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A Statistical Comparison of Nine Rock Failure Criteria

Today the rock failure criteria have a wide range of usage for determining the stress conditions around underground structures and slope stability methods .In this study we examine nine different failure criteria by comparing them to published polyaxial test data ($\sigma_1 > \sigma_2 > \sigma_3$) for five different rock types at a variety of stress states. several article have published about this subject with different criteria and rock data currently. For this purpose we need a factor named Least Mean Standard Deviation Misfits. In this paper we used all of them and try to gather them and present general result. Result of this article shows that polyaxial criteria has more accuracy.

Keywords: *Rock failure criteria, polyaxial test data, Misfits*

1. Introduction

A number of different criteria have been proposed to describe brittle rock failure. In this study we aim to find which failure criterion, best describes the behavior of each rock type by minimizing the mean standard deviation misfit between the predicted failure stress and the experimental data. several papers are published, Colmenares et al compared seven criteria for five different rock types at a variety of stress states by associated misfits[1]. Thomas Benz et al [2] introduced the quantitative comparison of the six failure criteria follows the methodology introduced in[1]. In [2] more criteria and rock types are evaluated. In this article at first we present 9 rock failure criteria. They are Mohr–Coulomb (MC), The orginal Hoek–Brown, The Hoek–Brown criterion- edition 2002, HBMN, Mogi (1967), Mogi (1971), Drucker–Prager(DP), Modified Wiebols and Cook(MW), Modified Lade(ML). We also introduce Least mean standard deviation misfits and strength data of five rock types. Finally we calculate misfit of rock types for nine criteria and Interpret results and compare all of criteria with [1] and [2].

2. Nine Rock Failure Criteria

2.1. Mohr-Coulomb Criterion

The Mohr–Coulomb (MC) failure criterion is one of the earliest and most trusted failure criteria for soils and rocks [3]. Failure is assumed when in any (failure) plane the shear stress reaches the failure shear stress $\tau_{\max}$ which is given by a functional relation of the form Eq (1):

$$|\tau| = S_0 + \mu_i \sigma_n \tag{1}$$

Where S_0 is the shear strength or cohesion of the material and μ_0 is the coefficient of internal friction of the material. Since the sign of τ only affects the sliding direction, only the magnitude of τ matters. The linearized form of the Mohr failure criterion may also be written as Eq (2):

$$|\sigma_1| = C_0 + q\sigma_3 \tag{2}$$

Where

$$q = [(\mu_i^2 + 1)^{.5} + \mu_i]^2 = \tan^2(\frac{\pi}{4} + \frac{\phi}{2}) \tag{3}$$

where σ_1 is the major principal effective stress at failure, σ_3 is the least principal effective stress at failure, C_0 is the uniaxial compressive strength and φ is the angle of internal friction equivalent to $a\tan(\mu_0)$. This failure criterion assumes that the intermediate principal stress has no influence on failure. The yield surface of this criterion is a right hexagonal pyramid equally inclined to the principal-stress axes. The intersection of this yield surface with the p-plane is a hexagon.

2.2. The Hoek-Brown Criterion

This empirical criterion uses the uniaxial compressive strength of the intact rock material as a scaling parameter, and introduces two dimensionless strength parameters, m and S . After studying a wide range of experimental data, Hoek and Brown [4] stated that the relationship between the maximum and minimum stresses given by Eq (4):

$$\sigma_1 = \sigma_3 + C_0\sqrt{m\frac{\sigma_3}{C_0} + s} \tag{4}$$

where m and S are constants that depend on the properties of the rock and on the extent to which it had been broken before being subjected to the failure stresses σ_1 and σ_3.

While these values of m obtained from lab tests on intact rock are intended to represent a good estimate when laboratory tests are not available, we will compare them with the values obtained for the five rocks studied. For intact rock materials, $s = 1$. For a completely granulated specimen or a rock aggregate, $s = 0$ [5].

2.3. The HE Hoek-Brown Criterion (Eddition 2002)

At failure, the generalized HB criterion [5] relates the maximum effective stress, σ_1 to the minimum effective stress σ_3 through the functional relation:

$$\sigma_1 = \sigma_3 + (m_b \frac{\sigma_3}{\sigma_{ci}} + s)^a \tag{5}$$

where m_b extrapolates the intact rock constant m_i to the rock mass as Eq (6):

$$m_b = m_i \exp(\left(\frac{GSI - 100}{28 - 14D}\right) \tag{6}$$

σ_{ci} is the uniaxial compressive strength of the intact rock. s and a (Eq (7),(8)) are constants which depend upon the rock mass's characteristics:

$$s = \exp(\left(\frac{GSI - 100}{9 - 3D}\right) \tag{7}$$

$$a = \frac{1}{2} + \left(\frac{1}{6}(\exp(\left(\frac{-GSI}{15}\right) - \exp(\left(\frac{-20}{3}\right)\right) \tag{8}$$

The geological strength index (GSI), introduced by Hoek, provides a system for estimating the reduction in rock mass strength under different geological conditions. Finally, a factor which quantifies the disturbance of rock masses. It varies from 0 (undisturbed) to 1 (disturbed) depending on the amount of stress relief, weathering, and blast damage as a result of nearby excavations.

2.4. The HBMN Criterion

The HBMN criterion extends the generalized HB criterion as described above, with the spatial mobilized plane concept of Matsuoka and Nakai (MN) [6]. The deviatoric shape of the MN criterion is assigned to the HB criterion by setting:

$$f_{HBMN} = q - LM_{c,HB}(p - p_t) \qquad (9)$$

Where

$$M_{c,HB} = \frac{3f}{f+3} \text{ and } M_{e,HB} = \frac{3f}{2f+3} \qquad (10)$$

and

$$p_t = \frac{f}{Mc,HB} + \frac{f}{3} + \sigma_3 \qquad (11)$$

and L varies between 1 and $\delta = M_{e,MC} / M_{c,MC}$ for Triaxial compression and extension, respectively [13]:

$$L(\theta) = \frac{\sqrt{3}\delta}{2\sqrt{\delta^2 - \delta + 1}} \frac{1}{\cos \vartheta} \qquad (12)$$

with

$$\begin{Bmatrix} \vartheta = \frac{1}{6}\arccos\left(-1 + \frac{27\delta^2(1-\delta)^2}{2(\delta^2 - \delta + 1)}\sin^2(3\theta)\right) \textit{for } \theta < 0 \\ \vartheta = \frac{\pi}{3} - \frac{1}{6}\arccos\left(-1 + \frac{27\delta^2(1-\delta)^2}{2(\delta^2 - \delta + 1)}\sin^2(3\theta)\right) \textit{for } \theta > 0 \end{Bmatrix} \qquad (13)$$

where the lode angle θ is defined as

$$\theta = \frac{1}{3}\arcsin\left(\frac{-3\sqrt{3}J_3}{2J_2^{1.5}}\right) \qquad (14)$$

and J_2 and J_3 are the second and third invariants of the deviatoric stress tensor, respectively.

2.5. Mogi 1967 Empirical Criterion

Mogi [7] studied the influence of the intermediate stress on failure by performing confined compression tests ($\sigma_1 > \sigma_2 = \sigma_3$), confined extension tests ($\sigma_1 = \sigma_2 > \sigma_3$) and biaxial tests ($\sigma_1 > \sigma_2 > \sigma_3 = 0$) on different rocks. He recognized that the influence of the intermediate principal stress on failure is non-zero, but considerably smaller than the effect of the minimum principal stress .When he plotted the maximum shear stress ($(\sigma_1 - \sigma_2)/2$ as a function of $(\sigma_1 + \sigma_2)/2$ for failure of Westerly Granite, he observed that the extension curve lied slightly above the compression curve and the opposite happened when he plotted the octahedral shear stress τ_{oct} as a function of the mean normal stress $(\sigma_1+\sigma_2+\sigma_3)/3$ (for failure of the same rock. Therefore, if ($(\sigma_1+B\,\sigma_2+\sigma_3)/3$) is taken as the abscissa (instead of ($(\sigma_1+\sigma_3)/$) or ($(\sigma_1+\sigma_2+\sigma_3)$)), the compression and the extension curves become coincidental at a suitable value of b: Mogi argued that this b value is nearly the same for all brittle rocks but we will test this assertion. The empirical criterion has the following formula as Eq (15):

$$(\sigma_1-\sigma_3)/2 = f_1[(\sigma_1+B\sigma_2+\sigma_3)/2] \tag{15}$$

where β is a constant smaller than 1. The form of the function f_1 in Eq. (21) is dependent on rock type and it should be a monotonically increasing function. This criterion postulates that failure takes place when the distortional energy increases to a limiting value, which increases monotonically with the mean normal pressure on the fault plane. The term $b\,\sigma_2$ may correspond to the contribution of σ_2 to the normal stress on the fault plane because the fault surface, being irregular, is not exactly parallel to s_2 and it would be deviated approximately by $\arcsin(B)$

2.6. Mogi 1971 Empirical Criterion

Mogi [7]proposed two functional relationships for rock failure, of which only the latter (Mogi 1971 criterion [21]) is considered here. In this, Mogi relates the octahedral shear stress at failure to the sum of the minimum and maximum principal stresses:

$$\tau_{oct} = f\left(\frac{\sigma_1+\sigma_3}{2}\right) \tag{16}$$

where f is a monotonically increasing function. Plotting τ_{oct} against $\sigma_{m,2}=(\sigma_1+\sigma_3)/2$ for different experimental data reveals that a linear function f readily gives satisfactory results, e.g. [2]. The linear Mogi criterion can be written as:

$$\tau_{oct} = a + b\left(\frac{\sigma_1+\sigma_3}{2}\right) \tag{17}$$

Considering that in triaxial conditions $q=\sigma_1-\sigma_3$ and that $\tau_{oct}=\sqrt{2}/3q$,the linear Mogi parameters a and b relate to the Coulomb shear strength parameters c and φ in triaxial compression and extension as Eq(18):

$$a = \frac{2\sqrt{2}}{3} c \cos \phi \quad , \quad b = \frac{2\sqrt{2}}{3} \sin \phi \tag{18}$$

2.7. Drucker-Prager Criterion

The von Misses criterion may be written in the following way as (19):

$$j_2 = k^2 \tag{19}$$

where k is an empirical constant. The extended von Mises yield criterion or Drucker–Prager criterion was originally developed for soil mechanics [8].The yield surface of the modified von Mises criterion in principal stress space is a right circular cone equally inclined to the principal-stress axes. The intersection of the p-plane with this yield surface is a circle. The yield function used by Drucker and Prager to describe the cone in applying the limit theorems to perfectly plastic soils has the form

$$j_2^{\frac{1}{2}} = k + \alpha j_1 \tag{20}$$

Where α and k are material constants. The material parameters α and k can be determined from the slope and the intercept of the failure envelope plotted in the j_1 and $(j_2)^{1/2}$ space. α is related to the internal friction of the material and k is related to the cohesion of the material, in this way, the Drucker–Prager criterion can be compared to the Mohr–Coulomb criterion. When $\alpha = 0$; Eq. (24) reduces to the Von Mises criterion.

2.8. Modified Wiebols and Cook Criterion

Zhou [9] presented a failure criterion, which is an extension of the Circumscribed Drucker–Prager criterion (described later) with features similar to the effective strain energy criterion of Wiebols and Cook . The failure criterion described by Zhou predicts that a rock fails if

$$j_2^{\frac{1}{2}} = A + Bj_1 + Cj_1^2 \tag{21}$$

Where

$$j_1 = (1/3) \times (\sigma_1 + \sigma_2 + \sigma_3) \tag{22}$$

and

$$j_2^{\frac{1}{2}} = \sqrt{\frac{1}{6}((\sigma_1 - \sigma_2)^2 + (\sigma_3 - \sigma_2)^2 + (\sigma_3 - \sigma_1)^2} \tag{23}$$

where j_1 is the mean effective confining stress and $j_2^{1/2} = (3/2)^{1/2}\tau_{oct}$; where τ_{oct} is the octahedral shear stress. The parameters A, B, and C are determined such that Eq. (14) is constrained by rock strengths under triaxial ($\sigma_2 = \sigma_3$) and biaxial ($\sigma_1 = \sigma_2$) conditions.

2.9. Modified Lade Criterion

In the modified Lade criterion developed by Ewy[10], Doing all the modifications and defining appropriate stress invariants the following failure criterion was obtained by Ewy:

$$(I_1')^3 / I_3' = 27 + \eta \tag{24}$$

Where

$$I_1' = (\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S) \tag{25}$$

and

$$I_3' = (\sigma_1 + S)(\sigma_2 + S)(\sigma_3 + S) \tag{26}$$

where S and η are material constants. The parameter S is related to the cohesion of the rock, while the parameter η represents the internal friction. These parameters can be derived directly from the Mohr–Coulomb cohesion S_0 and internal friction angle φ.

3. Least Mean Standard Deviation Misfits

This factor is one of the most important in Statistic concepts. we can use this factor for comparison of criteria[11].The least mean standard deviation misfit within this study is calculated as follows: Let m be the number of test series (i.e., tests with identical (σ_3), n be the number of data points per series, and i and j the indices for test series and data point per series, respectively. Then, the standard deviation of a failure criterion in test series i is as Eq(27):

$$s_i = \sqrt{\frac{1}{n}\sum_j (\sigma_{1,j}^{calc} - \sigma_{1,j}^{test})^2} \tag{27}$$

where $\sigma_{1,j}^{test}$ is the tested maximum stress at failure for data point j and $\sigma_{1,j}^{calc}$ is the calculated one. Finally the mean standard deviation misfit is calculated as the arithmetic mean of all calculated standard deviations for a specific rock type:

$$\bar{s} = \frac{1}{m}\sum_i s_i \qquad (28)$$

4. Strength Data

The five rock types investigated were amphibolite from the KTB site, Dunham dolomite, Solenhofen limestone, Shirahama sandstone and Yuubari shale. The polyaxial data of these rocks were obtained from published works [1],[2].

5. Result

In this part we want to calculate and gather values of misfits in past works. The mean standard deviation misfit is a scalar indicator of how precisely a failure criterion can predict rock failure. An ideal criterion in an ideal test environment would yield no misfit. Generally, the higher the precision of a criterion, the lower the resulting misfit. With the help of the calculated misfits, the merits of non-linear and intermediate principal stress dependent failure criteria are now analyzed using direct comparisons. Table 1 and figure 1 show value misfits values for 5 rock types clearly.

Table 1.

criteria Rock	Dunham dolomite	Solenhofenlimeston	Shiahamasandston	Yuubari shale	KTB amphibolite
MC	56	37	11	13	113
Mogi(1967)	42.1	29.8	13.2	10.3	95.2
Mogi(1971)	27.9	19.4	14.1	11.5	112.6
Original HB	56.2	37.4	8.7	13	89.9
HB(2002)	56	37	9	13	89
HBMN	**21**	**21**	**7**	**9**	**64**
ML	27	23	13	13	90
MW	27	25	12	12	76
DP	51.6	35.9	28.3	21	161.54

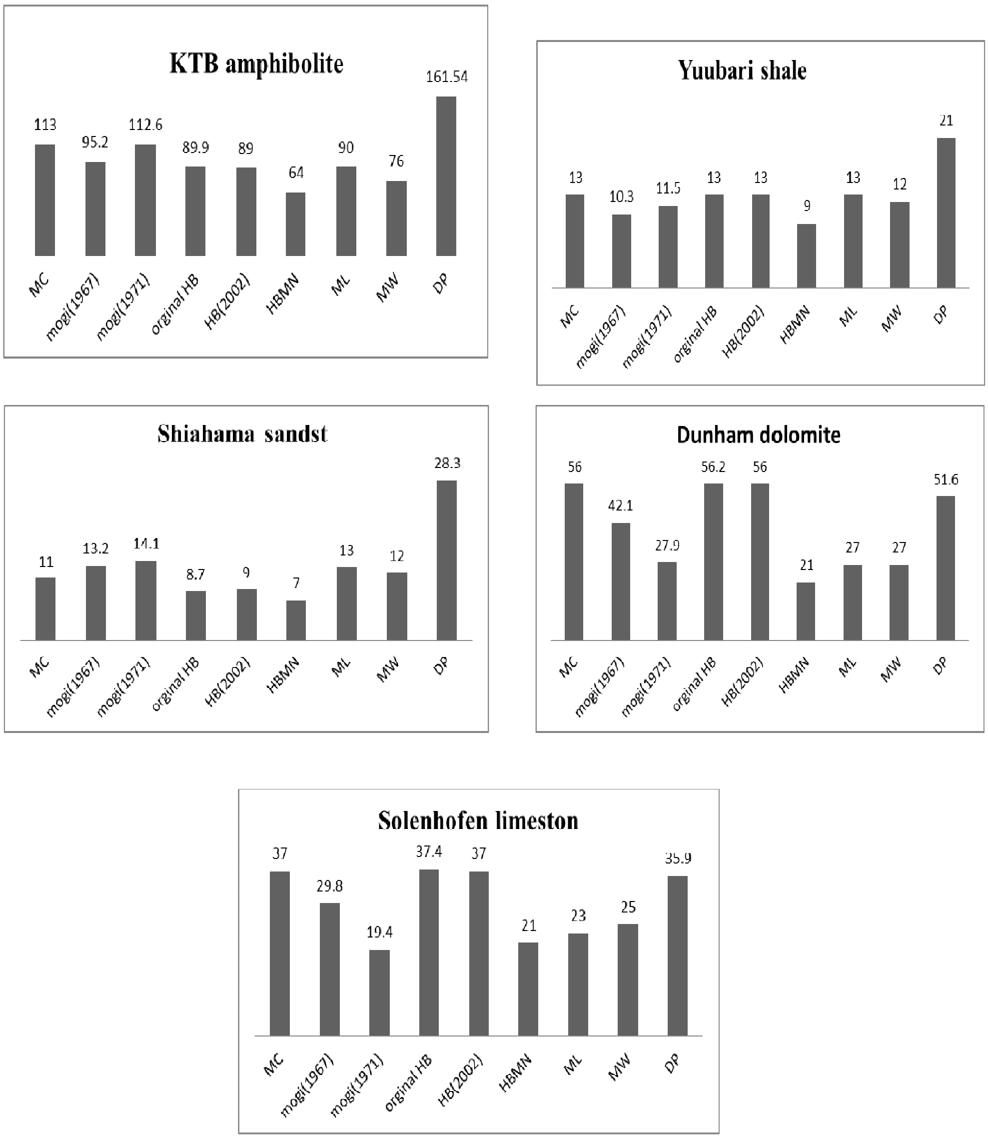

Figure. 6. Least mean standard deviation misfit for 5 rock types (MPa).

6. Summary and Conclusions

A comparison of these misfits revealed the enhancements that are possible in defining rock failure criteria as non-linear in $p-q$ space and dependent on intermediate principal stress. The new HBMN criterion which incorporates both of these features in one failure criterion consequently gave the overall least misfit in this study, original HB and HB(2002) have close misfits. The HB criterion is curved, whereas the MC criterion is linear. Comparing the misfits of the MC to the misfits of the HB criterion shows that the non-linear criterion always yields equal or less deviation from the test data. failure criteria MC,HB, Mogi(1971) and HBMN of have identical shapes in the $p-q$ plane and differ only in their deviatoric shape, a clear reduction of the misfit can be seen when the intermediate principal stress is considered in the failure criterion. Generally, the shape of the HBMN criterion is closer to experimentally observed rock failure behavior than the shape of the Mogi criterion. The criterion with the overall least misfit in the current benchmark test is the HBMN criterion.

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