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A.c. Power Measurement Using Power Analyzer Associated with External Transducers. Accuracy and Uncertainty Evaluation

The development of the digital signal processors and their implementation in measuring technique has led to the manufacturing of power analyzers used as multifunction meters in industry, automation, tests and laboratory activities, monitoring and control of processes, etc. The parameters of a three-phase system can be known if the phase currents, the phase voltages and the phase difference between them can be known.A power analyzer has six inputs for currents and voltages measuring signals. The paper presents a method of determination of errors and uncertainties of electrical quantities measurement using a power analyzer associated with external transducers. The best estimation of measured quantity and uncertainty of measurement are used to report the result of measurement process.

Keywords: measuring signal, transducer, power analyzer, accuracy, uncertainty

1. Introduction

The power analyzer can record simultaneous samples for three currents and three voltages and can calculate the parameters of the circuit and power or energy of system.

When the values of the measured quantities are inside of the measuring ranges of the analyzer, the measurement can be made using its internal transducers.

The sampled values are temporary stored in local memory buffer and they are used in digital algorithms for RMS values and phase difference values computing [1]. If the values of the measured currents and voltages are out of range then the analyzer can be associated with external transducers to extend the limits of the measurement ranges [2].

Using the readings of the voltage, current and phase angle for each phase, provided by the analyzer in association with external transducers, the active, reactive and apparent power can be measured and calculated.

The difference between the measured or calculated value and the true value of the quantity gives the value of deviation.

The paper presents an algorithm to calculate the accuracy and uncertainty of power measurement using only the current and voltage calibrations of power analyzer and its external transducers.

The second section shows the principle diagram of measuring system for three-phase balanced load. The external transducers are current and voltage instrument transformers.

The third section presents the errors and uncertainties of power measurement resulted from repeated readings of current, voltage and power factor.

The fourth section shows the errors and the uncertainties of power measurement resulted from the indirect method used to obtain the result of measured quantity.

In the fifth section are presented other sources of errors of power measurement on site or in laboratory tests and the next section presents the associated uncertainties of these errors.

The final section presents some conclusions resulted from the previous developed theory.

2. The principle diagram of measuring system for three-phase load

The primary terminals of VT transducers are connected in parallel across the load and the primary terminals of CT transducers are connected in series with the load.

The secondary terminals of VT transducers are connected to voltage inputs and the secondary terminals of CT transducers are connected to current inputs of the power analyzer [3].

If we denote by I_1, I_2, I_3, U_1, U_2, U_3 the line currents and the line to neutral voltages to load terminals and by k_{c1}, k_{c2}, k_{c3}, k_{v1}, k_{v2}, k_{v3} the CT ratios, respectively VT ratios, then the primary side currents and voltages can be expressed [4]:

$$I_i = k_{ci} I_i' \tag{1}$$

$$U_i = k_{vi} U_i' \tag{2}$$

where i=1...3.

U_i', I_i' – the actual secondary voltages and currents at primary values of input quantities.

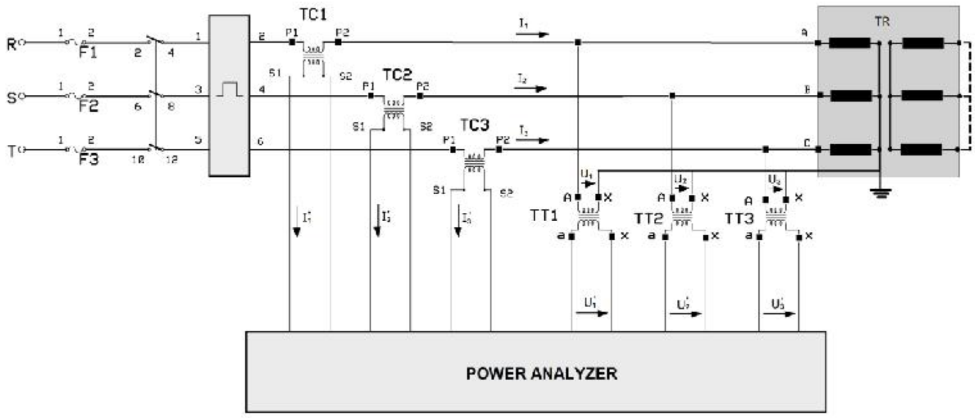

Figure 1. Block-diagram of measurement system for three-phase star load.

The calibration test report of the voltage and current transducers contains the actual value of transducer ratio (k_v or k_c), absolute error (Δk_v or Δk_c) and extended measurement uncertainty (u_{kv} or u_{kc}).

The calibration test report of the power analyzer includes the readings, absolute errors and extended measurement uncertainty u_v and u_c for all ranges.

With these conditions, the readings for voltages and currents on the primary side may have the values:

$$U_1 + \Delta U_1 = \left(k_{v1} + \Delta k_{v1}\right)\left(U_1' + \Delta U_1'\right) \tag{3}$$

$$I_1 + \Delta I_1 = \left(k_{c1} + \Delta k_{c1}\right)\left(I_1' + \Delta I_1'\right) \tag{4}$$

With the hypothesis that

$$I_1 + \Delta I_1 = \left(k_{c1} + \Delta k_{c1}\right)\left(I_1' + \Delta I_1'\right) \tag{5}$$

the absolute error of the voltage and current measurement , with external transducers and power analyzer, can be written:

$$\Delta U_1 = k_{V1} \Delta U_1^{'} + \Delta k_{V1} U_1^{'} \tag{6}$$

$$\Delta I_1 = k_{C1} \Delta I_1^{'} + \Delta k_{C1} I_1^{'} \tag{7}$$

The instantaneous values of the current and voltage for line 1:

$$i_1(t) = \sqrt{2}\, I_1 \sin(2\pi f t + \varphi_1) \tag{8}$$

$$u_1(t) = \sqrt{2}\, U_1 \sin(2\pi f t + \varphi_2) \tag{9}$$

$$u_{1s}(t) = \sqrt{2}\, U_1 \sin(2\pi f t + \varphi_2 + \pi/2) \tag{10}$$

$$p_1(t) = u_1(t)\, i_1(t) \tag{11}$$

$$q_1(t) = u_{1s}(t)\, i_1(t) \tag{12}$$

Similarly for the other two lines of power supply.

By analog to digital conversion, from the instantaneous values of the previous quantities, the samples $i_1(k)$, $u_1(k)$, $p_1(k)$ and $q_1(k)$, k=1...n are obtained.

Using these n samples the next parameters can be calculated [5]:

●RMS values of the current and voltage:

$$I_1 = \sqrt{\frac{\sum_{k=1}^{n} i_1^2(k)}{n}} \tag{13}$$

$$U_1 = \sqrt{\frac{\sum_{k=1}^{n} u_1^2(k)}{n}} \tag{14}$$

●Average values of the active or real power and reactive or imaginary power:

$$P_1 = \frac{\sum_{k=1}^{n} p_1(k)}{n} \tag{15}$$

$$Q_1 = \frac{\sum_{k=1}^{n} q_1(k)}{n} \tag{16}$$

$$S_1 = U_1 I_1 \tag{17}$$

●the value of the phase difference between current and voltage zero-crossing:

$$\varphi = \Delta k\, T_s\, \omega \tag{18}$$

Δk - the number of sampling period T_s between current and voltage zero-crossing;
ω - the voltage and current pulsation.

Because the calibration certificates of the current and voltage transducers and analyzer contain the informations about RMS values of current, voltage and transformer ratio or transfer factor, we will use the next equations for calculation of the active, reactive and apparent power [6]:

$$P_1 = U_1 I_1 \cos\varphi \tag{19}$$

$$Q_1 = U_1 I_1 \sin\varphi \tag{20}$$

$$S_1 = U_1 I_1 \tag{21}$$

These equations include the analyzer readings of current, voltage and phase difference or power factor.

The total power for unbalanced three-phase load is the sum of three-phase individual power:

$$P = P_1 + P_2 + P_3 \tag{22}$$

$$Q = Q_1 + Q_2 + Q_3 \tag{23}$$

$$S = S_1 + S_2 + S_3 \tag{24}$$

In the balanced three-phase load operation the total power can be calculated using short-form equations:

$$P = 3U_1 I_1 \cos\varphi \tag{25}$$

$$Q = 3U_1 I_1 \cos\varphi \tag{26}$$

$$S = 3U_1 I_1 \tag{27}$$

3. The errors and uncertainties of power measurement resulted from repeated readings

It is well known that the results of successive measurement may be different although measurement conditions are kept. Then the true value of measurand (P_k, Q_k or S_k, k=1,...,3) results from several statistical calculations using a sequence of n experimental data used as input data (U_i, I_i, φ_i, i=1...n). Further we consider only active power example. The samples of P_i result from input data:

$$P_i = U_i I_i \cos\varphi_i \tag{28}$$

The average of these samples is P_{med}:

$$P_{med} = \frac{\sum_{i=1}^{10} p_i}{10} \tag{29}$$

The experimental variance of the samples results from the equation,

$$s^2(P)=\frac{1}{n-1}\sum_{i=1}^{n}(P_i-P_{med})^2 \tag{30}$$

and the experimental standard deviation of the mean,

$$s(P_{med})=\sqrt{\frac{s^2(P)}{n}} \tag{31}$$

The type A of standard uncertainty, $u(P_{med})$ is equal to the experimental standard deviation of the mean, $s(P_{med})$:

$$u(P_{med})=s(P_{med}) \tag{32}$$

4. The errors and uncertainties of power measurement resulted from indirect method

In many cases the calibration certificate contain the errors and uncertainties only for currents and voltages and to calculate the active, reactive and apparent power values and errors can be used equations (19)-(21).

The output quantities P_k, Q_k and S_k are functions of input variables U_k, I_k and φ_k, k=(1,...,3). The deviations of output quantities are affected by the deviations of input quantities [7].

$$P_k=U_k I_k \cos\varphi_k \tag{33}$$

$$P_k+\Delta P_{kmm}=(U_k+\Delta U_k)(I_k+\Delta I_k)\cos(\varphi_k+\Delta\varphi_k) \tag{34}$$

$$\Delta P_{kmm}=(P_k+\Delta P_{kmm})-P_k \tag{35}$$

Using only the first order terms from Taylor serie of ΔP_{kmm} the following equation can be written:

$$\Delta P_{kmm}=\frac{\partial P_k}{\partial U_k}\Delta U_k+\frac{\partial P_k}{\partial I_k}\Delta I_k+\frac{\partial P_k}{\partial(\cos\varphi_k)}\Delta(\cos\varphi_k) \tag{36}$$

For k=1 the deviation of P_1 is:

$$\Delta P_{1mm}=\frac{\partial P_1}{\partial U_1}\Delta U_1+\frac{\partial P_1}{\partial I_1}\Delta I_1+\frac{\partial P_1}{\partial(\cos\varphi_1)}\Delta(\cos\varphi_1) \tag{37}$$

Similarly, can write the equations for Q_k and S_k.

The partial derivative of P_1 are the sensitivity coefficients of ΔP_{1mm}:

$$\frac{\partial P_1}{\partial U_1} = C_{PU} = I_1 \cos\varphi_1 \qquad (38)$$

$$\frac{\partial P_1}{\partial I_1} = C_{PI} = U_1 \cos\varphi_1 \qquad (39)$$

$$\frac{\partial P_1}{\partial(\cos\varphi_1)} = C_{P\varphi} = U_1 I_1 \qquad (40)$$

The deviations ΔU1 and ΔI1 result from equations (6) and (7) as a function of data included in calibration certificate ($\Delta U_1'$, $\Delta I_1'$, Δk_{V1}, Δk_{C1}).

To a steady-state point operation of a balanced load the power analyzer readings are P* for total power, I* for phase currents and U* for line to neutral voltages. The power factor cosφ* corresponding to this point results from the next ratio:

$$\cos\varphi^* = \frac{P^*}{3U^* I^*} \qquad (41)$$

If P is maximum active power measured at full scale of current I and at full scale voltage U,for the same power factor cosφ* results a load factor,

$$\eta = \frac{P^*}{P} 100[\%] \qquad (42)$$

Fig.2. shows the error ε(cosφ)=0,3% for cosφ*=0,15 and η=0,2 [2].

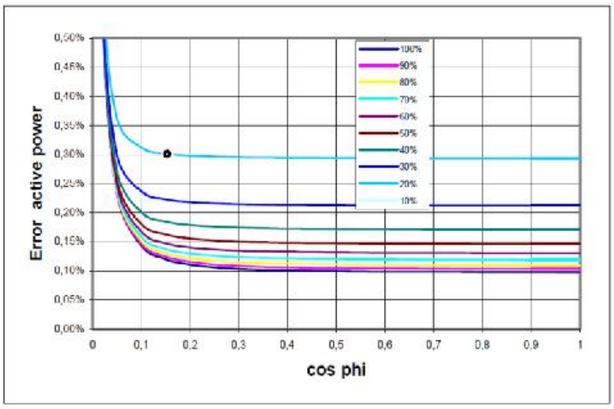

Figure 2.The active power measurement error as a function of power factor and load factor.

If the manufacturer does not gives the diagram from fig.2., then the deviation $\Delta(\cos(\varphi_1))$ results from the next equations:

$$\cos(\varphi_1 + \Delta\varphi_1) = \cos(\varphi_1)\cos(\Delta\varphi_1) - \sin(\varphi_1)\sin(\Delta\varphi_1) \qquad (43)$$

$$\cos(\varphi_1+\Delta\varphi_1)=\cos(\varphi_1)[\cos(\Delta\varphi_1)-\frac{\sin(\varphi_1)}{\cos(\varphi_1)}\sin(\Delta\varphi_1)] \tag{44}$$

$$\Delta\varphi_1\cong 0,...\sin(\Delta\varphi_1)\cong\Delta\varphi_1,...\cos(\Delta\varphi_1)\cong 1 \tag{45}$$

$$\cos(\varphi_1+\Delta\varphi_1)-\cos(\varphi_1)=\cos(\varphi_1)-\cos(\varphi_1)tg(\varphi_1)\Delta\varphi_1-\cos(\varphi_1) \tag{46}$$

$$\Delta(\cos(\varphi_1))=\cos(\varphi_1+\Delta\varphi_1)-\cos(\varphi_1)=-\cos(\varphi_1)tg(\varphi_1)\Delta\varphi_1 \tag{47}$$

If we take into account the active power values for all three phases (k=1,...,3) then the deviation of active power, ΔP_k, resulted from the deviation of power factor, $\Delta(\cos(\varphi_k))$, has the value,

$$\Delta P_k=U_kI_k\cos(\varphi_k+\Delta\varphi_k)-U_kI_k\cos(\varphi_k)=U_kI_k\Delta(\cos(\varphi_k)) \tag{48}$$

Similarly, the deviation of reactive power is affected by the deviations of input quantities:

$$\Delta Q_{kmm}=\frac{\partial Q_k}{\partial U_k}\Delta U_k+\frac{\partial Q_k}{\partial I_k}\Delta I_k+\frac{\partial Q_k}{\partial(\sin\varphi_k)}\Delta(\sin\varphi_k) \tag{49}$$

The sensitivity coefficients of reactive power are:

$$\frac{\partial Q_1}{\partial U_1}=C_{QU}=I_1\sin\varphi_1 \tag{50}$$

$$\frac{\partial Q_1}{\partial I_1}=C_{QI}=U_1\sin\varphi_1 \tag{51}$$

$$\frac{\partial Q_1}{\partial(\sin\varphi_1)}=C_{Q\varphi}=U_1I_1 \tag{52}$$

$$\sin(\varphi_1+\Delta\varphi_1)=\sin(\varphi_1)\cos(\Delta\varphi_1)+\sin(\Delta\varphi_1)\cos(\varphi_1) \tag{53}$$

$$\Delta\varphi_1\cong 0,...\sin(\Delta\varphi_1)\cong\Delta\varphi_1,...\cos(\Delta\varphi_1)\cong 1 \tag{54}$$

$$\Delta(\sin(\varphi_1))=\sin(\varphi_1+\Delta\varphi_1)-\sin(\varphi_1)=\Delta\varphi_1\cos(\varphi_1) \tag{55}$$

Also, the apparent power coefficients result from expressions:

$$\frac{\partial S_1}{\partial U_1} = C_{SU} = I_1 \tag{56}$$

$$\frac{\partial S_1}{\partial I_1} = C_{SI} = U_1 \tag{57}$$

and the deviation value is:

$$\Delta S_{kmm} = \frac{\partial S_k}{\partial U_k} \Delta U_k + \frac{\partial S_k}{\partial I_k} \Delta I_k \tag{58}$$

5. Other error sources

The measurement result can be affected by the active power consumed by:

●the connections conductors between primary terminals of transducers and load, ΔP_{pttl};

$$\Delta P_{kpttl} = R_{kpttl} I_k^2 \tag{59}$$

●the connections conductor between secondary terminals of transducers and terminals of power analyzer, ΔP_{sttpa};

$$\Delta P_{ksttpa} = R_{ksttpa} I_k'^2 \tag{60}$$

●the primary and secondary circuits of transducers, ΔP_{pst};

$$\Delta P_{kpst} = R_{kpt} I_k'^2 + R_{kst} I_k'^2 \tag{61}$$

●the internal transducers of power analyzer, ΔP_{itpac} – for current transducer, ΔP_{itpav} – for voltage transducer.

$$\Delta P_{kitpac} = R_{kitpac} I_k'^2 \tag{62}$$

$$\Delta P_{kitpav} = \frac{U_k'^2}{R_{kitpav}} \tag{63}$$

6. The uncertainties

Type A uncertainty has been presented in paragraph 3. This is the experimental standard uncertainty for a serie of repeated readings.

$$u_A = u(P_{med}) \tag{64}$$

The type B uncertainty has many components resulted from the contributions of the input quantities and the losses in circuits of setup. Generally, these contributions have low values compared to the power needed to load operation. The uncertainties associated with the previous deviations, Δ_n, have rectangular distributions and their values are:

$$u_{Bn} = \frac{\Delta_n}{\sqrt{3}} \tag{65}$$

The summary uncertainty budget contains:

- type A uncertainty, u_A;
- uncertainty associated with indirect measurement method, u_{kmm};

$$u_{kmm} = \frac{\Delta P_{kmm}}{\sqrt{3}} \tag{66}$$

- other uncertainties u_{Bn}.

Given these assumptions the combined uncertainty [8], [9] results from the equation:

$$u(P) = \sqrt{u_A^2 + u_{kmm}^2 + \sum_{i=1}^{n} u_{Bn}^2} \tag{67}$$

For a 95% level of confidence the coverage factor k has value 2 and the extended uncertainty is

$$U = ku(P) \tag{68}$$

The result of measurement can have the value

$$P = P_{med} \pm U = P_{med}(1 \pm U/P_{med})...[W] \tag{69}$$

or in percents,

$$P = P_{med}(100 \pm £)...[\%] \tag{70}$$

Similarly, we can find the true values of reactive and apparent power and can report the result of measurement using the same procedure.

7. Conclusion

The result of measurand, that is a function of more than one variable, depends on the deviation of each input measured quantity.

The power analyzer associated with external transducers are a powerful tool to measure various quantity: active, reactive and apparent power, resistance, reactance and impedance of load, currents, voltages and power factor in circuits.

Because of high cost of calibrating procedures for all output quantity, sometimes the calibrations are made only for currents and voltages measurands.

Then the errors and uncertainties for the other output quantities result from well known equations of electric sciences.

The values of deviations result from the calibrating certificate of the external transducers associated with the power analyzer.

The reported result of measurement depends on:

- the limited precision of the measurement tools;
- the method in which the measurement is performed;
- the human operator skills;
- the configuration of the setup used in the performed tests.

All of this factors must be taken into account to find the best estimate of measurand.

Because they perform multiple functions, the power analyzers are used both in laboratory tests and on site tests and measurement. Their usefulness is important in:

- no load and short circuit tests of three phase power transformers and machines;
- monitoring of three phase balanced or unbalanced loads;
- quality of the power and energy analysis in networks which operate in nonsinusoidal and asymmetric conditions;
- monitoring of variable frequency and voltage power supply for a.c. drives;

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