Contributions to the Transient Study of an Electric Motor with Shorted Moving Coil

In this article is studied the transient regime of an electric motor with a shorted moving coil, into alternating current supplied. The authors present their contributions related to the conceiving, achieving and testing of experimental stands of an electric motor with a shorted moving coil and to the identification of the mathematical model, indexical function parameters and the coefficients of the mathematical equation of this engine. Finally, the authors' conclusions are presented on the results of experimental studies and moving coil motor behavior in transient regime.

Keywords: electric motor with shorted moving coil, indexical function, transient regime

1. Time domain analysis of the elements of a system of automatic regulation

Any system is characterized by a certain functional dependence between the time variation of the output quantity $y(t)$ and the time variation of the input quantity $x(t)$. This relationship of dynamic regime can be expressed by differential equations obtained on the basis of the physical-chemical laws which characterize the functioning of some elements of the system. For a monovariabil linear system input/output, the differential equation in the general case has the form [2], [7], [8]:

$$a_n \frac{d^n y(t)}{dt^n} + ... + a_2 y(t) = b_m \frac{d^m x(t)}{dt^m} + ... + b_2 x(t)$$

(1)

in which the coefficients $a_n$, ..., $a_2$, $b_m$, ..., $b_2$ have physical significance, while the condition that the system is physically realizable is $n \geq m$.

By applying the Laplace transform to the equation (1), in null initial conditions, obtains the transfer function of the system:
\[
H(s) = \frac{y(s)}{x(s)} = \frac{b_0 s^m + b_{m-1} s^{m-1} + \ldots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \ldots + a_1 s + a_0} 
\]  
(2)

If common factors are given, the terms of \( a_0 \) and respectively \( b_0 \), is obtained the form "with time constants" of transfer function:

\[
H(s) = \frac{y(s)}{x(s)} = k \frac{T_0 s^n + T_{n-1} s^{n-1} + \ldots + T_1 s + 1}{T_n s^n + T_{n-1} s^{n-1} + \ldots + T_1 s + 1} = \sum_{i=0}^{n} \frac{T_i s^i + 1}{\sum_{i=0}^{n} T_i s^i + 1} 
\]  
(3)

where:

\[ k = \frac{b_0}{a_0} \] - the gain of the system;

\[ T_i = \frac{b_i}{a_i} \] - coefficients having dimension of time constants \((1 \leq j \leq m)\),

\((1 \leq i \leq n)\).

**Order** of a transfer function is given by order of the differential equation which is obtained by the Laplace transfer function. So for achievable physical systems, \( n > m \), the order coincides with the denominator polynomial of the transfer function \([2]\).

**Time analysis** is to determine the response time of the regarded systems at different types of input signals and determining of the main properties (stability, performance, etc..) \([2]\).

The performance of a dynamic system is described by synthetic indicators of quality that characterize the step response of the system \([2]\):

- overshoot \( \sigma \);
- the time of the first maximum or of achieving the maximum deviation of the output quantity in transient regime \( \tau_\sigma \);
- duration of transient regime defined by the time that elapses from the moment of application of excitation (input) on the reference channel and until the output entering a band of \( \pm (2 \pm 5)\) \([\%]\);
- oscillation index \( \Psi \) represents the relative variation of the amplitudes of two successive exceedance of the same sign of the value of steady state;
- oscillation period \( T \) for amortized oscillatory regime;
- number of oscillations \( N \) if the response across by a finite number of times the stationary component.

In addition to these main quality parameters, can also be define others such as:

- setting time: the time in which is reached the stationary value of the output
for the first time;
- rise time: the value of subtangent taken of \( y(t) \) to \( 0.5y_a \), the tangent being limited to the axis \( t \) and axis \( y_a \).

**The performance of stationary regime:**
- stationary error – the value of adjustment error in stationary regime (undisturbed, stabilized):

\[
u_s = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s y(s) H_s(s)
\]  

(4)

Appreciations of this quality indicator are based on the step response of the SRA, so of the closed loop transfer function. **The step response** is the response of a linear system when the input is of the step type (which could be considered, due to the linearity of the amplitude one, unitary step) [2].

A first-order element is described by an equation of the type:

\[
a_1 \frac{dy(t)}{dt} + a_0 y(t) = b_0 x(t)
\]  

(5)

which by applying the Laplace transform, leads to a transfer function of the type:

\[
H_s(s) = \frac{y(s)}{x(s)} = \frac{k}{Ts + 1}
\]  

(6)

where:

\[
T = \frac{a_1}{a_0} \quad \text{time constant,}
\]

\[
k = \frac{b_0}{a_0} \quad \text{amplification factor.}
\]

For \( k = 1 \), the system is described by transfer function:

\[
H(s) = \frac{1}{1 + Ts}, \quad T > 0
\]  

(7)

Considering the input system \( x(t) = 1(t) \), rezult \( x(s) = \frac{1}{s} \) and:

\[
y(s) = H(s) x(s) = \frac{1}{s(1 + Ts)} = \frac{1}{s} - \frac{T}{1 + Ts}, \quad \text{so } y(t) = (1 - e^{-\frac{t}{T}}) \delta(t)
\]  

(8)

which is represented in figure 1.
From relation (8) it is seen that: \( y_a = \lim_{t \to \infty} y(t) = \lim_{s \to 0} y(s) = 1 \) and how
\[
\frac{dy}{dt}(t) = \frac{1}{T} e^{t/T}, \quad t < 0,
\]
the tangent in origin to the graph of \( y(t) \) is:
\[
y(t) = y(0_+) t = \frac{1}{T} t \tag{9}
\]
For \( y(t_1) = y_a = 1 \) resulting \( t_1 = T \), that is the subtangent in the origin of the graph of the function \( y(t) \) determine the right of \( y_{st} \) on even segment equal with time constant \( T \). We can already say that, as the time constant increases, the system response is growing more slowly.

Conventionally it is considered that the transitional regime ceased when:
\[
|y(t) - y_a| \leq k_{st} y_a, \quad \forall t \geq t_i \tag{10}
\]
which defines the transient duration (or time transient) and where the usual:
\[
k_{st} = 0.05 \ [5\%] \text{ and } k_{st} = 0.02 \ [2\%] \tag{11}
\]
With this system, the relation becoming \( e^{-t/T} \leq k_{st}, \quad \forall t > t_i \), so \( t \geq -T \ln k_{st} \Rightarrow t_i = -T \ln k_{st} \equiv (3 \div 4)T \), where they took into account the constant value of \( k_{st} \). Should be retained and the relationship between pole system which is \( p = -1/T < 0 \) and the duration of the transient regime, which is expressed by:
and so, as the pole moves away from the imaginary axis (in the left half-plane) the transient regime is shorter.

2. Experimental contributions

For experimental measurements of alternating current, was using the electric motor with the shorted moving coil with variable height, presented in figure 2.a). In figure 2.b) are presented different constructive types of coils used in experiments.

![Figure 2. a) Electric motor with shorted moving coil; b) Different constructive types of coils. [19]](image)

In figure 3 is presented the experimental stand for testing the electrical linear motor with a shorted moving coil [9, 18, 19].

The motor has a solenoid mounted on the fixed column, having a number of adjustable turns by means of an outlet voltage of 300, 350, 450 to 500 turns which occupies only half the height of the column and electrodynamic forces acting through on a shorted moving coil, made of several rollers, mounted together with a number of turns also adjustable to 15, 30 to 45 turns (15 turns/roller), possibly inserting or removing the circuit of these rollers, which changes the height of the moving coil and therefore the eccentricity of the fixed coil and moving coil.

Based on the experimental data were plotted experimental features. For the interpretation of the characteristics was performed a comparative analysis of their results by identifying the theoretical characteristics indexical functions.
Figure 3. Experimental stand for testing the linear electric motor with shorted moving coil. [9], [19]

3. Theoretical contributions

The experimental results of linear electric motor with a shorted moving coil, made by combining the number of turns of the primary coil and the number of turns in the secondary coil, were transposed in table 1, which are shown indexical functions, experimental and theoretical, of the linear motor supplied at 220 V voltage in alternating current. The response of the motor is type PT1, characterized by a differential equation of the first order, as shown in table 2 [4, 7, 19].

As can be seen from table 1, the time constant during the transient thus, this type of electric motor for the moving coil, depending on the number of turns of the primary winding, the secondary, respectively. The time constant is large when using a reduced number of turns and introduce turns decreases as flow rule is valid for both the primary winding and the secondary winding.
<table>
<thead>
<tr>
<th>Experimental conditions</th>
<th>Type of process</th>
<th>Process equation</th>
<th>Indicial expression function</th>
<th>The values of the time constants [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>300 primary windings, 15 secondary windings — Inside</td>
<td>PT</td>
<td>$0.08 \times x(t) + x_i(t) = 9.81 \times x_i(t)$</td>
<td>$x_i(t) = 9.81 - e^{-0.03 t}$</td>
<td>$\gamma = 0.03$</td>
</tr>
<tr>
<td>300 primary windings, 15 secondary windings — Outside</td>
<td>PT</td>
<td>$0.033 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.033$</td>
</tr>
<tr>
<td>300 primary windings, 15 secondary windings — Middle</td>
<td>PT</td>
<td>$0.031 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.031$</td>
</tr>
<tr>
<td>300 primary windings, 30 secondary windings — Inside</td>
<td>PT</td>
<td>$0.027 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.027$</td>
</tr>
<tr>
<td>300 primary windings, 30 secondary windings — Outside</td>
<td>PT</td>
<td>$0.026 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.026$</td>
</tr>
<tr>
<td>300 primary windings, 45 secondary windings</td>
<td>PT</td>
<td>$0.028 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.028$</td>
</tr>
<tr>
<td>350 primary windings, 15 secondary windings — Inside</td>
<td>PT</td>
<td>$0.023 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.023$</td>
</tr>
<tr>
<td>350 primary windings, 15 secondary windings — Outside</td>
<td>PT</td>
<td>$0.018 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.018$</td>
</tr>
<tr>
<td>350 primary windings, 15 secondary windings — Middle</td>
<td>PT</td>
<td>$0.041 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.041$</td>
</tr>
<tr>
<td>350 primary windings, 30 secondary windings — Inside</td>
<td>PT</td>
<td>$0.036 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.036$</td>
</tr>
<tr>
<td>350 primary windings, 30 secondary windings — Outside</td>
<td>PT</td>
<td>$0.027 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.027$</td>
</tr>
<tr>
<td>350 primary windings, 45 secondary windings</td>
<td>PT</td>
<td>$0.036 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.036$</td>
</tr>
<tr>
<td>400 primary windings, 15 secondary windings — Inside</td>
<td>PT</td>
<td>$0.053 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.053$</td>
</tr>
<tr>
<td>400 primary windings, 15 secondary windings — Outside</td>
<td>PT</td>
<td>$0.076 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.076$</td>
</tr>
<tr>
<td>400 primary windings, 15 secondary windings — Middle</td>
<td>PT</td>
<td>$0.092 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.092$</td>
</tr>
<tr>
<td>400 primary windings, 30 secondary windings — Inside</td>
<td>PT</td>
<td>$0.052 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.052$</td>
</tr>
<tr>
<td>400 primary windings, 30 secondary windings — Outside</td>
<td>PT</td>
<td>$0.025 \times x(t) + x_i(t) = 9.73 \times x_i(t)$</td>
<td>$x_i(t) = 9.73 - e^{-0.03 t}$</td>
<td>$\gamma = 0.025$</td>
</tr>
<tr>
<td>400 primary windings, 30 secondary windings – inside</td>
<td>PT1</td>
<td>0.03 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.734x(t) )</td>
<td>( t_p = 0.031 )</td>
</tr>
<tr>
<td>400 primary windings, 30 secondary windings – outside</td>
<td>PT1</td>
<td>0.079 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.764x(t) )</td>
<td>( t_p = 0.079 )</td>
</tr>
<tr>
<td>400 primary windings, 45 secondary windings</td>
<td>PT1</td>
<td>0.032 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.756x(t) )</td>
<td>( t_p = 0.032 )</td>
</tr>
<tr>
<td>450 primary windings, 15 secondary windings – inside</td>
<td>PT1</td>
<td>0.085 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.755x(t) )</td>
<td>( t_p = 0.065 )</td>
</tr>
<tr>
<td>450 primary windings, 15 secondary windings – middle</td>
<td>PT1</td>
<td>0.039 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.761x(t) )</td>
<td>( t_p = 0.039 )</td>
</tr>
<tr>
<td>450 primary windings, 15 secondary windings – outside</td>
<td>PT1</td>
<td>0.031 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.760x(t) )</td>
<td>( t_p = 0.024 )</td>
</tr>
<tr>
<td>450 primary windings, 30 secondary windings – inside</td>
<td>PT1</td>
<td>0.025 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.773x(t) )</td>
<td>( t_p = 0.025 )</td>
</tr>
<tr>
<td>450 primary windings, 30 secondary windings – outside</td>
<td>PT1</td>
<td>0.058 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.765x(t) )</td>
<td>( t_p = 0.05 )</td>
</tr>
<tr>
<td>450 primary windings, 45 secondary windings</td>
<td>PT1</td>
<td>0.076 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.756x(t) )</td>
<td>( t_p = 0.074 )</td>
</tr>
<tr>
<td>500 primary windings, 15 secondary windings – inside</td>
<td>PT1</td>
<td>0.021 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.754x(t) )</td>
<td>( t_p = 0.051 )</td>
</tr>
<tr>
<td>500 primary windings, 15 secondary windings – middle</td>
<td>PT1</td>
<td>0.041 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.761x(t) )</td>
<td>( t_p = 0.041 )</td>
</tr>
<tr>
<td>500 primary windings, 15 secondary windings – outside</td>
<td>PT1</td>
<td>0.022 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.751x(t) )</td>
<td>( t_p = 0.023 )</td>
</tr>
<tr>
<td>500 primary windings, 30 secondary windings – inside</td>
<td>PT1</td>
<td>0.022 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.752x(t) )</td>
<td>( t_p = 0.023 )</td>
</tr>
<tr>
<td>500 primary windings, 30 secondary windings – outside</td>
<td>PT1</td>
<td>0.047 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.756x(t) )</td>
<td>( t_p = 0.047 )</td>
</tr>
<tr>
<td>500 primary windings, 45 secondary windings</td>
<td>PT1</td>
<td>0.084 ( \frac{dx(t)}{dt} + x(t) )</td>
<td>( x(t) = 9.761x(t) )</td>
<td>( t_p = 0.084 )</td>
</tr>
</tbody>
</table>
Table 2. Graphical representation of indexical functions, experimental and theoretical, obtained in the case of linear electric motor with shorted moving coil with variable height. [19]

300 primary turns, 15 secondary turns - inside

300 primary turns, 15 secondary turns - middle

300 primary turns, 15 secondary turns - outside
4. Conclusions

If in automatic system research is using the differential equation subject to adjustment object, the approximate values of the coefficients can be determined by experimental dynamic characteristics of the object: indexical function or transfer function. The form of experimental characteristic usually allows to appreciate the necessary degree and character of the equation.

For experimental measurements of a.c. electric motor was used one electric linear motor with a short moving coil having as input supply voltage level signal.

The preliminary experimental data were extrapolated based on theoretical characteristic function deducted from the expression of an indexical function of an PT1 type system. The response of the motor is of type PT1 characterized by an I order differential equation.

Mathematical model of an electric motor with a short moving coil, presented in this paper, may be studied as a static object I order. The time constant of the motor is high when we use a reduced number of turns and introduce turns decreases as flow rule is valid for both the primary winding and the secondary winding.

References


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