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Corneliu Botan, Florin Ostafi

Optimal Control of an Electrical Drive System with Variable Torque

The optimal control from the energetic point of view of the transient state of electrical drive systems is presented. A step variation of the load torque in the optimization interval is considered. The performance criteria consider only Joule losses, since they significantly overcome other ones in the transient state. The paper refers to a structure with voltage control of the drive. The study is performed in continuous time domain for a fixed end point problem.

Keywords: motor drives, variable load torque, linear quadratic optimal control, fixed end point

1. Introduction

The optimal control [1], [2] from energetic point of view of the electrical drive systems is an important approach for energy saving, taking into account that more than 60% of the consumption of the electrical energy is for these systems. Moreover, it is possible to reduce in such cases the rated power of the drive motors since this is chosen from heat consideration and the optimal control leads just to a diminished Joule losses.

There are numerous studies dedicated to the optimal control of the electrical drive systems, for different types of motors, control strategies, criteria, or used methods (for instance, we mention [3], [4], [5], [6], [7], but many other papers can be indicated). The optimization is appreciated as a main direction of the developing of the electrical drive systems in the future [8].

The control of an electrical drive must be chosen so that to obtain a small energy losses and an acceptable behaviour of the system. The demands and conditions for different applications are not the same and therefore, one can formulate different optimal control problems for an electrical drive system. Some considerations for various applications are indicated below.

First it should be noted that the optimization can refer to the steady state (more frequently), or to the optimal control of the transient state. The criterion expresses the power in the first case and the energy in the dynamic optimization problems. In the last case, the criterion refers only to the cooper losses, since they significantly overcome all other losses, because of very great values of currents.

Another aspect refers to the controlled system. The problems can present differences depending on the used motor type, but a general approach can be adopted. Also, the structure of the optimal drive system may be with current or with voltage control, depending on the type of the power electronic converter. The first one can be easier implemented, but different other considerations can influence the choice of the structure. The features of the load torque are also important in the implementation of the optimal solutions.

Finally, the problems can differ depending on control aspects. For instance, it is possible to use one or two variables in the dynamic optimization problems. The last variant is useful for optimal control if the drive operates in many situations with a reduced load. However, the most used cases refer to a system with one control variable. The adopted criteria can be different, depending on the considered components, or on the terminal conditions (for instance, the optimal problem can be with free or fixed final time or with free or fixed final states variables).

The authors have presented in some previous paper different variants for optimal control problems for transient period of the electrical drives. For example, the voltage control for the drives with constant load torque was studied in [6] and [9]. The current control variant was presented in [10] for constant load torque and in [11] for a variable one. The fixed end point optimal control for constant torque is discussed in [12]. A control problem for a voltage controlled drive system with variable torque, for a free end point problem is presented in [13].

The paper deals with optimal control of a voltage controlled drive system with variable load torque. Of course, a general algorithm for a variable torque can be established, but the implementation is significantly simpler for a constant load torque. Therefore, a suboptimal solution can be obtained if the variation of the load torque in the transient period is approximated with a step function. Such situations are frequently met in the electrical drive systems, when the no-load or small load torque is succeeded by a great one. Examples for such operation are the rolling mills, or cutting processes.

The studied problem refers to one with fixed end point and with fixed final time and it is performed in continuous time domain. A similar study in discrete time domain was presented in [14].

Only the case of one control variable is considered. The results are valid for different motor types, because the mathematical model is the same with adequate assumptions.

2. Problem formulation

A linear electrical drive system is described by the state equation

$$\dot{\tilde{x}}(t) = A\tilde{x}(t) + Bu(t) + \tilde{w}(t)\,, \tag{1}$$

where $\tilde{x}(t) = [\mu(t)\ \ i(t)]^T$ is the state vector (T denotes the transposition), $\tilde{w}(t) = Wm(t)$ is the disturbance variable (with $m(t)$ the load torque) and $\mu(t)$ is the rotor speed. The variables i and u are the rotor current and voltage in the case of a brushed d.c. motor (the flux is constant). For synchronous and asynchronous motors, i and u correspond to the q current and voltage components and similar equations may be adopted with adequate assumptions (mainly, the i_d component is constant or very small). The matrices $A,\ B$ and the vector $\tilde{w}(t)$ are in the form

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},\ B = \begin{bmatrix} 0 \\ b_2 \end{bmatrix},\ \tilde{w}(t) = Wm(t) = \begin{bmatrix} w_1 \\ 0 \end{bmatrix} m(t)\,, \tag{2}$$

and they depends on the constant parameters of the drive system.

In the case of the electrical drive system, the aim of the optimal control is to obtain small energy consumption and steady state and transient errors as smaller as possible for the state variables.

In order to achieve the mentioned goals and taking into account that the problem is with fixed end point (we have to obtain exactly the desired speed μ_d at the final moment t_f) it is adopted the performance index

$$J = \frac{1}{2}\int_{t_0}^{t_f} q_2[\mu(t) - \mu_d]^2 + q_1 i^2(t) + pu^2(t) + \ell u(t)i(t)\,, \tag{3}$$

which penalizes, by means of first and third terms, the transient error of the speed and the great values of the control variable. Also, the cooper energy losses are penalized by the second term and the last term refers to the global energy consumption. The presence of the last term in the criterion is not mandatory because the cooper losses significantly overcame all other losses in the transient period.

The performance index (3) can be written in the general form

$$\begin{aligned} J = \frac{1}{2}\int_{t_0}^{t_f} \{&\tilde{x}^T(t)Q_1\tilde{x}(t) + [\tilde{x}(t) - x_d]^T Q_2[\tilde{x}(t) - x_d] + \\ &+ u^T(t)Pu(t) + [\tilde{x}(t) - x_d]^T Lu(t) + u^T(t)L^T[\tilde{x}(t) - x_d]\}, \end{aligned} \tag{4}$$

with

$$Q_1 = diag(0, q_1) \geq 0, Q_2 = diag(q_2, 0) \geq 0, P = p > 0, L = \begin{bmatrix} 0 & \ell/2 \end{bmatrix}. \tag{5}$$

and the desired state vector $x_d = [\mu_d\ \ 0]^T$. Some details about L matrix will be indicated in the next section.

The fixed end-point optimal control problem is usually solved for the final state in the origin ($\tilde{x}(t_f) = 0$) or, more general, if $\tilde{x}(t_f)$ is placed on target set defined by the restrictions $C\tilde{x} = 0$, where C is a mxn matrix with linear independent rows. A variables change must be introduced because in many cases (especially in the problems with perturbations) the desired vector is $x_d \neq 0$. With $x(t) = \tilde{x}(t) - x_d$ the system equation becomes

$$\dot{x}(t) = Ax(t) + Bu(t) + w(t) \, , \tag{6}$$

where

$$w(t) = \tilde{w}(t) + Ax_d \, . \tag{7}$$

This last vector includes all exogenous variables (disturbance and desired values).

With the previous translation the performance index is

$$\begin{aligned} J = \frac{1}{2}\int_0^{t_f} \{ [x(t) + x_d]^T Q_1 [x(t) + x_d] + x^T(t) Q_2 x(t) \\ + u^T(t) P u(t) + x^T(t) L u(t) + u^T(t) L^T x(t) \} dt. \end{aligned} \tag{8}$$

The fixed end-point optimal control problem is to find the closed loop control $u(x(t))$ which transfers the system (6) from the initial state $x(0)$ in the final state $x(t_f) = 0$ and minimizes the performance criterion (8).

3. Optimal Controller Design

One obtains from the necessary conditions for optimality [7]

$$u(t) = -P^{-1}[B^T \text{ª}(t) + L^T x(t)] \, , \tag{9}$$

where $\text{ª}(t) \in \mathbf{R}^n$ is the co-state vector and

$$-\dot{\text{ª}}(t) = Qx(t) + Lu(t) + A^T \text{ª}(t) + Q_1 x_d \, , \quad Q = Q_1 + Q_2 . \tag{10}$$

Using (9), the equations (6) and (10) can be rewritten as

$$\text{¥}(t) = K\text{¥}(t) + r(t) \, , \tag{11}$$

with

$$\text{¥}(t) = \begin{bmatrix} x(t) \\ \text{ª}(t) \end{bmatrix} \in \mathbf{R}^{2n}, r(t) = \begin{bmatrix} r_1(t) \\ r_2(t) \end{bmatrix} = \begin{bmatrix} w(t) \\ -Q_1 x_d \end{bmatrix} \in \mathbf{R}^{2n}, K = \begin{bmatrix} \tilde{A} & -N \\ -\tilde{Q} & -\tilde{A}^T \end{bmatrix}, \tag{12}$$

$$N = BP^{-1}L^T, \quad \tilde{A} = A - BP^{-1}L^T, \quad \tilde{Q} = Q - LP^{-1}L^T .$$

The L matrix must be chosen so that $\tilde{Q} > 0$.

The solution to the equation (11) is

$$¥(t) = \mathrm{d}\,(t,0)¥^{0} + g(t), \quad ¥^{0} = ¥(0), \tag{13}$$

where

$$\mathrm{d}\,(t,0) = \begin{bmatrix} \mathrm{d}_{11}(t,0) & \mathrm{d}_{12}(t,0) \\ \mathrm{d}_{21}(t,0) & \mathrm{d}_{22}(t,0) \end{bmatrix} \in \mathbf{R}^{2nx2n}, \quad \mathrm{d}_{ij} \in \mathbf{R}^{nxn}, i, j = 1,2 \tag{14}$$

is the transition matrix for K, and the vector $g(t) \in \mathbf{R}^{2n}$ depends on the exogenous variables

$$g(t) = \int_0^t \mathrm{d}\,(t,²)r(²)d² = \begin{bmatrix} g_1(t) \\ g_2(t) \end{bmatrix}, \quad g_1, g_2 \in \mathbf{R}^{n}. \tag{15}$$

The initial value $¥(0)$ is unknown but it can be computed from $\tilde{x}(t_f) = 0$ replaced in (13) for $t = t_f$:

$$ª\,(0) = -\mathrm{d}_{12}^{-1}(t_f, 0)[\mathrm{d}_{11}(t_f, 0)x(0) + g_1(t_f)]. \tag{16}$$

The variables x and ª can be expressed in terms of $x(0)$ and the optimal control becomes

$$u(t) = -P^{-1}[L^T(\mathrm{d}_{11} - \mathrm{d}_{12f}^{-1}\mathrm{d}_{11f}) + B^T(\mathrm{d}_{21} - \mathrm{d}_{22}\mathrm{d}_{12f}^{-1}\mathrm{d}_{11f})]x(0) + f_1, \tag{17}$$

where $\Gamma_{12} = \Gamma_{12}(t,0)$, $\Gamma_{12f} = \Gamma_{12}(t_f,0)$, $x = x(t)$ and so on. The term f_1 depends on g_1, g_2, g_{1f}, x_d. It is proved that Γ_{12} is a nonsingular matrix if the system (6) is completely controllable [8].

Although the optimal control (17) can be easily computed in real time, the main disadvantage is that the control law is in open loop. The state feedback control can be obtained if x(0) is expressed in terms of x from (11) and replaced in (17). The real time calculus of control law is more complicated since it is necessary to compute the inverse of a time variant matrix.

In order to avoid the mentioned difficulty, it is proposed another method that supposes a change of variables:

$$\begin{gathered} ¥(t) = U\,°(t), \quad °(t) = \begin{bmatrix} x(t) \\ v(t) \end{bmatrix} \in \mathbf{R}^{2n}, \\ U = \begin{bmatrix} I_n & 0 \\ R & I_n \end{bmatrix} \in \mathbf{R}^{2n\times 2n}, \quad U^{-1} = \begin{bmatrix} I_n & 0 \\ -R & I_n \end{bmatrix}, \end{gathered} \tag{18}$$

where R is a constant symmetric positive definite matrix and I_n is the nxn identity matrix. The supplementary vector v(t) contains terms depending on the exogenous variables and a corrective component which compensates the fact that R is a constant and not a time variant matrix (as in classical procedures). With this change of variable, the system becomes

$$\dot{\rho} = H\rho + U^{-1}r\,, \tag{19}$$

with

$$H = U^{-1}KU = \begin{bmatrix} \tilde{A} - NR & -N \\ RBP^{-1}B^T R - R\tilde{A} - \tilde{A}^T R - \tilde{Q} & -\tilde{A}^T + RN \end{bmatrix}. \tag{20}$$

If we choose

$$RBP^{-1}B^T R - R\tilde{A} - \tilde{A}^T R - \tilde{Q} = 0\,, \tag{21}$$

the matrix from (20) can be written in the form

$$H = \begin{bmatrix} \tilde{F} & -N \\ 0 & -\tilde{F}^T \end{bmatrix}, \quad \tilde{F} = \tilde{A} - NR\,. \tag{22}$$

Note that (21) is the Riccati algebraic equation for the linear quadratic optimal control problem with infinite final time.

The transition matrix of H is

$$\Omega(t,\tau) = \begin{bmatrix} \Psi(t,\tau) & \Omega_{12}(t,\tau) \\ 0 & \Phi(t,\tau) \end{bmatrix}, \tag{23}$$

where $\Psi(t,\tau), \Phi(t,\tau)$ are the transition matrices for $\tilde{F}$, and $-\tilde{F}^T$, respectively and

$$\Omega_{12}(t,\tau) = \int_t^{\tau} \Psi(t,\sigma) N \Phi(\sigma,\tau)\, d\sigma\,. \tag{24}$$

A similar relationship with (20) can be applied for the transition matrix of H and one obtains:

$$\begin{aligned} d_{11}(t,\tau) &= \Psi(t,\tau) - \Omega_{12}(t,\tau)R, \quad d_{12}(t,\tau) = \Omega_{12}(t,\tau), \\ d_{21}(t,\tau) &= R\Psi(t,\tau) - R\,\Omega_{12}(t,\tau)R - \Phi(t,\tau)R, \\ d_{22}(t,\tau) &= \Phi(t,\tau) + R\,\Omega_{12}(t,\tau). \end{aligned} \tag{25}$$

The solution to the system (19) is

$$\rho(t) = \Omega(t,0)\rho(0) + \int_0^t \Omega(t,\tau) U^{-1} r(\tau)\, d\tau \tag{26}$$

or

$$\begin{bmatrix} x(t)\\ v(t)\end{bmatrix} = \begin{bmatrix} \mathrm{v}\,(t,0) & \mathrm{t}_{12}(t,0)\\ 0 & \mathrm{c}\,(t,0)\end{bmatrix}\begin{bmatrix}x(0)\\ v(0)\end{bmatrix} + \int_0^t \begin{bmatrix}\mathrm{v}\,(t,^2) & \mathrm{t}_{12}(t,^2)\\ 0 & \mathrm{c}\,(t,^2)\end{bmatrix}\begin{bmatrix} r_1(^2)\\ -Rr_1(^2)+r_2(^2)\end{bmatrix} d^2 . \tag{27}$$

From (27) for $t = t_0$ and (16), yields

$$v(0) = (-\mathrm{d}_{12f}^{-1}\mathrm{d}_{11f} - R)x(0) - \mathrm{d}_{12f}^{-1} g_1(t_f) \tag{28}$$

and taking into account (25), we obtain

$$v(0) = -\,\mathrm{t}_{12f}^{-1}\mathrm{v}_{\,f}\, x(0) - \mathrm{t}_{12f}^{-1} g_{1f}\,. \tag{29}$$

Now we can compute the supplementary vector v(t) from (27) with

$$v(t) = -\mathrm{c}\,(t,0)(\,\mathrm{t}_{12f}^{-1}\Psi_f x^0 + \mathrm{t}_{12f}^{-1} g_{1f}) + \int_0^t \mathrm{c}\,(t,^2)(-Rr_1(^2)+r_2(^2))d^2 . \tag{30}$$

Finally, the optimal control (6) becomes

$$u(t) = u_f(t) + u_s(t)\,, \tag{31}$$

where

$$u_f(t) = -P^{-1}(B^T R + L^T)x(t) \tag{32}$$

is the feedback component and

$$u_s(t) = -P^{-1}B^T v(t) \tag{33}$$

is a supplementary component, depending on v(t) given by (30). This component contains a component depending on the initial state $x(0)$ and one depending on the exogenous vector r(t).

This last component can be computed only if $m(t)$ is beforehand known on the interval $[t_0, t_f]$, because (30) can be computed only in this case. The problem can also be solved in the case when it is known the shape of $m(t)$, and its magnitude is measured or estimated at the beginning of the optimization interval. The simplest case (but frequently met in electrical drives) $m(t) = m = constant$ was discussed in [12]. The simplification in the constant disturbance case consists in the possibility to extract w(t) from above mentioned integrals. The paper deals with the case when the load torque has a step variation during the transient period, from a small (no-loaded) operation to another great value. Such situations are frequently met in different applications and, in many situations, it is possible to know beforehand the two values of the torque and the switching moment.

We shall suppose that the load torque is $m_1 = constant$ for $t \in [t_0, \bar{\ }]$ and $m_2 = constant$ for $t \in [\bar{\ }, t_f]$. Consequently, the components of the vector r given by (12) have the values r_{11}, r_{12}, and r_{21}, r_{22} on the mentioned intervals.

In this case, the integral from (30) can be computed with:

$$h(0,t)=\begin{cases}\int_{t_0}^{t} \mathrm{c}\,(t,^2)d^2[-Rr_{11}+r_{21}], \quad for\ t_0 \le t \le ^{-} \\ \int_{t_0}^{-} \mathrm{c}\,(t,^2)d^2[-Rr_{11}+r_{21}]+\int_{-}^{t} \mathrm{c}\,(t,^2)d^2[-Rr_{12}+r_{22}], \quad for^{-} < t \le t_f\end{cases} \tag{34}$$

The constant vector g_{1f} can be obtained from (15) and (25):

$$\begin{aligned} g_1(t_f) &= \int_0^{t_f}[\mathrm{d}_{11}(t,^2)r_1(^2)+\mathrm{d}_{12}(t,^2)r_2(^2)]d^2 = -\int_0^{t_f} \mathrm{t}_{12}(t,^2)d^2\, Q_1 x_d + \\ &+\int_{t_0}^{-}[\mathrm{v}\,(t,^2)-\mathrm{t}_{12}(t,^2)R]d^2\, r_{11} + \int_{-}^{t_f}[\mathrm{v}\,(t,^2)-\mathrm{t}_{12}(t,^2)R]d^2\, r_{11}. \end{aligned} \tag{35}$$

One can remark that the integrals from (34) and (35) can be easily computed:

$$\int_{t_m}^{t_n} \mathrm{v}\,(t_{m,}{}^2)d^2 = e^{Ft_m}\int_{t_m}^{t_n} e^{-F^2}d^2 = e^{Ft_m}(e^{-F^2}-I_n)\Big|_{t_m}^{t_n}(-F)^{-1} = [\mathrm{v}\,(t_m,t_n)-I_n](-F)^{-1} \tag{36}$$

and

$$\int_{t_m}^{t_n} \mathrm{c}\,(t,^2)d^2 = [I_n - \mathrm{c}\,(t_m,t_n)]F^{-T}, \quad F^{-T}=(F^T)^{-1}. \tag{37}$$

Taking into account that $\mathrm{t}\,(.)$ is the transition matrix for H given by (22), we can obtain the integral from (35) using the property $\dot{\mathrm{t}}\,(.) = G\,\mathrm{t}\,(.)$. From this 2nx2n matrix equation, we extract the differential equation referring to $\dot{\mathrm{t}}_{12}(.)$. Integrating this relation and using (37), we have:

$$\int_0^{t_f} \mathrm{t}_{12}(t,^2)d^2 = F^{-1}[-\mathrm{t}_{12}(0,t_f)+N(\mathrm{c}\,(0,t_f)-I_n)F^{-T}]. \tag{38}$$

Remark 1: The procedure indicated above can be extended for a drive system with a more general form of the variation of the load torque. In this case, it is possible to approximate this variation with a step function on several subintervals and the problem is solved in a similar manner.

Remark 2: The above presented relations are quite complicated, but the most part of the computing is performed off-line, in the stage of the controller design. This stage implies to establish the solution to the discrete Riccati equation

(21) (the command lqr from Matlab can be used), and to compute the constant matrices and vectors. For this stage, the computing of the integrals can be avoided, as it was indicated above.

The real time computing of the optimal control vector implies the relations (30)...(33), where only the matrix $\text{c}\,(t,0)$ and the vector $h(0,t)$ are time variant. Both elements can be iteratively computed. Indeed, one can write:

$$\text{c}_{k+1} = \text{a}\text{c}_k, \text{c}_k = \text{c}\,(k¢,0), \text{c}_0 = \text{c}\,(0,0) = I, \text{a} = e^{-F^T ¢}, ¢ - \textit{sampling period.}$$

The vector $h(0,t)$ is computed only for sampling moments $t_k, k = 0,1,2,...$ and $h(0,t) = h(0,t_k) = h_k$, for $t_k \le t \le t_{k+1}$. From (34) and (37), it results

$$h_k = h_{k-1} + \text{c}\,(t_0, t_{k-1}) E\,° \quad , \tag{39}$$

where

$$E = (I_n - \text{a}) F^{-T} \;\; and \;\; ° = \begin{cases} °_1 = -Rr_{11} + r_{21}, & for\ 0 < t \le ¯ \\ °_2 = -Rr_{12} + r_{22}, & for\ t > ¯ \end{cases} , \tag{40}$$

The variable elements from the supplementary component of the optimal control vector given by (32) can be now easily computed.

Remark 3: The adopted optimal control ensures that the desired value x_d will be reached at the final moment t_f. The behaviour after the moment t_f is not reflected by this control. Therefore one must adopt another control law in order to obtain a convenient behaviour of the drive system for $t > t_f$. A possibility in this direction can be found in [12].

4. Simulation results

A set o simulation test was performed for a drive system with the matrices in (2) having the values:

$$A = \begin{bmatrix} 0 & 20 \\ -3.5 & 19 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 6.25 \end{bmatrix}, \; W = \begin{bmatrix} -35 \\ 0 \end{bmatrix}$$

These matrices correspond to a drive system with a d.c. motor with rated data U=110V and I=3.3A. The sampling period is $² = 0.002s$ and the final time is 0.3s. The desired speed is $\mu_d = 25 rad/s$.

Figure 1 shows the behaviour of the drive system for a step variation of the load torque from 0.2 Nm to 1.5 Nm (double of the rated value) at the moment $¯ = 0.2s$. Since the optimal control $u(t)$ is computed based on a beforehand computed „mean” value of the load torque, the optimal variation of the voltage and of the current are not affected by the step variation of the load torque. Only a small variation of the acceleration can be observed at the moment $\overline{\ } = 0.2s$.

Figures 2, 3 and 4 indicate the effect of an erroneous estimation of the load torque and of the switching moment, respectively. A comparison between the behaviour of the cases of correct estimation (continuous curves) and erroneous estimation (doted curves) is indicated in both cases.

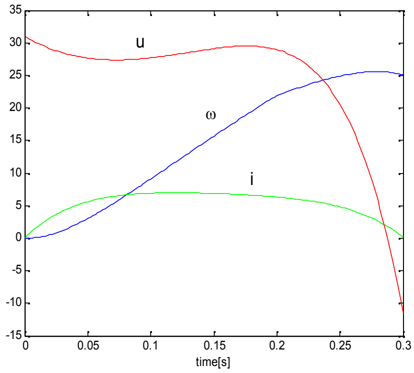

Figure 1. Behaviour of the optimal system.

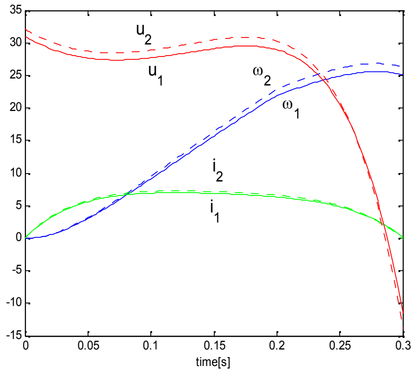

Figure 2. Behaviour in the case of an erroneous estimation of the load torque – greater then the true values.

In figure 2, the results of estimation are considered the values 0.3 and 0.8 Nm (instead of real values 0.2 and 1.5Nm).

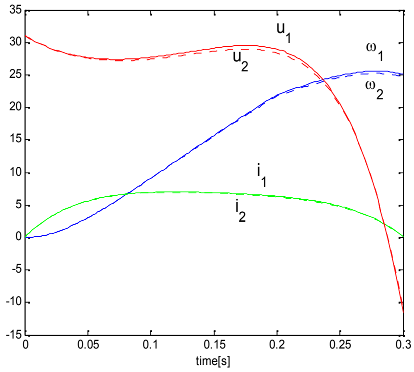

Figure 3. Behaviour in the case of an erroneous estimation of the load torque – having a true mean value.

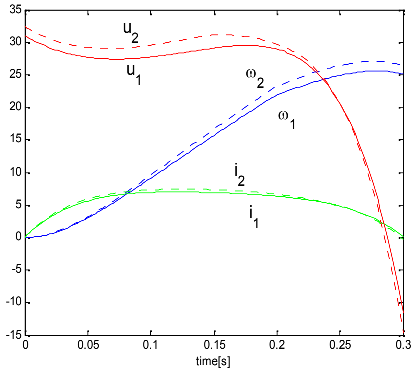

Figure 4. Behaviour in the case of an erroneous estimation of the switching moment.

The disturbance estimated values considered in figure 3 have about the same mean value as the true one (0.3 and 1.3Nm). The switching moments are the same as above in both figures.

In figure 4, the switching moment was anticipated at 0.15s instead of 0.2s.

The variations of currents, voltages, velocities, energy losses and performance indices were compared for different estimation errors.

The amplitude and sign of the mentioned variations are influenced by the sign and size of estimation errors. In any case, the variations of the mentioned entities around its optimal values are acceptable for reasonable size of estimation errors.

The analysis of the energy losses shows a small decrease in comparison with conventional cascade structure when the mean load torque is closely related to the rated one. But the reduction of the energy losses is up to 25% in the case of small mean load torque.

4. Conclusion

A new method for the optimal control problem with fixed end point for an electrical drive system with variable load torque is presented. The proposed algorithm can be easier implemented then the classical procedure for the LQ optimal control.

The adopted criterion ensures a good behaviour for the system and a significant decrease of the energy consumption, especially in the transient states that appears at the changing of the imposed value of speed and the mean load torque is small in comparison with the rated one.

The described optimal control is useful especially for the medium and high power electrical drives with frequent changing of the speed.

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Addresses:

- Prof. Dr. Eng. Corneliu Botan, "Gh. Asachi" Technical University of Iasi, Blvd. D. Mangeron, nr. 27, 700050, Iasi, cbotan@gmail.com
- Dr. Eng. Florin Ostafi, "Gh. Asachi" Technical University of Iasi, Blvd. D. Mangeron, nr. 27, 700050, Iasi, fostafi@ac.tuiasi.ro