Study on the Identification of an Actuator

The paper presents the results obtained from the study conducted on an execution element in view of its modelling and identification. The mathematical model obtained from the studied execution element constitutes the basic information serving to the design of the adjustment algorithm of the automatic system where the actuator will be introduced, as well as the establishment of the technical parameters of the static converter for the supply of the electric motor.

Keywords: actuator, identification, transfer function, modelling, indicial response

1. Introduction

In order to design the command algorithm of the adjustment system we need to know the mathematical model of the fixed section of the system as well as the calculation method of the regulator in correlation with the specific performances and the technical characteristics of the process.

The execution element subjected to the analysis drives a syringe and is part of the structure of a piece of medical equipment requiring the control of the injecting speed of a fluid.

The analytical modelling of an automatic system element, based on the laws of physics, leads to the obtaining of a set of mathematical relations between the inputs / entries and outputs / exits of the studied element.

In the paper we proceeded to the identification of the non-parametric dynamic model of the actuator based on the recording of the indicial response.

The experimental results obtained were used through their comparison with the analytical mathematical model of the element resulted from the processing of the transfer functions of each component section of the element.

The experimental identification of the actuator supposed the completion the following stages [1]:

a) Construction and attachment to the actuator of a revolution incremental transducer;
b) Experimental determination of the static characteristic of the execution element;
c) Preparation of the recording / measurement of signals at the entry and exit of the actuator;
d) Establishment of the structure of the actuator model;
e) Determination of the model parameters;
f) Validation of the model.

The knowledge of the automatic system elements by analytical modelling or by identification is currently used by many researchers, and the results of the researches are published in various papers: [1], [2], [3], [4], [5].

2. Theoretical considerations

The actuator is considered to be an element of linear automatic system obtained by the interconnection of several mechanical and electrical sub-assemblies. By means of its indicial response we aim at determining the mathematical model in the operational range, as well as the establishment of the parameters intervening in the model.

2.1. Preparation of the signals recording

The main diagram of the actuator identification is presented in figure 1.a., where as figure 1.b. shows the experimental installation used by the authors.

![Diagram](image)

**Figure 1.a.** Diagram related to the actuator identification.

where: 1- electric power supply source;
2- oscilloscope;
3- actuator;
4- revolutions transducer;
5- PC.

The unitary step signal $u(t)$, applied at the entry of the actuator is a voltage signal obtained from a source of variable direct current/voltage source ($u(t)=15/15=1$). The indicial function $n(t)$, the function form the actuator exit, is
measured with a high-revolution laser digital transducer, without contact. The indicial function $n(t)$ is recorded with the help of a PC connected to the transducer through the USB interface.

**Figure 1.b.** View of the experimental installation.

### 2.2. Establishment of the model’s structure

The structural diagram of the actuator model is presented in figure 2

![Figure 2. Structure diagram of the actuator.](image)

where: $H_1(s)$ - transfer function of the static converter; $H_2(s)$ - transfer function of the electric motor; $H_3(s)$ - transfer function of the tachogenerator; $H_4(s)$ - transfer function of a mechanical transmission; $H_5(s)$ - transfer function of incremental transducer.

The structural diagram of the adjustment system (SRA) where the actuator is included is shown in figure 3.
Fig. 3. Structural diagram of S.R.A.

where: 
- \( H_R(s) \) - transfer function of the regulator;
- \( H_E(s) \) - transfer function of the electric motor;
- \( H_P(s) \) - transfer function of the process.

The actuator can be considered as being obtained by the series connection of 5 elements of the linear actuator system, each characterised through its own transfer function:

a) the static converter supplying the electric motor with a filtered direct voltage ranging between \((0 \div 15) \text{ V}\), with the transfer function \( H_1(s) \);

b) the direct current motor, with transfer function \( H_2(s) \);

c) the direct current tachogenerator, with the transfer function \( H_3(s) \);

d) the mechanical transmission (reducing gear) with the transfer function \( H_4(s) \);

e) the incremental transducer with the transfer function \( H_5(s) \).

The analytic modelling allowed the establishment of the expressions of the transfer functions of each element as follows:

a) The modelling of the static converter

The static converter appropriate for the supply of the direct current motor is a four-quadrant one, with the transfer function:

\[
H_1(s) = \frac{K_1}{1 + T_1 \cdot s}, \tag{1}
\]

where: 
- \( K_1 \) is the transfer coefficient of the converter;
- \( T_1 \) is the dead time constant \((T_1 = f \cdot p)\);
- \( f \) - frequency of the redressed voltage, \( f=50\text{Hz} \);
- \( p \) - number of pulses of the voltage redressed for a period.

b) Modelling of the electric motor

The direct current electric motor with separate excitation has the transfer function [7] :

\[
H_2(s) = \frac{K_2}{T_m \cdot T_e \cdot s^3 + T_m \cdot s + 1} \tag{2}
\]
where: \( K_2 = \frac{1}{K} \) is the transfer coefficient of the motor, \( K \) being the constant of the direct current motor;

\[
T_m = \frac{J \cdot R}{K^2}
\]

is the time constant of mechanical delay, \( J \) being the inertia moment of the parts under rotation motion, \( R \) the resistance of the rotor winding and of the elements of the direct current motor mounted in series with the former;

\[
T_e = \frac{L_A}{R_A}
\]

is the time constant of electromagnetic delay, \( L_A \) being the dispersion inductivity of the rotor winding.

The working range [8] of the D.C. motor is presented in figure 4.

Figure 4. Working range of the direct current motor

c) Tachogenerator modelling

The revolution transducers is a direct current tachogenerator with the transfer function [9]:

\[
H_3(s) = \frac{K_3}{(1+T_G \cdot s)(1+T_{RG} \cdot s)}
\]

where: \( K_3 = K_{TG} \cdot K_f \) is the transfer coefficient of the tachogenerator together with the RC philtre at its entry;

\( T_{RG} = R \cdot C \) is the delay time constant of the exit philtre;

\( T_G \) is the mechanical time constant of the tachogenerator.

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d) Modelling of the mechanical transmission

The mechanical transmission between the direct current motor shaft and syringe is done through two mechanisms connected in series:
- a toothed wheeled reducing gear with the transmission ratio \( i = 500 \);
- a screw with bolt transforming the rotation motion (n) into the translation motion (v).

The transfer function of the mechanical transmission is:

\[
H_4(s) = \frac{K_4}{1 + T_s \cdot s} \tag{4}
\]

where: \( K_4 \) is the transfer coefficient of the mechanical transmission;
\( T_s \) is the delay time constant of the mechanical transmission.

e) Modelling of the incremental transducer

The transfer function of the incremental transducer is:

\[
H_5(s) = K_5 \tag{5}
\]

The transfer function of the process can be modelled with the relation:

\[
H_p(s) = \frac{K_p}{1 + T_p \cdot s} \tag{6}
\]

where: \( K_p \) is the transfer coefficient of the process;
\( T_p \) is the time constant of the process.

The transfer function of the actuator is:

\[
H_E(s) = H_1(s) \cdot H_2(s) \cdot H_3(s) \cdot H_4(s) \cdot H_5(s) \tag{7}
\]

\[
H_E(s) = \frac{K}{(1 + T_s \cdot s)(1 + T_m \cdot s + T_m \cdot T_e \cdot s^2)(1 + T_G \cdot s)(1 + T_{G2} \cdot s)(1 + T_r \cdot s)} \tag{8}
\]

where: \( K = K_1 \cdot K_2 \cdot K_3 \cdot K_4 \cdot K_5 \) are the transfer coefficients of the actuator;
\( T_s, T_m, T_e, T_G, T_{G2}, T_r \) are the time constants of the actuator.

The sum of the small time constants present in the actuator structure is the following:

\[
T_E = T_s + T_e + T_G + T_{G2} + T_r \tag{9}
\]

Thus, the transfer function of the execution element obtained is:

\[
H_E(s) = \frac{K}{(1 + T_E \cdot s)(1 + T \cdot s)} \tag{10}
\]
3. Experimental results and conclusions

3.1. Recording of the indicial response

Following the recording of the actuator’s indicial response we obtained the curve presented in figure 5.

![Figure 5. Indicial response in revolution of the actuator.](image)

By recording the value of the number of revolutions at the modification of the supply voltage of the actuator $U_i$ we obtained the results presented in table 1 below.

<table>
<thead>
<tr>
<th>$U_i$ [V]</th>
<th>0.0</th>
<th>0.5</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
<th>4.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$n$ [r.p.m.]</td>
<td>0</td>
<td>0.6</td>
<td>1.3</td>
<td>2.2</td>
<td>3.0</td>
<td>3.9</td>
<td>4.7</td>
<td>5.2</td>
<td>6.1</td>
<td>6.9</td>
</tr>
<tr>
<td>$U_i$ [V]</td>
<td>5.0</td>
<td>5.5</td>
<td>6.0</td>
<td>6.5</td>
<td>7.0</td>
<td>7.5</td>
<td>8.0</td>
<td>8.5</td>
<td>9.0</td>
<td>9.5</td>
</tr>
<tr>
<td>$n$ [r.p.m.]</td>
<td>7.7</td>
<td>8.3</td>
<td>9.0</td>
<td>9.9</td>
<td>10.7</td>
<td>11.4</td>
<td>12.2</td>
<td>13.0</td>
<td>13.7</td>
<td>14.5</td>
</tr>
<tr>
<td>$U_i$ [V]</td>
<td>10.0</td>
<td>10.5</td>
<td>11.0</td>
<td>11.5</td>
<td>12.0</td>
<td>12.5</td>
<td>13.0</td>
<td>13.5</td>
<td>14.0</td>
<td>15.0</td>
</tr>
<tr>
<td>$n$ [r.p.m.]</td>
<td>15.2</td>
<td>15.9</td>
<td>16.9</td>
<td>17.6</td>
<td>18.4</td>
<td>19.1</td>
<td>20.2</td>
<td>20.8</td>
<td>21.6</td>
<td>22.9</td>
</tr>
</tbody>
</table>
By using these values, we built the real static characteristic of the actuator, shown in figure 6.a., and linearised static characteristic presented in figure 6.b.

![Diagram](image)

**Figure 6.a.** Real static characteristic of the actuator; **6.b.** Linearised static characteristic

### 3.2. Experimental identification

The actuator experimental identification leads to an element of the PT2 type with the following transfer function:

\[
H_E(s) = \frac{K}{(1+T_z \cdot s)(1+T \cdot s)} \tag{11}
\]

where: 
- \(K\) is the transfer coefficient; 
- \(K = \frac{9.56}{22.9} = 0.417\)
- \(T\) is the main time constant; 
- \(T = 10\) sec.
- \(T_z\) is the sum of small time constants present in the actuator structure; 
- \(T_z \approx 0.1 T = 1\) sec.

The parameters \(K\), \(T\) and \(T_z\) resulted from the grapho-analytical processing of the indicial response curve from figure 5.

### 3.3. Fixed section of S.R.A

The fixed section of S.R.A. has the transfer function:

\[
H_f(s) = \frac{K}{(1+T_z \cdot s)(1+T \cdot s)(1+T_p \cdot s)} \tag{12}
\]

which allows the design of the PID-type regulator, according to the module criterion.
4. Observations and conclusions

Following the study conducted on the execution element the following conclusions could be drawn:

4.1 Personal contributions

- Construction and attachment to the execution element of a revolution incremental transducer with very small delay moment (≈0). This transducer allowed the building of the indicial response of the element without introducing additional delays;
- Determination of the static characteristic of the element and validation of the linearity hypothesis of the direct current motor;
- Establishment of the transfer function of the execution element and determination of the parameters based on the indicial response;
- Interpretation of results.

4.2 Conclusions

- The execution element respects the characteristic of a linear system element obtained by assembling several simpler system elements (direct current motor, D.C. tachogenerator, mechanical reducing gear). The experimental results obtained by measurements (static characteristic) validate the hypothesis of linearity made at the analytical study of the elements and the establishment of the transfer function, valid for linear elements;
- The indicial response in revolution indicates a PT2 element, in critical damped regime, which is indicated for medicine applications;
- Determination of the parameters from the mathematical model in operational based on the indicial response allows the synthesis of an adjustment system, as well as the establishment of the accord parameters of regulators;
- The analysed execution element corresponds to an element able to ensure a good rapidity / safety in exploitation ratio.

References

[3] Yuh-Shzan Hwang, Bo-Han Hwang, Ho-Cheng Lin, Jiann-Jong Chen., PLL-Based Controllers Energy Transfer Analog RSK Demodulator Using High-

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