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Calculation of Industrial Power Systems Containing Induction Motors

The current paper proposes two methods and algorithms for determining the operating regimes of industrial electrical networks which include induction motors. The two methods presented are based on specific principles for calculating electrical networks: Newton-Raphson and Backward-Forward for iteratively determining currents and voltages. The particularity of this paper is how the driven load influences the determination of the motors operating regimes. For the industrial machines driven by motors we take into account the characteristic of the resistant torque depending on speed. In this way, at the electrical busbars to which motors are connected, the active and the reactive power absorbed are calculated as a function of voltage as opposed to a regular consumer busbar. The algorithms for the two methods are presented. Finally, a numerical study for a test network is realized and the convergence is analyzed.

Keywords: load flow analysis, induction motors, industrial power systems

1. Introduction

Calculating the power flow of a network containing induction motors implies modeling circuits containing non-linear elements. The induction motor may behave differently, according to the characteristics of the resistant torque of the industrial machine as a function of speed, TR(n). Because the operating regimes of the induction motor are characterized by the slip s, it's easier to express the industrial machines characteristics as TR(s):

$$T_R(s) = T_0 + T_{va} \cdot (1-s)^{\alpha} \qquad (1)$$

where T_0 and T_{va} are constants and exponent α indicates the type of industrial machine.

Paper [1] presents the newest and most efficient method to determine the periodic steady-state solution for three-phase induction machines based on the amalgamation of the Poincaré map. Using the on-load machine performance measurement as the input data, in paper [2] a less complicated yet accurate technique is proposed, which is formulated in an iterative algorithm with few assumptions. The results obtained clearly show the influence of the skin effect and the saturation on the rotor resistance, on the magnetizing and leakage reactance. The algorithm used in paper [3] for calculating networks containing induction motors considers a constant mechanical power. Paper [4] is focused on medium voltage distribution systems analysis and proposes to identify a new model of the fixed voltage nodes (PV nodes). The model is used within the backward/forward (b/f) analysis method applied to solve radial and weak interconnected systems. Paper [5] also presents models for induction motors. Paper [6] shows methods for calculating the electric network using the Newton-Raphson algorithm.

In [7] the back-forward method is presented. It iteratively determines the currents and voltages applied to radial networks. We must point out that, usually, the networks which contain induction motors are radial. Reference [8] shows a method for calculating the nonsymmetrical regimes of networks containing induction motors which applies the methods specific to nonsymmetrical power systems. Paper [9] deals with the efficient use of the electric power supply of an industrial network. As the active and reactive power depend on the type of the voltage and industrial machine, the method proposed in the present work is relevant.

The method for calculating the symmetrical regime of networks proposed in this paper takes into consideration the regime of the induction motors depending on the applied voltage and on the driven load characteristic.

2. Modeling Induction Motors

The induction motors from the network are modeled by the equivalent diagrams given in figure 1.

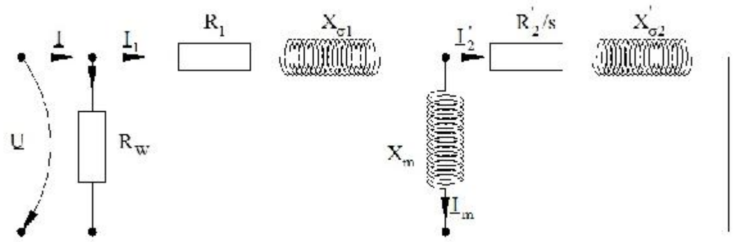

Figure 1. Equivalent diagram of the induction motor

From figure 1 we deduce the current through the rotor:

$$I_2^{'}(s)=\frac{s \cdot X_m \cdot U}{\sqrt{[R_1^2 \cdot X_2^{'2}+\left(X_1 \cdot X_2^{'}-X_m^2\right)^2] \cdot s^2+2 \cdot R_1 \cdot R_2^{'} \cdot X_m^2 \cdot s+\left(R_1^2+X_1^2\right) \cdot R_2^{'2}}} \tag{2}$$

where:

- U – phase voltage at the motor terminals;
- RW - equivalent resistance for losses through iron;
- R1 - stator winding resistance;
- R'2 - rotor winding resistance related to the stator;
- Xσ1 - reactance corresponding to the stator losses flow;
- X'σ2 - reactance corresponding to the rotor losses flow related to the stator;
- Xm - reactance corresponding to the magnetizing flow.

The electromagnetic torque is obtained from the power balance equation:

$$T_e(s)=\frac{\Delta P_2(s)}{s \cdot \Omega_0}=\frac{3 \cdot R_2^{'} \cdot I_2^{'2}(s)}{s \cdot \frac{\mu}{p}} \tag{3}$$

where Ω_0 is the angular speed of the rotating field, in [rad/s], ω=2·π·f is the pulsation of the alternate current, and f the frequency of the applied voltage.

$$\Omega_0=\frac{\mu}{p} \tag{4}$$

p is the number of pole pairs.

The slip in a certain regime is determined based on the torque balance equation:

$$T_e(s)=\Delta T_{fv}(s)+T_R(s) \tag{5}$$

where $\Delta T_{fv}(s)$, is the friction and ventilation torque considered proportional to the motor speed:

$$\Delta T_{fv}(s)=K_{fv} \cdot \Omega=K_{fv} \cdot \left[\frac{\mu}{p} \cdot (1-s)\right] \tag{6}$$

where Ω is the angular speed of the rotor, and K_{fv} is a proportionality coefficient determined from the power balance equation in the nominal regime.

From figure 1 results the regime impedance of the motor, $\underline{Z}(s)$.

3. Determining the Operating Regime of the Induction Motors

Determination of the induction motors operating regimes is done on each calculation iteration of the system load flow.

The motor operating regime is calculated as follows:

a. The parameters of the motor equivalent diagram are established from figure 1, using [2] or [5].

b. The dependence $T_R(s)$ of the driven industrial machine is established.

c. The phase voltage, U, is imposed by computing the regime of the network (section 4 or 5).
d. Equation 5 is solved and the regime slip, s, is determined.
e. The power absorbed by the motor is calculated:

$$\underline{S}_{mot} = 3 \cdot \frac{U^2}{\underline{Z}^*(s)} = P_{mot} + j \cdot Q_{mot} \tag{7}$$

Z* - is the complex conjugate of the motor impedance
f. The useful power at the motor shaft is determined:

$$P_{load} = T_R(s) \cdot \Omega = T_R(s) \cdot \frac{\mu}{p} \cdot (1 - s) \tag{8}$$

g. The active power losses in the motor are determined:

$$\Delta P_{mot} = P_{mot} - P_{load} = \mathrm{Re}\left(\underline{S}_{mot}\right) - P_{load} \tag{9}$$

4. Calculating the Power Flow of the Power System

As mentioned before, this calculation can be done by several methods. The Newton-Raphson method (applicable in the general case) and the backward-forward method for iterative determination of currents and voltages (applicable only to radial power systems) are presented below.

A. Newton-Raphson method

The power flow of the network is calculated as follows:
a. Reading the network data and imposing the slack busbar voltage U_s.
b. Establishing a set of types of busbars for each network n: load busbar (l), motor busbar (m) and voltage controlled busbar (g).
c. Forming the nodal admittance matrix $[\underline{Y}_n]$.
d. Initializing the iterative process:
- Current iteration number k=0
- Voltages of nodes equal to the rated phase voltages $U_i = \frac{U_{ratedi}}{\sqrt{3}}, i = 1,2,..,n, \quad i \neq s$
- Angles in busbars, δ_i=0, $i \in n$

e. Calculation of operating regimes and absorbed power by the induction motors, P_{moti}, Q_{moti}, $i \in m$. The methodology from section 3 is applied.
f. Calculation of active and reactive power injected into nodes depending on voltage and on angle:

$$P_i = 3\sum_{j=1}^{N} U_i \cdot U_j \cdot \left[G_{ij} \cdot \cos\left(\delta_i - \delta_j\right) + B_{ij} \cdot \sin\left(\delta_i - \delta_j\right)\right], i \in n \setminus s \tag{10}$$

$$Q_i = 3\sum_{j=1}^{N} U_i \cdot U_j \cdot \left[G_{ij} \cdot \sin\left(\delta_i - \delta_j\right) - B_{ij} \cdot \cos\left(\delta_i - \delta_j\right)\right],\ i \in n \setminus s \tag{11}$$

N – total number of busbars

$\underline{Y}ij$=Gij+jBij – matrix elements $[\underline{Y}n]$.

g. Deviation calculation of active and reactive power:

$$\begin{aligned}
\Delta P_i &= P_{loadi} - P_i,\ i \in l \\
\Delta P_i &= P_{gi} - P_i,\ i \in g \\
\Delta P_i &= -P_{moti} - P_i,\ i \in m \\
\Delta Q_i &= Q_{loadi} - Q_i,\ i \in l \\
\Delta Q_i &= -Q_{moti} - Q_i,\ i \in m
\end{aligned} \tag{12}$$

P_{loadi}, Q_{loadi} – active and reactive power injected into load busbars

P_{gi}– active power injected into voltage controlled busbars

h. Verification of convergence:

$$\begin{aligned}
\left|\Delta P_i\right| &\le \varepsilon,\ i \in n \setminus s \\
\left|\Delta Q_i\right| &\le \varepsilon,\ i \in l \cup m
\end{aligned} \tag{13}$$

ε – is the permissible power error.

If conditions (13) are satisfied then go to step o.

i. If k<2 then go to step k

j. Voltage controlled busbar type nodes are handled.

k. Calculation of the elements in the Jacobian matrix

$$\left[\frac{\partial P}{\partial \delta}\right], \left[\frac{\partial P}{\partial U} \cdot U\right], \left[\frac{\partial Q}{\partial \delta}\right], \left[\frac{\partial Q}{\partial U} \cdot U\right].$$

l. The Newton type equations system is solved

$$\begin{aligned}
[\Delta P] &= \left[\frac{\partial P}{\partial \delta}\right] \cdot [\Delta \delta] + \left[\frac{\partial P}{\partial U} \cdot U\right] \cdot \left[\frac{\Delta U}{U}\right] \\
[\Delta Q] &= \left[\frac{\partial Q}{\partial \delta}\right] \cdot [\Delta \delta] + \left[\frac{\partial Q}{\partial U} \cdot U\right] \cdot \left[\frac{\Delta U}{U}\right]
\end{aligned} \tag{14}$$

and deviations are determined $\Delta \delta_i,\ i \in n \setminus s,\ \frac{\Delta U_i}{U_i},\ i \in l \bigcup m$.

m. New values for the voltages and angles are determined

$$\begin{aligned}
\delta_i &= \delta_i + \Delta \delta_i,\ i \in n \setminus s \\
U_i &= U_i + \frac{\Delta U_i}{U_i} \cdot U_i,\ i \in l \cup m
\end{aligned} \tag{15}$$

n. The iteration number is incremented k=k+1 and go to step e.

o. If k<2 then go to step j.

p. The power and current flows are calculated, as well as the power losses in the power system.

The algorithm presented above shows that, in comparison with the traditional algorithm, the power absorbed by the busbars to which the motors are connected is determined in each iteration, depending on the voltage.

B. Backward-forward method for iterative determination of currents and voltages

As shown, this method can only be used for radial power systems. For simplicity, the consideration that there are no voltage controlled nodes in the network is made. The operating regime of the network is calculated as follows:

a. Reading the network data and imposing the slack busbar voltage Us.

b. Establishing a set of types of nodes in the network, n: load bus node type (l) and motor node type (m).

c. Calculation of line impedance $\underline{Z}_{ij}$, $i, j \in n$.

d. Initializing the iterative process:

- Current iteration number k=0
- Voltages of nodes equal to the nominal phase voltages

$$U_i = \frac{U_{ratedi}}{\sqrt{3}}, i = 1,2,..,n, \quad i \neq s \tag{16}$$

- Angles in nodes, δ_i=0, $i \in n$

e. The voltages values for the convergence control are retained: $\underline{U}_{memi} = \underline{U}_i$, $i \in n \setminus s$

f. Calculation of operating regimes and of power absorbed by the motors, P_{moti}, Q_{moti}, $i \in m$. The methodology from section 3 is applied.

g. The current absorbed in the nodes is calculated:

$$\underline{I}_i = -\frac{\underline{S}^*_{loadi}}{3 \cdot \underline{U}^*_i}, i \in l \tag{17}$$

h. Line currents are calculated as the sum of downstream currents plus currents due to line admittance (see figure 2):

$$\underline{I}_{ij} = \sum_{p \in j} \underline{I}_{jp} + \underline{I}_j + \underline{U}_j \cdot \left(\underline{Y}_{ji0} + \sum_{p \in j} \underline{Y}_{jp0} \right) \tag{18}$$

$$\underline{I}_i = \frac{\underline{S}^*_{moti}}{3 \cdot \underline{U}^*_i}, i \in m \tag{19}$$

i. The node voltages are calculated starting from the slack busbar:

$$\underline{U}_j = \underline{U}_i - \underline{Z}_{ij} \cdot \underline{I}_{ij} \tag{20}$$

j. Verification of voltages convergence:

$$\left\| \underline{U}_i \right| - \left| \underline{U}_{memi} \right\| = £, i \in n \setminus s \tag{21}$$

If convergence is not satisfied go to step e.

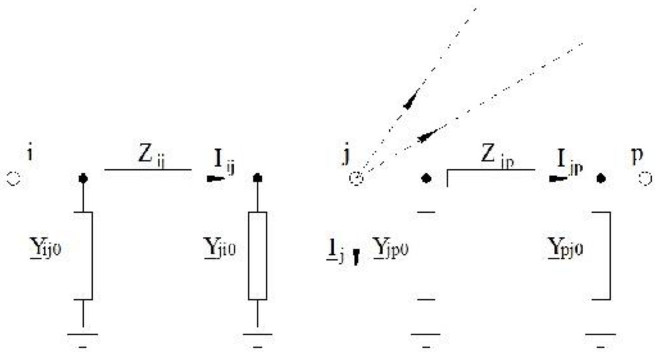

Figure 2. Explanatory diagram for calculating the lines

k. The power and current flows are calculated, as well as the power losses in the network.

5. Example of Applying the Algorithm

A. Characteristics of installations

For algorithm testing the industrial power system from figure 3 was used. The diagram contains 5 motors, 2 connected to 6 kV (M1, M2) and 3 connected to 0.4 kV (M3, M4, M5).

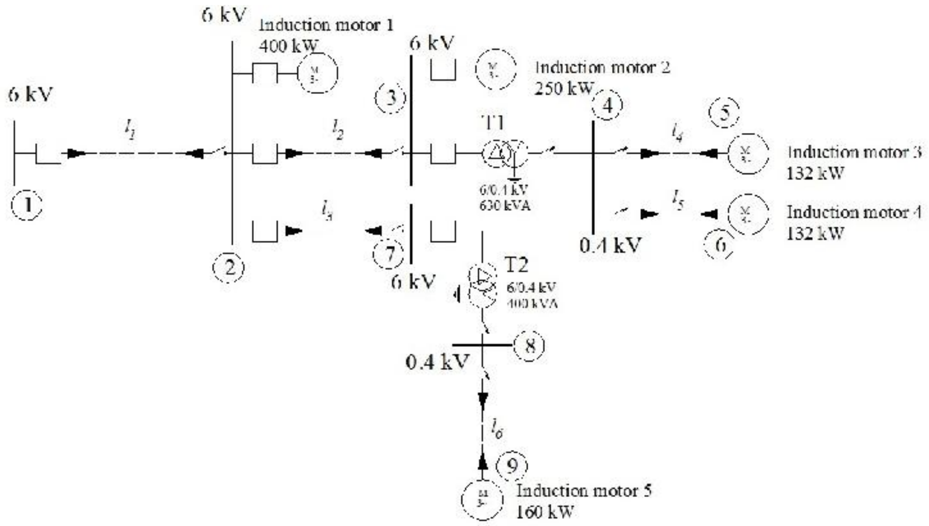

Figure 3. Single line diagram of the test system

Motors characteristics are given in table 1.

Table 1. Motors characteristics

No.	Motor	P_n [kW]	R_W [Ω]	R_1 [Ω]	R'_2 [Ω]	$X_{\sigma 1}$ [Ω]	$X'_{\sigma 2}$ [Ω]	X_m [Ω]
1	M1	400	4800	1.2259	1.1555	4.5525	6.2412	151.75
2	M2	250	7348.44	2.1055	1.9844	5.6146	12.2121	187.154

In table 2 there are presented the characteristics of the resistive torque of the driven industrial machine:

Table 2. Characteristics of the resistive torque for the industrial machine

No.	Motor	Characteristic $T_R(s)$ [Nm]	Working machine
1	M1	$1000 + 2500 \cdot (1 - s)^2$	Centrifugal pump
2	M2	$500 + 2300 \cdot (1 - s)$	DC Electric Generator
3	M3,M4	1810.23	Conveyor
4	M5	$200 + \frac{1050}{1 - s}$	Winder

B. Results obtained by Newton-Raphson method

The initial considered voltages are 6000 / √ 3 V and 400 / √ 3 V (nominal voltages of the network). Table 3 presents, for each iteration, the calculated errors for the power in the nodes with motors. Table 4 presents the voltages between phases at motor terminals and the slip for each iteration.

Table 3. Deviations of power at motor terminals on Newton iterations

Itera tion k	Motor M1		Motor M2		Motor M4		Motor M5	
	ΔP [kW]	ΔQ [kVAR]	ΔP [kW]	ΔQ [kVAR]	ΔP [kW]	ΔQ [kVAR]	ΔP [kW]	ΔQ [kVAR]
0	-388	-269.5	-234.	-216.3	-120.1	-112.3	-142.3	-95.79
1	-0.99	1.068	4.339	-3.055	-5.425	0.164	-4.649	-0.761
2	0.0	0.007	-0.01	0.0	-0.001	0.109	-0.004	0.057
3	0.0	0.0	0.0	0.0	0.0	-0.004	0.0	-0.001
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

Table 4. Voltages at motor terminals and slip on newton iteration

Itera tion k	Motor M1		Motor M2		Motor M4		Motor M5	
	U [V]	s [%]	U [V]	s [%]	U [V]	s [%]	U [V]	s [%]
0	6000	1.3241	6000	1.361	400	1.2409	400	1.2669
1	5990.67	1.3284	5986.6	1.3674	379.60	1.3906	386.5	1.3683
2	5990.56	1.3285	5986.4	1.3675	379.27	1.3932	386.2	1.3708
3	5990.56	1.3285	5986.4	1.3675	379.29	1.3931	386.2	1.3708
4	5990.56	1.3285	5986.4	1.3675	379.28	1.3931	386.2	1.3708

C. Results obtained through the back-forward method for iterative determination of currents and voltages

Table 5 presents voltages between phases at motor terminals and the slip for each iteration.

Table 5. Voltages at motor terminals and slip – backward-forward method

Itera tion k	Motor M1		Motor M2		Motor M4		Motor M5	
	U [V]	s [%]	U [V]	s [%]	U [V]	s [%]	U [V]	s [%]
0	6000	1.3241	6000	1.361	400	1.2409	400	1.2669
1	5990.49	1.3285	5986.32	1.3675	379.10	1.3941	386.13	1.3712
2	5990.56	1.3285	5986.41	1.3675	379.27	1.3932	386.22	1.3708

6. Conclusions

From the data presented the following conclusions can be drawn:

- The operating regime of induction motors considering the characteristic of the driven industrial machine, as a resistant torque function TR (s) , leads to a change of motor slip and of absorbed power (by about 0.24% for the active power and by 6.5% for the reactive power in the case of motor 4).
- Depending on the imposed accuracy, the convergence for the Newton-Raphson method is achieved in 3-4 iterations.
- Depending on the imposed accuracy, the convergence for the backward-forward method is achieved in 2-3 iterations.
- For both methods, the characteristic parameters vary significantly only in the first iteration.
- The advantage of the methods presented is that the operating regime of the induction motors was obtained as a result of the calculation of the operating regime of the industrial power system.

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