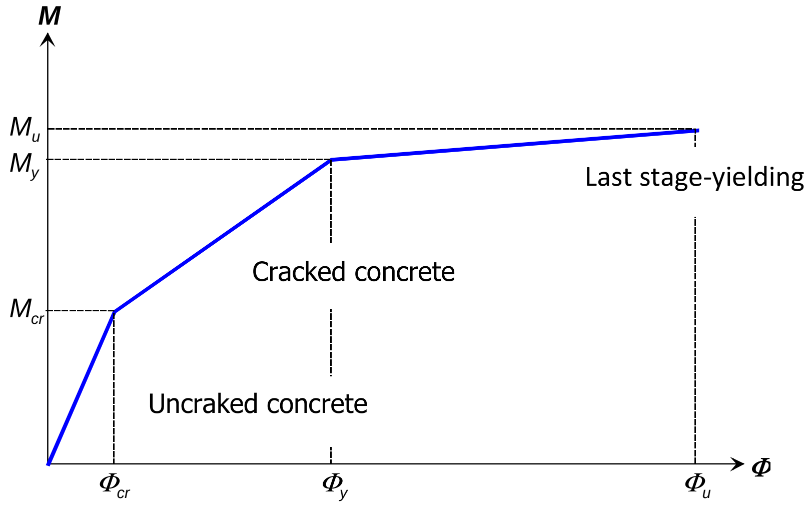

Figure 1. Schematic moment-curvature graphic

In a control cross sections a curved bar stiffness is governed by the relationship between bending moment and curvature.

As mentioned before, one of the main goals in order to achieve direct stiffness matrix method through an algorithm is to create control cross section that will be defining the moment curvature relation. Nevertheless to mention that this step is one of the main factor that will define the accuracy of a certain calculus and is of great importance For a given area, it can be built through the mesh in straight segments defined by pairs of points in the plane MO- Φ . Identification of coordinates (MO, Φ) must start mainly with identifying key points that define the behavior of an element . These are (see figure 1.), according to Melosh [2]:

-Concrete cracking in far fiber-stretched;

-Reinforcement yielding point;

-Reaching extreme resistance to compression compressed layer;

-Achieve specific ultimate strain in extreme compression layer.

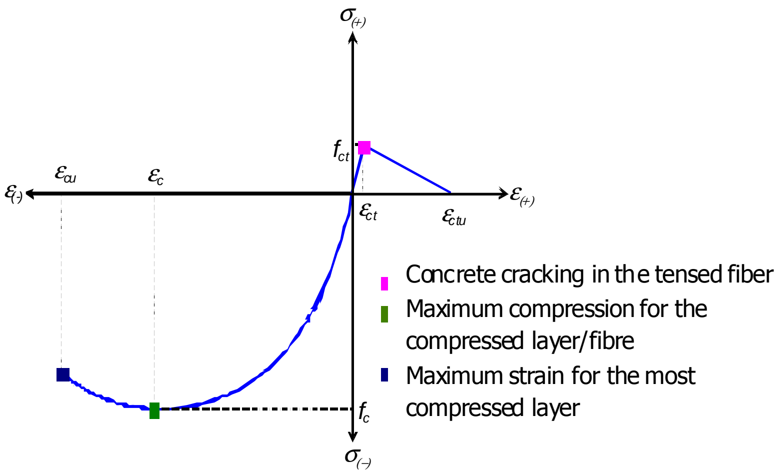

Figure 2. Schematic characteristic diagrams for concrete

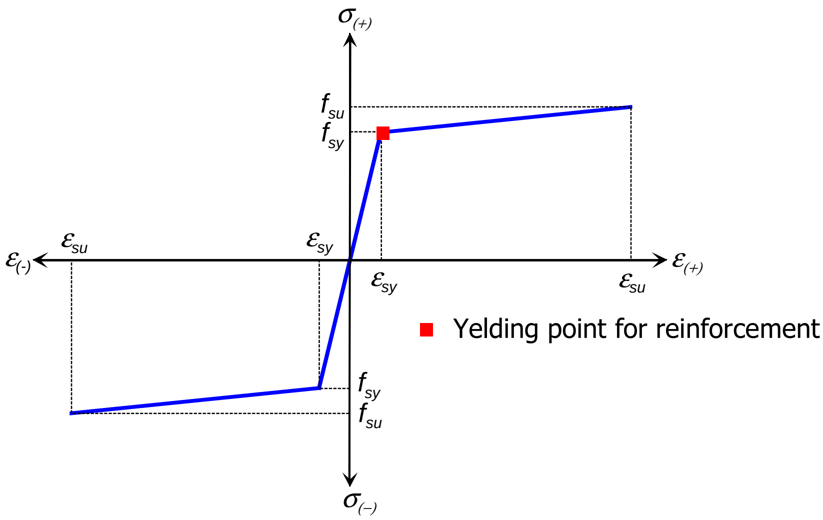

Figure 3. Schematic characteristic diagrams for reinforcement steel

As seen in figure 2 and in figure 3, the concrete has distinctive points due to geometry that is involved, especially due to the fact that the section becomes devided by layers that are either tensioned either compressed. As for the RC section the key points are imported from both concrete and reinforcement and obviously the values are distinctive as in figure 4, according to Bonora [3].

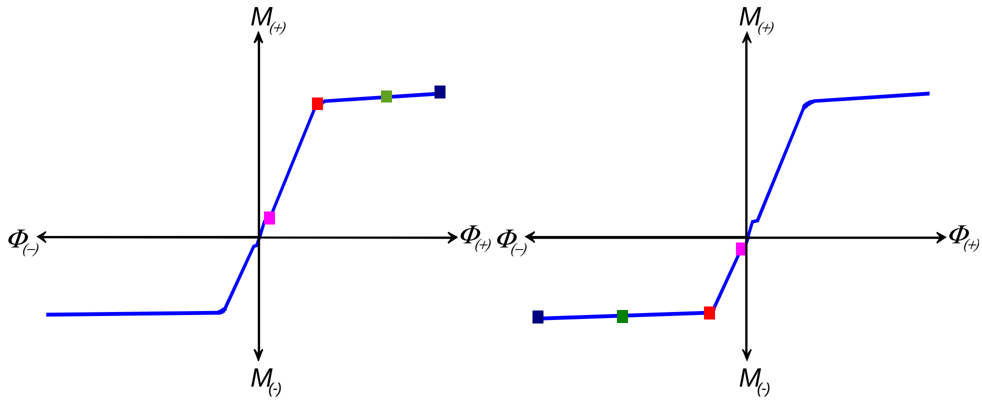

Figure 4. Schematic characteristic diagrams for RC elements

2. Iteration methods

The calculation of the key points is performed by a series of preliminary calculations as seen in figure 5 and figure 6.

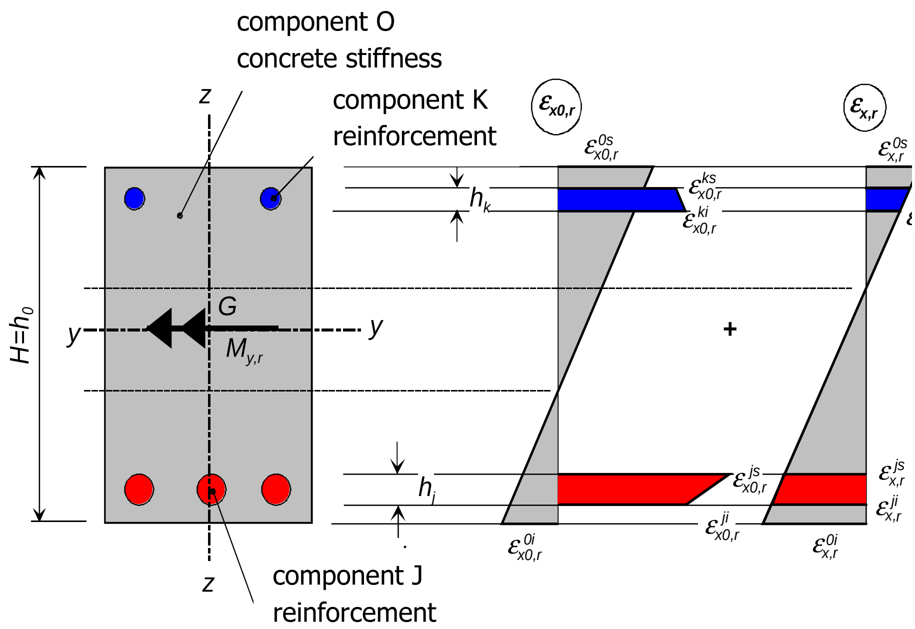

Figure 5. Cross RC section with initial strain along with strain from exterior efforts in "r" step analyses

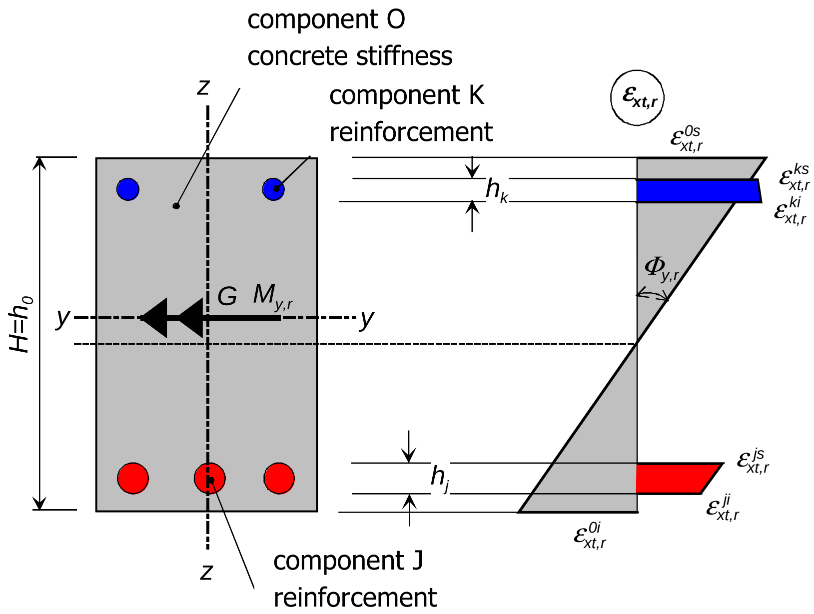

Figure 6. Cross RC section with initial strain in "r" step analyses

The calculation of the overall sectional analysis conducted by the method of direct iterations. Since direct stiffness method itself can be implemented in successive biographical analysis, conducted by constitution relations on updated models (in terms of time and effort, etc.), updating diagrams constituents and consequently moment-curvature diagrams reference should be made at the beginning of each stage of "r" load.

If is considered an element having a matrix component as embedded components "0" and "n". Prior to explaining the approach developed further please note that the initial implementation of efforts in the section analyses can identify several curves theoretically each component can have its own sectional curvature. I enter the following notations as Liu [4]:

-The vector of initial strains for lower fiber/layer of each component

$$\left\{\varepsilon^{i}_{0,r}\right\} = \begin{bmatrix} \varepsilon^{0i}_{0,r} \\ \varepsilon^{1i}_{0,r} \\ \varepsilon^{2i}_{0,r} \\ \vdots \\ \varepsilon^{ni}_{0,r} \end{bmatrix} \tag{1}$$

The vector of initial strains components fibre/layer at the bottom of each component.

$$\left\{\varepsilon_{0,r}^{s}\right\}=\begin{bmatrix}\varepsilon_{0,r}^{0s}\\ \varepsilon_{0,r}^{1s}\\ \varepsilon_{0,r}^{2s}\\ \vdots\\ \varepsilon_{0,r}^{ns}\end{bmatrix} \tag{2}$$

-Zz axis direction height of the vect. of all n +1 components for sectional analyses:

$$\left\{h\right\}=\begin{bmatrix}h_0\\ h_2\\ h_3\\ \vdots\\ h_n\end{bmatrix} \tag{3}$$

-Minimum rates vect. Zz axis direction of all n+1 components for sectional analyses

$$\left\{z_{min}\right\}=\begin{bmatrix}z_{min}^{0}\\ z_{min}^{1}\\ z_{min}^{2}\\ \vdots\\ z_{min}^{n}\end{bmatrix} \tag{4}$$

Given prestressed concrete practice (see figure 7.), further curvature will be considered as the main material sectional curvature, which provides embedding matrix of other components (for reinforced and prestressed concrete). Please note that the constituent relations are expressed as matrix formulations.

Applying the methodology of implementation of constitutional models developed by Mircea and collective [5], when given the interior bending moment for a certain deformability condition (cumulative initial stress and appropriate outside response) is calculated

$$M_{y,r} = \iint_D \sigma_{x,r}(\varepsilon_{x,r}) z dA = \iint_D \sigma_{x,r}(\varepsilon_{x,r}) z dy dz = \sum_k \iint_{D_k} \sigma_{x,r}^k(\varepsilon_{x,r}^k) z dy dz \quad (5)$$

where

$$\varepsilon_{x,r}^k(z) = \varepsilon_{x,r}^{0i} + z\frac{\varepsilon_{x,r}^{0s} - \varepsilon_{x,r}^{0i}}{H} + \varepsilon_{x0,r}^{ki} + (z - z_{min}^k)\frac{\varepsilon_{x0,r}^{ks} - \varepsilon_{x0,r}^{ki}}{h_k} \quad (6)$$

As expected, finding key points defining moment - curvature diagrams to step "r" load consists of a series of iterative processes , setting a specific strain of fibers from one extreme (stretched or compressed , depending on the nature of the key point). Figures 5 and figure 6 expresses iteration on how sectors convergence taking place from both sides of the solution, and specific deformations can be seen in table 1, that provides key points extreme control values.

Table 1. Specific strain/deformations for key points values

Key points	**Imposed static strain** $\varepsilon_{fix,r}$		**Start strain** $\varepsilon_{var,r}$	
	layer (+)	**layer (-)**	**layer (+)**	**layer (-)**
Concrete cracking	ε_{ct}	-	-	$\varepsilon_{cu}/2$
Yielding of reinforcement*	ε_{sy}	-	-	-
Maximum compression concrete effort	-	ε_c	$\varepsilon_{su}/2$	-
Maximum strain for compressioned concrete	-	ε_{cu}	$\varepsilon_{su}/2$	-

$\varepsilon_{fix,r}$ specific deformation , in "r" is considered to have the center of gravity right were tensioned reinforcement groups (layers) are positioned. At section level, by neglecting the adhesion, there is a variation in the specific strains and the nodal forces associated with the one shown in figure 6. Give initial strain/deformation vectors of relations (1) and (2) directly the assemble and calculation efforts and nodal forces associated by applying relations (5) and (7). Please note that this kind of efforts is easy to implement in thermostatic calculus as well as calculus that are related to certain chemical factors or dynamic factors.

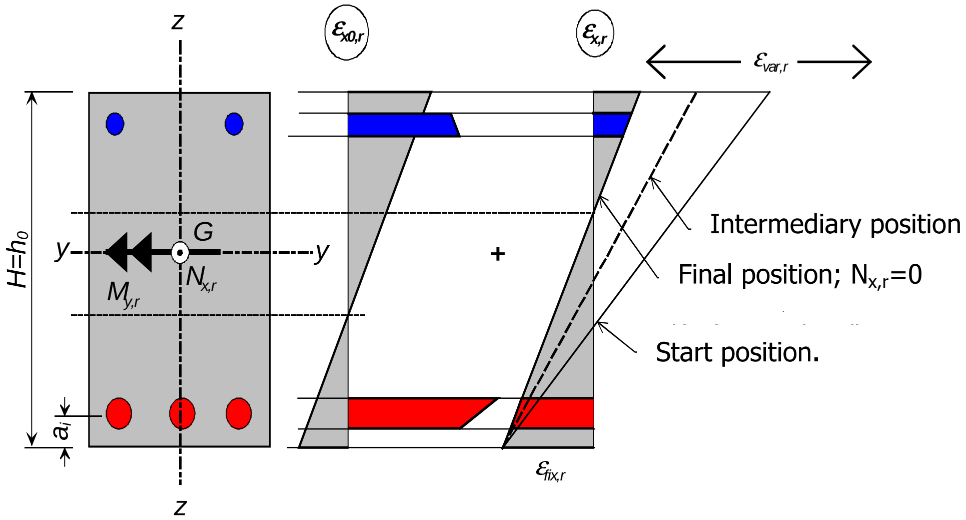

Figure 7. Schematic bending moment calculus with respect to strain from maximum stretched layer-left convergence

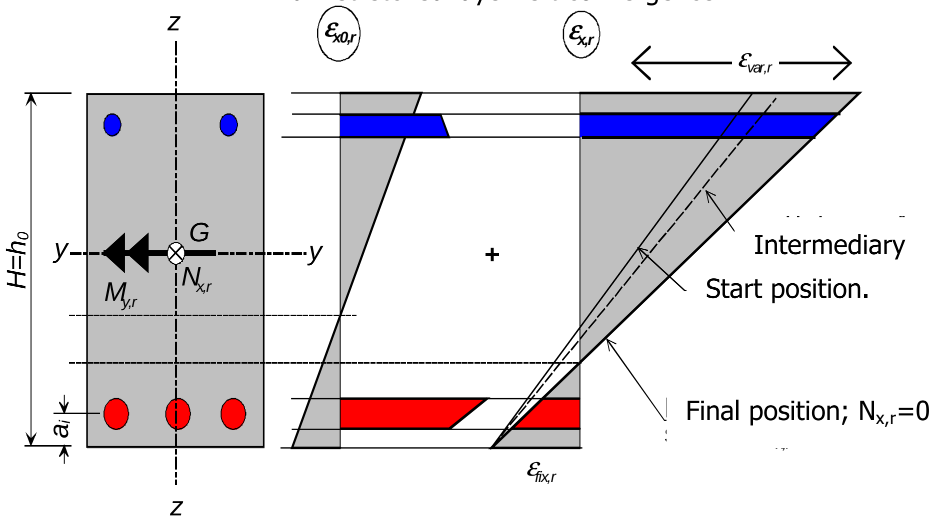

Figure 8. Schematic bending moment calculus with respect to strain from maximum stretched layer-right convergence

Convergence is achieved when equilibrium is obtained horizontally, that is to say in the present case, "r"=0, where the axial force Nx at each iteration is calculated by the relation (7) as follows, according to Wen [6].

$$N_{x,r} = \iint_D \sigma_{x,r}(\varepsilon_{x,r})dA = \iint_D \sigma_{x,r}(\varepsilon_{x,r})dydz = \sum_k \iint_{D_k} \sigma^k_{x,r}(\varepsilon^k_{x,r})dydz \qquad (7)$$

The elements subject to eccentric compression, convergence is obtained when axial interior effort is equal to the effort of compression applied.

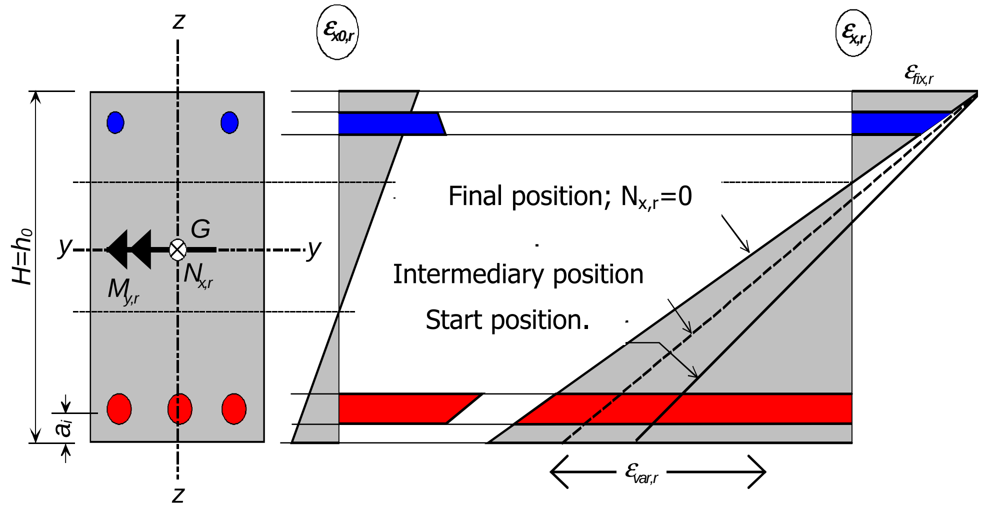

Figure 9. Schematic bending moment calculus with respect to strain from maximum compressed layer-left convergence

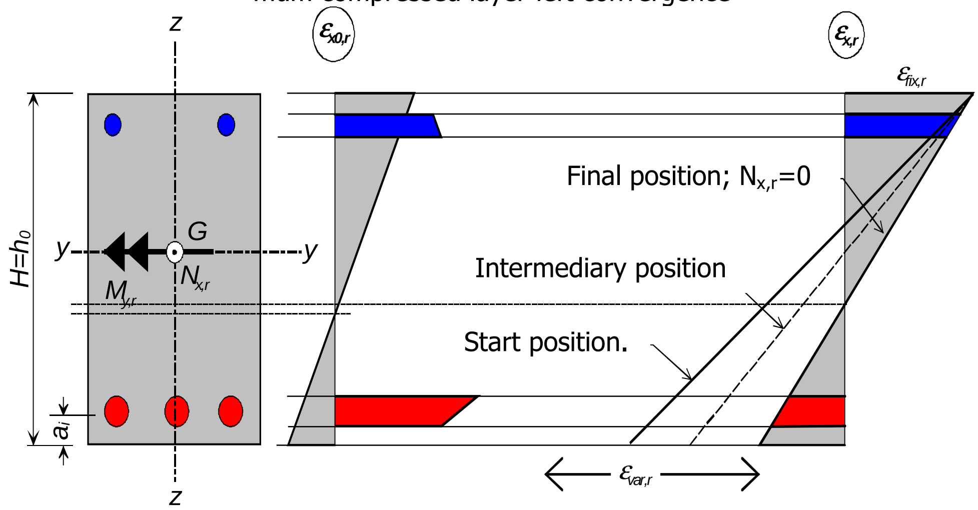

Figure 10. Schematic bending moment calculus with respect to strain from maximum compressed layer-right convergence

In addition to the checkpoints, you can identify other pairs (M, Φ) by taking other deformations imposed as fixed , especially in the compression area .The proposed method generally involves the following steps:

-Assuming constitutive models of materials from the cross section components (eg concrete, mild steel, high strength steel, etc.);

- Specific control is assumed for deformations of the key points $\varepsilon_{fix,r}$, r and deformations/strains $\varepsilon_{var,r}$, as well as "r" for starting iterative procedure ;

- Calculate axial relationship (7) and compared with external effort (curved sections it is null);

- If the convergence is recalculated $\varepsilon_{var,r}$ and restore the previous step, to achieve convergence;

- After convergence is obtained, the flexural bending moment is to be calculates according to the relationship (5) and the associated curve with the relationship as expressed in (8), as in figure 9.

$$\Phi_{x,r} \cong tan\left(\Phi_{x,r}\right) = \frac{\varepsilon^{0i}_{xt,r} - \varepsilon^{0s}_{xt,r}}{H} \tag{8}$$

- After obtaining checkpoints, to increase the refinement of the diagram is assumed other fixed values for specific control deformations $\varepsilon_{fix,r}$ in compressed area with $\varepsilon_{var,r}=\square_{su}/2$ and calculated in other sections of the moment-curvature diagram, with a repetitive structure as seen for key points.

- By comparison algorithm checkpoints are inserted in the string of points (M, Φ) to give the final shape of moment-curvature relationship through discrete points; any other point in the chart can be obtained with sufficient accuracy interpolating linearly between the nearest points on the chart.

There is the possibility that convergence is not achieved if there is a large flow of steel bars, in which case this checkpoint is missing in the chart. This situation is rare and is specific for sections with too large amount of reinforcement steel.

Building moment-curvature diagrams is a preoccupation of about half a century for scientists. In 1964 Pfrang and staff [7] proposed an analytical approach based on equal curvature interpolation of curves embedded in axial interaction diagram - time along the line corresponding to a constant axial effort. This is achieved by having previously analytical calculation of axial force and bending moment assuming a constant curvature and different values for specific extreme deformations/strains, as explained in figure 11. The method is very ingenious and required a huge amount of computing its implementation in practice through a series of charts reported quality of concrete, steel and section. Implementing the method in numerical applications is easy and can have a high degree of applicability, but may encounter difficulties in accurate assessment of key points.

Many other analytical methods, iterative and/or analytical methods were developed along past time. The method proposed in this paper brings in terms of new elements the integration components that make up the outline of a cross section and iterative determination of the points needed to build the chart as Mircea and collective [5] suggested the same way but integration diagrams obtained through an incremental approach - Raphson Newton [8], but using tangential sectional rigidity as Hognestad [9] proposed.

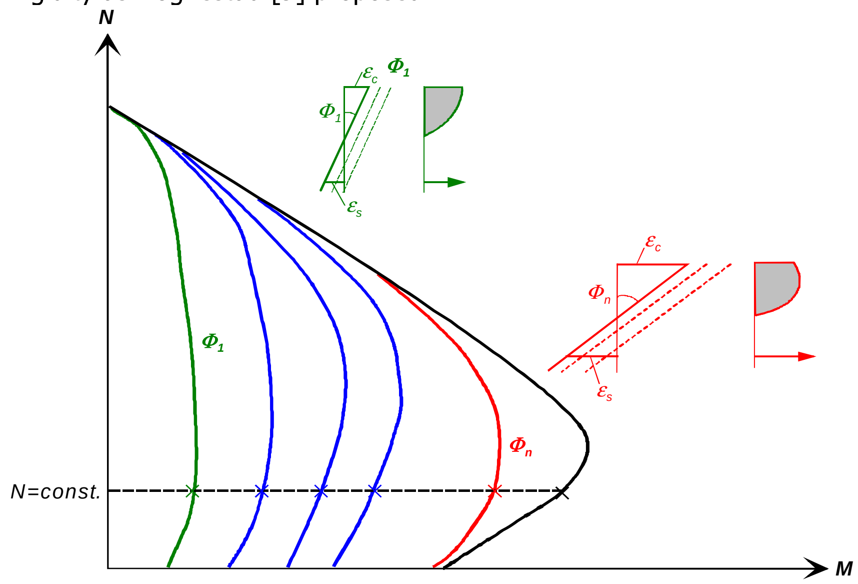

Figure 11. Building of moment-curvature graphic using equal curb lines as Pfrang –Siess – Sozen [7] sugested

And this approach, however, has difficulty in assessing the exact same control points. Numerous other applications build moment-curvature diagrams using numerical integration of layers.

3. Proposed algorithm

Algorithm is essential in understanding the method of work. The algorithm presents a new method to estimate the deformations that occur in a reinforced concrete element. It should be noted that the algorithm is quite difficult and difficult to be presented in full detail, so this algorithm will generally work and will present schematic working that were used to achieve secondary objectives of the computer soft program to be created. The claim that is necessarily required to develop algorithms that work properly was needed is a careful study of literature, to achieve adequate documentation.

Please note that this algorithm represents a new element of nonlinear method of calculation. It is observed that incorporation methods must be used to reduce the number of iterations. This approach shows a sustainable algorithm and delivers fast results. Besides this I would mention that this kind of approach is easier for programmers' point of view and are easy to understand.

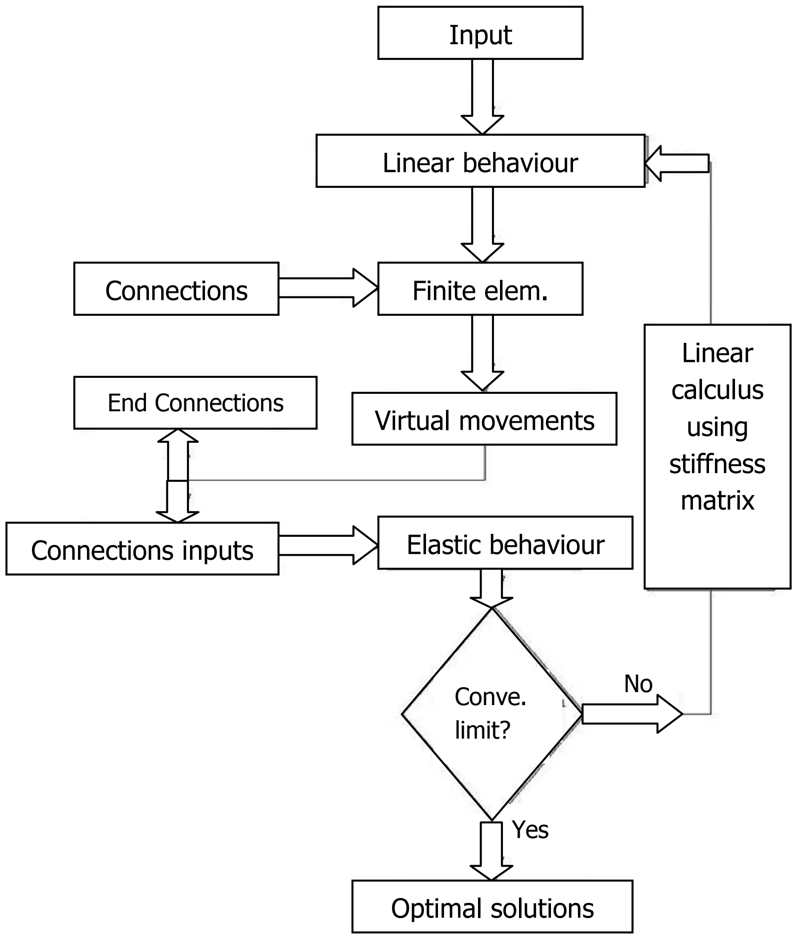

Figure 12. Algorithm suggested for static linear/nonlinear calculus by direct stiffness method

Please note that similar algorithms can be created and this algorithm can be improved as the available databases can be created so that more elements can be considered like aging, initial strain/deformations and new materials as well.

4. Conclusions

Based on the results expressed in this paper and the theoretical study some conclusions can be drawn:

1. Reliable algorithms can be created in order to address the static analyse for a reinforced concrete element;
2. The algorithm proposed in this paper is easy to understand and practical, and furthermore it was transformed in a functional computer soft program;
3. The idea of creating reliable methods for static calculus can be applied easily to dynamic calculus through the change of the stiffness matrix;
4. For a reliable computer soft is necessary a good calibration of the algorithm proposed in this paper in order to eliminate mathematical errors that are accumulated.

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