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Modal Analysis of a Building with Dynamic Behavior of an Elastic Constrained Rigid, Subjected to Seismic Actions

This paper presents the modal analysis for different kinds of buildings having symmetry around Y-axis. This study includes the dynamic behavior of buildings considered as elastic constrained rigid subjected to seismic actions. The results are interpreted for two different kinds of buildings – one with symmetry around X-axis and another one non-symmetric.

Keywords: *seismic actions, dynamic behavior, elastic constraints*

1. Introduction

The method of buildings base isolation is considered to be a widely used procedure for about two decades with remarkable performance in the field of seismic protection of structures.

Building supporting using elastomeric isolators that allow lateral displacements of about 0.3 - 1.2 m represents the most favourable technical solution to increase the dynamic isolation in the horizontal direction so that the seismic motion loads acting from the ground upon the building to be decreased till their total reduction.

For this, the solution adopted in the building design phase using the elastic support assured by supporting groups, each group being composed of a number of elastomeric isolators, necessarily implies dynamic analysis of building behaviour under the assumption that its deformations are negligible compared with displacements generated by the seismic motion, so it may be considered as a rigid with 6 DOF.

This paper is part of this context because of its objective to determine the correlation between inertial parameters, stiffness and position of groups of support, so the resonance frequencies may be determined, i.e. the specific eigenvalues intrinsically depending only on the geometric structure, mass distribution, the supports elasticity and location.

2. The dynamic model of a building with rigid motion supported by elastomeric isolators

It is considered the rigid represented by Fig. 1, elastic supported in four points of its inferior base, having *yCz* plane as longitudinal vertical plane of symmetry.

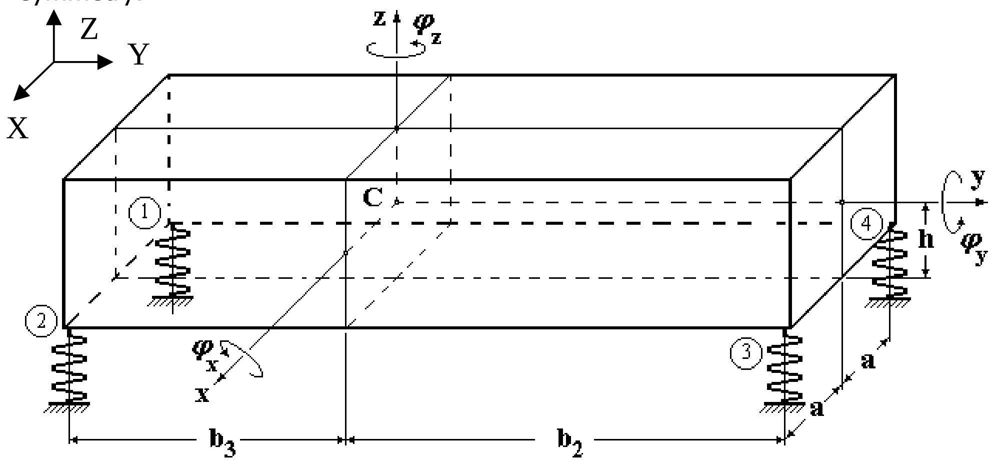

Figure 1. Building elastic supported in four points of its inferior base, having *yCz* plane as longitudinal vertical plane of symmetry

The assumptions used for the symmetry property of the rigid are based on the following features:

- Mass distribution;
- Dimensional symmetry;
- Identical elastic constraints with symmetrical positions and location on the same horizontal plane.

Because of the mentioned symmetry, some coupling terms of the stiffness matrix were considered zero and we had:

$$\sum k_{iy} x_i = 0$$
$$\sum k_{iz} x_i = 0$$
$$\sum k_{iz} x_i y_i = 0$$
$$\sum k_{iy} z_i x_i = 0,$$

and the stiffness matrix became:

$$\underline{C} = \begin{bmatrix} 4k_x & 0 & 0 & 0 & -4hk_x & -2k_x(b_3-b_2) \\ 0 & 4k_y & 0 & 4hk_y & 0 & 0 \\ 0 & 0 & 4k_z & 2k_z(b_3-b_2) & 0 & 0 \\ 0 & 4hk_y & 2k_z(b_3-b_2) & 2\left[k_z\left(b_2^2+b_3^2\right)+2h^2k_y\right] & 0 & 0 \\ -4hk_x & 0 & 0 & 0 & 4\left(h^2k_x+a^2k_z\right) & 2hk_x(b_3-b_2) \\ -2k_x(b_3-b_2) & 0 & 0 & 0 & 2hk_x(b_3-b_2) & 2\left[2a^2k_y+k_x\left(b_2^2+b_3^2\right)\right] \end{bmatrix} \quad (1)$$

In a tabular form, the stiffness matrix may be presented as follows:

$\underline{C}$=	X	Y	Z	φ_x	φ_y	φ_z	
	c_{11}	0	0	0	c_{15}	c_{16}	X
	0	c_{22}	0	c_{24}	0	0	Y
	0	0	c_{33}	c_{34}	0	0	Z
	0	c_{42}	c_{43}	c_{44}	0	0	φ_x
	c_{51}	0	0	0	c_{55}	0	φ_y
	c_{61}	0	0	0	0	c_{66}	φ_z

where the figured coefficients c_{ij} are non-zero, having the expressions from matrix (1), all the other being zero.

In the tabular form of the stiffness matrix it was noticed that the system was decoupled in two subsystems described by the coordinates *(Y, Z,* φ_x*)* and *(X,* φ_y, φ_z*)*, characterized by specific matrices, by the elimination of the coupling terms (zero terms).

• the subsystem *(Y, Z,* φ_x*)* has the following matrices:

- The matrix of inertia:

$$\underline{A}_3 = \begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & J_x \end{bmatrix} \quad (2)$$

- Stiffness matrix:

$$\underline{C}_3 = \begin{bmatrix} 4k_y & 0 & 4hk_y \\ 0 & 4k_z & 2k_z(b_3-b_2) \\ 4hk_y & 2k_z(b_3-b_2) & 2\left[k_z\left(b_2^2+b_3^2\right)+2h^2k_y\right] \end{bmatrix} \quad (3)$$

• the subsystem *(X,* φ_y, φ_z*)* has the following matrices:

– The matrix of inertia:

$$\underline{A}_4 = \begin{bmatrix} m & 0 & 0 \\ 0 & J_y & 0 \\ 0 & 0 & J_z \end{bmatrix} \quad (4)$$

– Stiffness matrix:

$$\underline{C}_4 = \begin{bmatrix} 4k_x & -4hk_x & -2k_x\left(b_3 - b_2\right) \\ -4hk_x & 4\left(h^2 k_x + a^2 k_z\right) & 2hk_x\left(b_3 - b_2\right) \\ -2k_x\left(b_3 - b_2\right) & 2hk_x\left(b_3 - b_2\right) & 2\left[2a^2 k_y + k_x\left(b_2^2 + b_3^2\right)\right] \end{bmatrix} \quad (5)$$

3. Eigen values determination for coupled modes

The two subsystems (*Y, Z,* φ_x) and *(X,* φ_y, φ_z*)* are characterized each of them by three coupled dynamic coordinates (3 DOF).

Free vibrations of systems with three dynamic degrees of freedom

It was considered the system with elastic constraints and three dynamic degrees of freedom described by the followings: the generalized coordinates vector

$$\underline{\underline{q}} = \left[q_1, q_2, q_3\right]^{\mathrm{T}}, \quad (6)$$

The generalized velocities vector

$$\underline{\underline{\dot{q}}} = \left[\dot{q}_1, \dot{q}_2, \dot{q}_3\right]^{\mathrm{T}} \quad (7)$$

The generalized accelerations vector

$$\underline{\underline{\ddot{q}}} = \left[\ddot{q}_1, \ddot{q}_2, \ddot{q}_3\right]^{\mathrm{T}}. \quad (8)$$

Quadratic forms of the system energies are expressed as follows:

- Kinetic energy

$$2E = a_{11}\dot{q}_1^2 + a_{22}\dot{q}_2^2 + a_{33}\dot{q}_3^2 + 2a_{12}\dot{q}_1\dot{q}_2 + 2a_{23}\dot{q}_2\dot{q}_3 + 2a_{13}\dot{q}_1\dot{q}_3; \quad (9)$$

- Potential energy

$$2V = c_{11}q_1^2 + c_{22}q_2^2 + c_{33}q_3^2 + 2c_{12}q_1q_2 + 2c_{23}q_2q_3 + 2c_{13}q_1q_3. \quad (10)$$

Using *Lagrange equations of the second kind,* we obtained the equations of motion in matrix form as follows:

$$\underline{\underline{A}}\ddot{\underline{\underline{q}}}+\underline{\underline{C}}\underline{\underline{q}}=\underline{\underline{0}}, \tag{11}$$

where:

$\underline{A}=\left\{a_{ij}\right\}_{i,j=\overline{1,3}}=\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$ is the inertia matrix;

$\underline{C}=\left\{c_{ij}\right\}_{i,j=\overline{1,3}}=\begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$ is the stiffness matrix;

$\underline{\underline{0}}^{\mathrm{T}}=\left[0,0,0\right]$ is the null vector with three components

Searching for the system (11) synchronous and sin-phase solutions having the following form:

$$\underline{\underline{q}}=\underline{\underline{a}}\sin pt=\begin{Bmatrix} a_1 \\ a_2 \\ a_3 \end{Bmatrix}\sin pt=\begin{Bmatrix} a_1 \\ \mu_2 a_1 \\ \mu_3 a_1 \end{Bmatrix}\sin pt \tag{12}$$

and taking into account that $\ddot{\underline{\underline{q}}}=-p^2\underline{\underline{q}},$ it was obtained the next matrix equation

$$\left(\underline{C}-p^2\underline{A}\right)\underline{\underline{a}}=\underline{\underline{0}}. \tag{13}$$

Equation (13) has non-zero solutions only if the determinant of the matrix $\underline{F}=\underline{C}-p^2\underline{A}$ is zero, i.e.:

$$\Delta=\det\left(\underline{C}-p^2\underline{A}\right)=0. \tag{14}$$

Third degree polynomial equation in p^2 is the eigen pulsations equation of a dynamic system with 3 DOF.

Since the matrix $\underline{F}$ may be written as

$$\underline{F} = \underline{C} - p^2 \underline{A} = \begin{bmatrix} c_{11} - p^2 a_{11} & c_{12} - p^2 a_{12} & c_{13} - p^2 a_{13} \\ c_{21} - p^2 a_{21} & c_{22} - p^2 a_{22} & c_{23} - p^2 a_{23} \\ c_{31} - p^2 a_{31} & c_{32} - p^2 a_{32} & c_{33} - p^2 a_{33} \end{bmatrix}, \tag{15}$$

it follows that its determinant is

$$\Delta = \begin{vmatrix} c_{11} - p^2 a_{11} & c_{12} - p^2 a_{12} & c_{13} - p^2 a_{13} \\ c_{21} - p^2 a_{21} & c_{22} - p^2 a_{22} & c_{23} - p^2 a_{23} \\ c_{31} - p^2 a_{31} & c_{32} - p^2 a_{32} & c_{33} - p^2 a_{33} \end{vmatrix}, \tag{16}$$

and we'll have by developing:

$$\begin{aligned} \Delta = &\left(c_{11} - p^2 a_{11}\right)\left(c_{22} - p^2 a_{22}\right)\left(c_{33} - p^2 a_{33}\right) + \left(c_{13} - p^2 a_{13}\right)\left(c_{21} - p^2 a_{21}\right)\left(c_{32} - p^2 a_{32}\right) + \\ &+ \left(c_{12} - p^2 a_{12}\right)\left(c_{23} - p^2 a_{23}\right)\left(c_{31} - p^2 a_{31}\right) - \left(c_{13} - p^2 a_{13}\right)\left(c_{22} - p^2 a_{22}\right)\left(c_{31} - p^2 a_{31}\right) - \\ &- \left(c_{11} - p^2 a_{11}\right)\left(c_{23} - p^2 a_{23}\right)\left(c_{32} - p^2 a_{32}\right) - \left(c_{12} - p^2 a_{12}\right)\left(c_{21} - p^2 a_{21}\right)\left(c_{33} - p^2 a_{33}\right). \end{aligned}$$

Since the inertia matrix is diagonal ($a_{ij} = a_{ji} = 0,\ \forall i, j = \overline{1,3},\ a_{ii} \neq 0$) and the stiffness matrix is symmetric and non-diagonal ($c_{ij} = c_{ji},\ \forall i, j = \overline{1,3},\ c_{ii} \neq 0$ $\exists i \neq j$ a.î. $c_{ij} \neq 0$), the system being elastic coupled, it follows that the matrix $\underline{F}$ may be formulated as follows:

$$\underline{F} = \underline{C} - p^2 \underline{A} = \begin{bmatrix} c_{11} - p^2 a_{11} & c_{12} & c_{13} \\ c_{12} & c_{22} - p^2 a_{22} & c_{23} \\ c_{13} & c_{23} & c_{33} - p^2 a_{33} \end{bmatrix}. \tag{17}$$

With this, the eigen pulsations equation has the expression

$$\begin{aligned} \Delta = &\left(c_{11} - p^2 a_{11}\right)\left(c_{22} - p^2 a_{22}\right)\left(c_{33} - p^2 a_{33}\right) + 2c_{12}c_{23}c_{13} - \\ &- c_{23}^2\left(c_{11} - p^2 a_{11}\right) - c_{13}^2\left(c_{22} - p^2 a_{22}\right) - c_{12}^2\left(c_{33} - p^2 a_{33}\right) = 0. \end{aligned} \tag{18}$$

Equation (12.182) may be solved analytically or numerically, yielding three eigen pulsations p_1, p_2 and p_3.

Eigenvalues were determined using the relationships

$$\mu_{2i} = \frac{c_{12}c_{13} - c_{23}\left(c_{11} - p_i^2 a_{11}\right)}{c_{12}c_{23} - c_{13}\left(c_{22} - p_i^2 a_{22}\right)} \quad i = \overline{1,3} \tag{19}$$

$$\mu_{3i} = \frac{\left(c_{11} - p_i^2 a_{11}\right)\left(c_{22} - p_i^2 a_{22}\right) - c_{12}^2}{c_{12}c_{23} - c_{13}\left(c_{22} - p_i^2 a_{22}\right)} \quad i = \overline{1,3}. \tag{20}$$

The laws of motion for free vibration are expressed by:

$$\begin{cases} q_1(t) = C_1 \sin(p_1 t + \theta_1) + C_2 \sin(p_2 t + \theta_2) + C_3 \sin(p_3 t + \theta_3) \\ q_2(t) = \mu_{21} C_1 \sin(p_1 t + \theta_1) + \mu_{22} C_2 \sin(p_2 t + \theta_2) + \mu_{23} C_3 \sin(p_3 t + \theta_3) \\ q_3(t) = \mu_{31} C_1 \sin(p_1 t + \theta_1) + \mu_{32} C_2 \sin(p_2 t + \theta_2) + \mu_{33} C_3 \sin(p_3 t + \theta_3), \end{cases} \tag{21}$$

where C_1, C_2, C_3, θ_1, θ_2 and θ_3 are constants of integration.

4. Case study for a rigid building supported on rigid elastomeric isolators

According to Fig.1 it was chosen two kinds of buildings with Y-symmetry:

Case I. Non-symmetrical around X-axis
a=7.5m
b3= 12 m
b2= 8 m
h=7 m
Jx=42x 106 kgm^2
Jy=25x 106 kgm^2
Kz=32x 106 N/m
Ky= Kx=8x 106 N/m

Case II. Symmetrical
a=7.5m
b3= 10 m
b2= 10 m
h=7 m
Jx=35x 106 kgm^2
Jy=25x 106 kgm^2
Kz=32x 106 N/m
Ky= Kx=8x 106 N/m

Table 1 presents the results of the study of the dynamic analysis for the above cases:

- The eigen values for all cases computed using MAPLE program (Eq.14);
- The eigen pulsations (Eq. 19);
- The eigen vectors (Eq. 20).

Table 1.

Type of building symmetry	Subsystems	λ eigen values	p eigen pulsations	μ_{2i} (19)	μ_{3i} (20)
Non-symmetric	Subsystem I	20.20	4.494441	-0.60068	-
		87.64	9.361624	0.94248	-
		430.4	20.74608	1.10482	-
Non-symmetric	Subsystem II	18.13	4.257934	-	-0.03751
		203.1	14.25132	-	2.30787
		357.4	18.90503	-	-0.041717
Symmetric	Subsystem I	91.43	9.561904	0.95167	-
		413.2	20.32732	1.10323	-
		20.2	4.494441	-0.27443	-
Symmetric	Subsystem II	200.0	14.14214	-	2.34272
		18.54	4.30581	-	-0.000126
		355.0	18.84144	-	0.000603

5. Conclusion

The presented model may schematize the dynamic behaviour of a rigid building supported by elastomeric isolators featuring geometric, mass and elastic support symmetry, in relation to the median longitudinal plane.

Also, it was considered the case in which the building CG is at the intersection of two vertical median planes, respectively the longitudinal plane and the transverse one.

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