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Educational Approach to Information Transmission Channels of Data from Two Different Viewpoints: Combinatorial and Probabilistic

For future engineers is important to know how to make the transfer of information, data and tasks between different systems and thus we can say that the problem of communication is as important as the determination of the amount of information. Elements of information theory and communications are useful to all students of the electrical engineering specializations who must be able to collect, store, process and interpret information of any kind. In order to determine the amount of information we can use a mathematical tool based on elements of probability theory and algebra.

Keywords: amount of information, data transmission channel, bit coding, signal.

1. Introduction

The information is any message that makes a specification in a matter involving a degree of uncertainty [1]. Information theory deals with, among other things, how to make and organize a system for transmitting information on different channels from a source to a receiver. Having the beginning in the late 1940s through the work of Claude Shannon and Bell Labs, the information theory "concerns itself with various questions about information, including different ways of storing and communicating messages" [2].

A signal can be seen as a carrier of information about the evolution of a physical system. The amount of information is very important physical size. In this theory is important to establish the structure of a data channel (or telecommunications), the mode of transmission of information from a transmitter to a receiver able to receive this information without errors and with as few errors, the possibility of having a good data channel signal noise ratio and which are the most convenient ways to coding and decoding a transmitted signal so that the probability of full reception to be as high.

In information theory the coding is essential. The code is understood like "a set of signs and elementary symbols, along with a set of rules according to which these symbols are combined (from source) and analyzed (at the receiver) into the expression and interpretation of that information. Codification is the process of establishing a code"[1].

1.1 Algebraically approach for the quantity of information

Let L = {a, b, c} be a set of three letters (symbols). We intend to realize messages made from two letters (symbols). We see that we can form 3x3=9 messages. In this case the cardinal of the set L is 3 and the length of the message is n = 2. In the general case when |L| = N can form N^n messages, where |L| is the cardinal number of the set L.

To describe the amount of information we can not accept an exponential function, the mathematical function describing the amount of information (and in general, any physical quantity) is desired to be as linear as possible. It would also mean that as the number N of elements of a system (set) is higher and a message length is bigger, the amount of information increases exponentially, which is not true in reality! Therefore for the amount of information I we try to adopt a mathematical relations of the following type:

$$I = kn \tag{1}$$

In previous relation k is a constant we need to determine and n is the length of a message.

To determine the constant k we do the following: We consider two systems of signals characterized by numbers N_1 and N_2 (the cardinal numbers of these systems or sets). This signals systems are not chosen randomly. The choice is made so that $N_1^{n_1} = N_2^{n_2}$ and

$$I_1 = k_1 \cdot n_1 = k_2 \cdot n_2 = I_2 \tag{2}$$

From (2) we have:

$$\frac{k_1}{k_2} = \frac{n_2}{n_1} = \frac{\log_b N_1}{\log_b N_2}, \tag{3}$$

where the logarithms base b is a random one.

Relation (3) can be generalized and rewritten such as:

$$\frac{k_1}{\log_b N_1} = \frac{k_2}{\log_b N_2} = \ldots = \frac{k}{\log_b N} = k_0 \tag{4}$$

where k_0 is a constant that can be determined by imposing some conditions.

We made the determination of k such as:

$$k = k_0 \cdot \log_b N \text{ , } I = k \cdot n = k_0 \cdot n \cdot \log_b N \qquad (5)$$

Considering the constant k_0 = 1, the previous relation defines the amount of information I.

If b = 10, the measuring unit of the amount of information is called Hartley or dit (from decimal units). Whether you work in binary as almost all electronic digital computing machines working, the unit of measurement for the amount of information is called bit (from binary unit). If we use natural logarithms, the base b=e, the unit of measurement for the amount of information is called nat, nit or nepit.

Remark 1: Because in digital electronics, computing, programming we are working in binary base, we will use the relation:

$$I = n \cdot \log_2 N \qquad (6)$$

and the unit of the function I will be the bit .

Knowing the amount of information I, we ask ourselves how many bits do we need to represent a symbol? What is the minimum number of bits to represent that symbol? This is very important, because in this era when the microprocessors are modern and the computing speed increasing, it is essential to not have any redundant bit in representing any kind of symbol!

We have two particular situations:

A) When the number of elements N in a system that we propose to encode in binary is a power of 2, $N = 2^k$, in this particular case the minimum number of bits required for representation is I^{min}, so I^1 = I.

B) If $N \neq 2^k$ the minimum number of bits required for binary representation of any symbol from the system N, is $I^{min} = [I] + 1$ where [I] is the integer part of I.

So, the amount of information given by (6) may be expressed by the minimum of I^{min} binary symbols

$$I^{min} = \begin{cases} I \text{,when...} N = 2^k \\ [I] + 1\text{,when...} N \neq 2^k \end{cases} \qquad (7)$$

In (7) to the number of k is considered a set of natural numbers, because we can not work with fractions bit.

The amount of information given by (6) may be expressed by $I^r = nI^{min}$ binary symbols.

Example 1.1

If we have a message of n letters in a language and we intend to encode it in binary alphabet (considering the alphabet of that language consisting of 29 letters), we have:

$$I = n \cdot ([\log_2 29]+1) = 5n \text{ bits}$$

Example 1.2

We want to find what amount of information have the number® represented, say, with 705 decimal places. As we know, decimals after the point in the number ® are numbers from 0 to 9 and have no periodicity. So our set is characterized by N = 10. So:

$$I = 705 \cdot ([\log_2 10]+1) = 2980 \text{ bits.}$$

1.2. Probabilistic approach for quantity of information

This theory is based on the probability of occurrence of signals. Consider X a set of signals and A matrix that corresponds to the set. In this matrix each element of the set of signals x_i has associated the occurrence probability of that signal $p(x_i)$:

$$A = \begin{pmatrix} x_1 & x_2 & \ldots\ldots\ldots & x_{n-1} & x_n \\ p(x_1) & p(x_2) & & p(x_{n-1}) & p(x_n) \end{pmatrix} \quad (8)$$

The amount of information is evaluated according to the frequency of the signal. So the information is a function which depends on the occurrence probability of signal:

$$I(x_i) = f[p(x_i)] \qquad i = \overline{1,\ldots,n} \quad (9)$$

Experimentally, the study of information processes long established that if $p(x_i) > p(x_j)$, then $I(x_i) > I(x_j)$, where x_i and x_j and are two distinct signals from the initial set of signals X. As a signal occurs rarely, it is more charged with information. Instead, in case of very high frequency, we have more "redundant information", so the load of information is small.

If the signals (or events) x_i and x_j from the set X (or the space X of events) are independent it was established that the amount of information possess the property of additive:

$$I(x_i, x_j) = I(x_i) + I(x_j) \quad (10)$$

In 1928 R.V. Hartley gave the formula for determining the amount of information:

$$I = \log_b \frac{1}{p} \quad (11)$$

where I the quantity of information contained in any signal, symbol or message, p is the probability of this signal, symbol or message at the source.

To evaluate the amount of information that gives us the source we use the entropy. Claude Shannon used the signals distribution name or simply entropy for

$$H(X) = \sum_{x} p(x) \cdot I(x) \qquad (12)$$

$p(x)$ is the probability of event x from the set of events X.

The average amount of information that exists in the source is

$$H(X) = -\sum_{i} p(x_i) \cdot \log_2 p(x_i) \qquad (13)$$

where $p(x_i)$ is the occurrence probability of that signal x_i.

2. Applications

Next we give two examples to try to associate to the theoretical relationship (11) a physical sense.

Example 2.1. We toss a coin. The occurrence of one of the two sides of the coin will raise some uncertainty, so will contain some information. Tossing a coin which implies the occurrence of one of the two sides of the coin with equal probability 1/2 :

$$X = \begin{pmatrix} YES & NO \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} \qquad (12)$$

In the matrix of events, YES means the appearance of the coin face when it falls and NOT for the coin reverse appearance.

$$I(YES) = \log_2\left(\frac{1}{\frac{1}{2}}\right) = 1\text{bit} \quad \text{and} \quad I(NO) = \log_2\left(\frac{1}{\frac{1}{2}}\right) = 1\text{bit}$$

In digital electronics, YES event can be assigned to logical 1 and NO for logical 0 .

Example 2.2. Now take the example 1.2 presented in section 1.1 we write this time probabilistic model of the number π, the matrix for 705 decimal, as follows:

$$® = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ \frac{74}{705} & \frac{76}{705} & \frac{73}{705} & \frac{73}{705} & \frac{74}{705} & \frac{63}{705} & \frac{70}{705} & \frac{55}{705} & \frac{71}{705} & \frac{79}{705} \end{pmatrix} \qquad (13)$$

We achieve the binary encoding of the number π wanting to prove that the probabilistic model can be achieved by adopting a code on a much smaller number of bits than the combinatorial model.

First, we write the decimal numbers {0, 1, ... , 9} involved in the representation of the number π with 705 decimal, in decreasing order of their probabilities of occurrence as follows: 9 , 1, 0, 2 , 3, 4, 8 , 6, 5 and 7. After this, start to encode digits that appear with the highest probability (frequency). The numbers that appear most frequently are coded with shorter codewords.

So, as previously stated, we encoding the ten decimal digits arranged in descending order of likelihood:

$$\begin{cases} 9.......010 \\ 1.......011 \\ 0.......100 \\ 2.......101 \\ 3.......110 \\ 4.......111 \\ 8.......0000 \\ 6.......0001 \\ 5.......0010 \\ 7.......0011 \end{cases}$$

According to the above coding, the length of the number π (denoted $l(\text{®})$) written in binary code, with 705 decimals is:

$$l(\text{®}) = 3\cdot\left(705\cdot\frac{79}{705}+705\cdot\frac{76}{705}+705\cdot\frac{74}{705}+705\cdot\frac{74}{705}+\frac{73}{705}+\frac{73}{705}\right)+$$
$$4\cdot\left(705\cdot\frac{71}{705}+705\cdot\frac{70}{705}+705\cdot\frac{63}{705}+705\cdot\frac{55}{705}\right)=2371\text{ bits}$$

Note that in Example 1.2 where is presented the combinatorial approach to the quantity of information, the length of the natural binary representation of the number π was 2820 bits. Much higher than in this case. We can say that in this case of the probabilistic approach, the signal is fully loaded with information.

Referring to the two models presented above, the model that was imposed is the probabilistic one. It is a flexible mathematical model, much easier to use in encoding theory and allows the study of optimal encoding being applicable to both discrete spaces and to continuous spaces.

Example 2.3. The amount of information I(x) can not exist without entropy. The data transmission channels, the amount of information and entropy are opposite physical quantities. One is the degree of organization of a system (data transmission, electric drive, etc.) and the other the degree of disorder, of degradation and show us the multitude of errors which occur in a system. Therefore, in a data communications channel (of any type) entropy calculation is very important. The value of entropy depends on the accuracy of data transmission. Almost all parameters of data transmission channel depends on the H(X) and hence I(X). (transmission rate, efficiency, capacity of the data channel, etc.).

The structure of a data transmission channel with noise involved is presented in Figure 1.

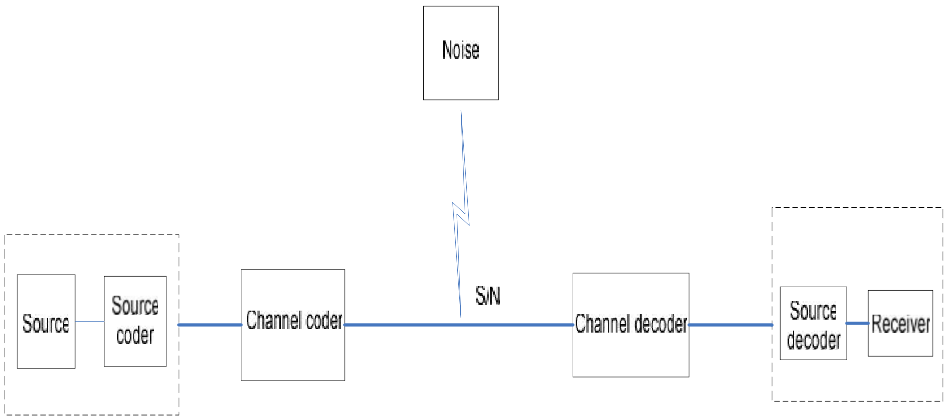

Figure 1. Data transmission channel with noise

Data transmission, speed and accuracy with which they reach their destination, speed of encoding and decoding the data is very important nowadays in many areas: telecommunications, radio, computers and even in electric drives in structural control scheme and digital control of various components and industrial equipment. Such a control block diagram of a logic and control industrial system using shift registers is shown in Figure 2.

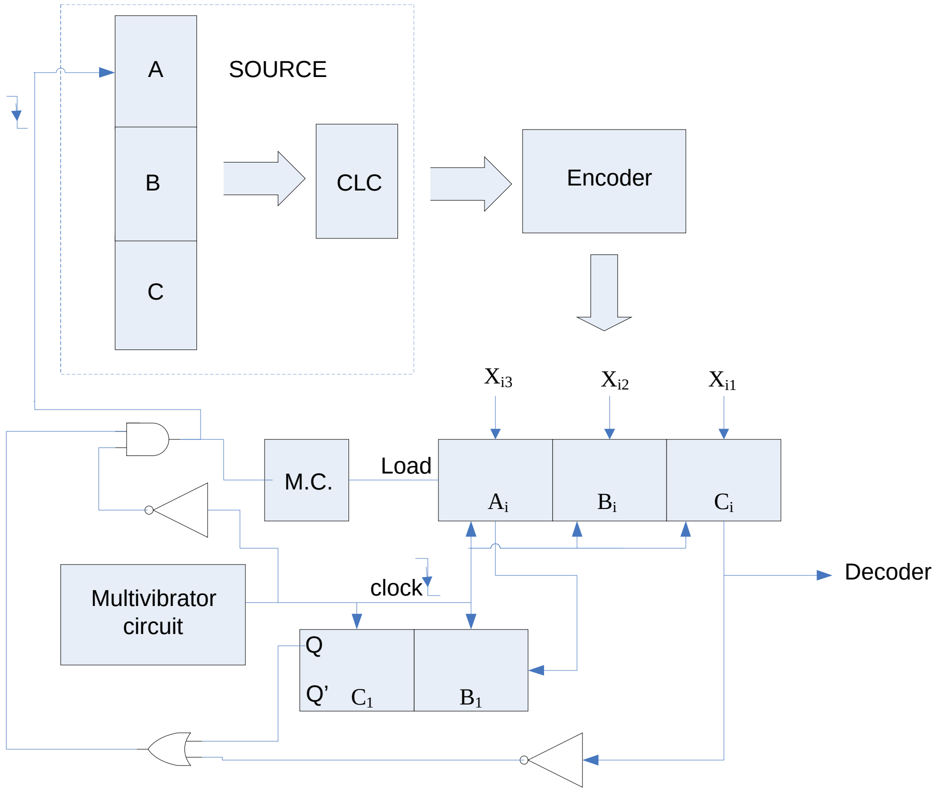

Figure 2. A control block diagram [7]

The block diagram represents the encoding of a series of bits that is transmitted through a channel data transmission without interference or disturbance very small. We are used in encoding three bits since we have a maximum of eight code words.

If the probability distribution of the signals source S, $s_i, i = 1...4$ is:

$$\begin{pmatrix} s_1 & s_2 & s_3 & s_4 \\ \frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{pmatrix}$$

the entropy of the signals source is given by (13)

$$H(S) = -\sum_{i=1}^{4} p(s_i) \cdot \log_2 p(s_i) = \frac{1}{2} + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 3 = 1{,}75 \text{ bit / event}$$

This is a good encoding and data compression for such a logical scheme. Obviously, this is possible in the sense that the data transmission channel is no interference, it actually found in any field of digital electronics and electrical (in electric drives too).

4. Conclusion

Establishing the mathematical definition of the amount of information I(x) and the unit of measure for this is of great importance given the fact that the size of I(x) are also important from the point of view of data transmission channels of the communications signals, not only in terms of information theory and codes, but also in telecommunications, radio etc. It characterizes the efficiency of a data transmission channel, channel capacity to transmit data unaltered, speed data transmission, the signal/noise (S/N) of a data channel etc., things of real importance in the fields of electronics and telecommunication, radio, propagation and transmission of signals, computers (especially in the field of data compression), etc. Probabilistic model approach to the amount of information I(x) is obvious, because the sizes involved in the transmission data are probabilistic. They're hard to know exactly all the phenomena occurring in a data transmission channel; thus they are addressed probabilistically. As such, defining the amount of information I(x) and setting the unit for this is very important for teaching and learning.

Information theory is useful to all students of engineering faculties because it can be applied in thermodynamics, quantum physics, in computing, in probability theory, communications networks, and in the electrical engineering, in electrotechnics and signal processing field.

Students may become more interested in mathematics classes, at these times, especially if there are chapters not only theoretical but also with applications in engineering. So besides using elements of information theory in signal processing and electrical courses may do so even in special mathematics courses for electrical and computer engineers.

References

[1] Raicu Ş., Roşca E., Burciu Ş., Teoria Informaţiei şi Coduri, Editura Agir, Bucureşti, 2012.
[2] Nelson M., The Data Compression Book, Second Edition, 1995.
[3] Angheloiu A., et al., Introducere în sisteme tehnice mari, Editura Militară, Bucureşti, 1980.
[4] Răduca E., Răduca M., Ungureanu Anghel D., Circuite Digitale, Editura Eftimie Murgu, Resita 2010.
[5] Răduca E., Electronică Aplicativă, “Orizonturi Universitare” Press, Timisoara 2003.

[6] Lee E.A., Messerschmitt D.G, Digital Communication, IEEE Press, 1994.
[7] Draghici S., Anghel Drugarin C.V., Raduca E., Study of Encoders with data transmission binary channels without interference. Simulation of the Encoder functioning using the Electronic Workbench software, 6th IEEE International Workshop on Soft Computing Applications SOFA, 24-27 July, Timisoara, 2014.

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