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Correlation of the Process of Concrete Compaction through Vibration with the Internal Dissipation of the Energy

Through the correlation of the compacting process with the vibration parameters, the compaction of the fresh concrete is optimized. The employed dynamic model is a linear viscous one. The dissipative energy in the vibrated concrete is given by the vibration parameters and the hysteresis loop is determined analyzing the dissipative energy in correlation with the degree of concrete compaction.

Keywords: *hysteresis loop, vibration parameters, concrete compaction*

1. Introduction

The compaction of fresh concrete can be optimized by effectively correlating the degree of compaction with the vibration parameters (amplitude, frequency). In this case compliance with the required viscosity, which determines the internal energy dissipation, must ensure the viscous damping coefficient c for the fresh concrete put into practice and subjected to an adequate vibration regime.

In the context of coherence, relations were established between the amplitude of the vibrating support excited by the force $F = m_0 r \omega^2 \sin \omega t$, in which $F_0 = m_0 r \omega^2$, and the mass of the fresh concrete from the mold (pattern).

2. Dynamic Model

The dynamic model is shown in Figure 1 and is characterized by the linear, viscous connection, with a damping coefficient c, chosen between the concrete mass and the vibrating platform, meaning that the viscous connection force is of the type $F(x) = cv = c(\dot{x}_1 - \dot{x}_2)$.

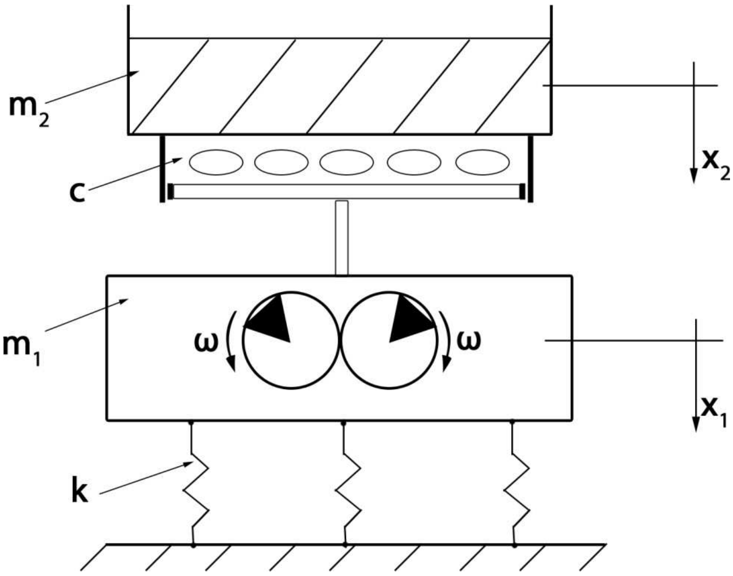

Figure 1. Dynamic Model

In Figure 1, the vibrating platform with mass m_1 and instantaneous displacement $x_1 = x_1(t)$ is excited by an inertial vibrator with eccentric masses having the static moment $m_o r$ and the angular velocity ω.

Mass m_2 of the fresh concrete is considered as a whole and has the instantaneous displacement $x_2 = x_2(t)$ and the coefficient of viscosity of the fresh concrete c. The entire assembly rests elastically on a system of springs with the elastic constant k.

Deformation equations in the complex domain are of the form [1]:

$$\begin{aligned} & m_1\ddot{z}_1 + c(\dot{z}_1 - \dot{z}_2) + kz_1 = F_0 e^{j\omega t} \\ & m_2\ddot{z}_2 + c(\dot{z}_2 - \dot{z}_1) = 0 \end{aligned} \tag{1}$$

where: $z_i = A_i e^{j(\omega t - \varphi_i)}$, with i=1,2 and $j = \sqrt{-1}$.

Thus:

$F(t) = F_0 e^{j\omega t}$ with R_e $F(t) = F_0 \cos \omega t$

$$\begin{aligned} & x_1 = R_e z_1 = A_1 \cos(\omega t - \varphi_1) \\ & x_2 = R_e z_2 = A_2 \cos(\omega t - \varphi_2) \end{aligned} . \tag{1'}$$

Replacing z_1, z_2 and $\dot{z}_1, \dot{z}_2$ in (1), in the end, the following relations are obtained:

$$A_1 = \frac{F_0}{R(\omega)}\sqrt{c^2 + m_2^2\omega^2}$$
$$A_2 = \frac{F_0}{R(\omega)}c \qquad (2)$$

where $F_0=m_0r\omega^2$ and represents the amplitude of the perturbing force and $R(\omega) = \sqrt{r_6\omega^6 + r_4\omega^4 + r_2\omega^2 + r_0}$ is the transfer function of attenuation of the dynamic force, applied in amplitudes of the instantaneous displacements.

The ratio of the amplitudes $\lambda = \frac{A_1}{A_2}$ is obtained in the form of:

$$\lambda = \frac{A_1}{A_2} = \frac{\sqrt{c^2 + m_2^2\omega^2}}{c} \qquad (3)$$

or

$$\lambda = \sqrt{1+\nu^2} \qquad (3')$$

where $\nu(\omega) = \nu = \frac{m_2\omega}{c}$ is the damping ratio.

3. Internal Energy Dissipation

The dissipative energy in the vibrated concrete is given by [2]:

$$W_d = \pi c\omega A_2^2 \qquad (4)$$

where $A_2 = \frac{A_1}{\lambda} = \frac{A_1}{\sqrt{1+\nu^2}}$.

Thus:

$$W_d = \pi c\omega\frac{A_1^2}{1+\nu^2}. \qquad (4')$$

If one replaces $\nu = \nu(\omega) = \frac{m_2\omega}{c}$ in (4'), it results:

$$W_d = \pi A_1^2\frac{\omega c^3}{c^2 + \omega^2 m_2^2} \qquad (5)$$

For the continuous variation of ω, the maximum dissipative energy $W_d^{\max}$ can be found considering the condition $\frac{dW_d}{d\omega} = 0$ or

$$\frac{dW_d}{d\omega} = \pi c^3 A_1^2 \frac{c^2 - m_2^2\omega^2}{(c^2 + \omega^2 m_2^2)^2} = 0 \quad (6)$$

from which $\omega_d = \pm\frac{c}{m_2}$, with the only physical solution $\omega_d = \frac{c}{m_2}$.

The curve has two points of extension, namely a point of minimum $Min\left(-\frac{c}{m_2}, -W_d^*\right)$ and a point of maximum $Max\left(+\frac{c}{m_2}, +W_d^*\right)$, and the curvature sense (concave, convex) is given by $\frac{d^2W}{d\omega^2} = 0$, that is [3]:

$$\frac{d^2W}{d\omega^2} = -2m_2^2\omega(m_2^2\omega^2 + c^2)(3c^2 - m_2^2\omega^2) = 0 \quad (7)$$

The roots are: $\omega_1 = -\frac{c}{m_2}\sqrt{3}$, $\omega_2 = 0$, $\omega_2 = +\frac{c}{m_2}\sqrt{3}$ and have the sign of

$\frac{d^2W}{d\omega^2} \prec 0$ for $\omega \in \left[-\infty, -\frac{c}{m_2}\sqrt{3}\right] \cup \left[0, +\frac{c}{m_2}\sqrt{3}\right]$ and

$\frac{d^2W}{d\omega^2} \succ 0$ for $\omega \in \left[-\frac{c}{m_2}\sqrt{3}, 0\right] \cup \left[+\frac{c}{m_2}\sqrt{3}, +\infty\right]$.

The curve representation $W_d = W_d(\omega)$ is shown in Figure 2 with the significant points: $Min\left(-\frac{c}{m_2}, -W_d^*\right)$, $Max\left(+\frac{c}{m_2}, +W_d^*\right)$,

inflection $I_1\left(-\frac{c}{m_2}\sqrt{3}, -W_i\right)$, $I_1\left(+\frac{c}{m_2}\sqrt{3}, +W_i\right)$,

and in which $W_d^* = W_d^{\max}$ represents the maximum energy.

The maximum energy $W_d^{\max} = W_d^*$, results from the condition that $\omega_d = \frac{c}{m_2}$:

$$W_d^{\max} = W_d^*(\omega_d) = \frac{\pi}{2} A_1^2 \frac{c^2}{m_2} \quad (8)$$

The corresponding energy of the inflection point I, $\omega_I = \frac{c}{m_2}\sqrt{3}$, is:

$$W_I(\omega_I) = \pi A_1^2 \frac{\sqrt{3}\frac{c}{m_2}c^3}{c^2 + 3\frac{c^2}{m_2^2}m_2^2} = \frac{\pi}{4}\sqrt{3}A_1^2\frac{c^2}{m_2} \quad (9)$$

or

$$W_I(\omega_I) = \frac{\sqrt{3}}{2}W_d^* \quad (9')$$

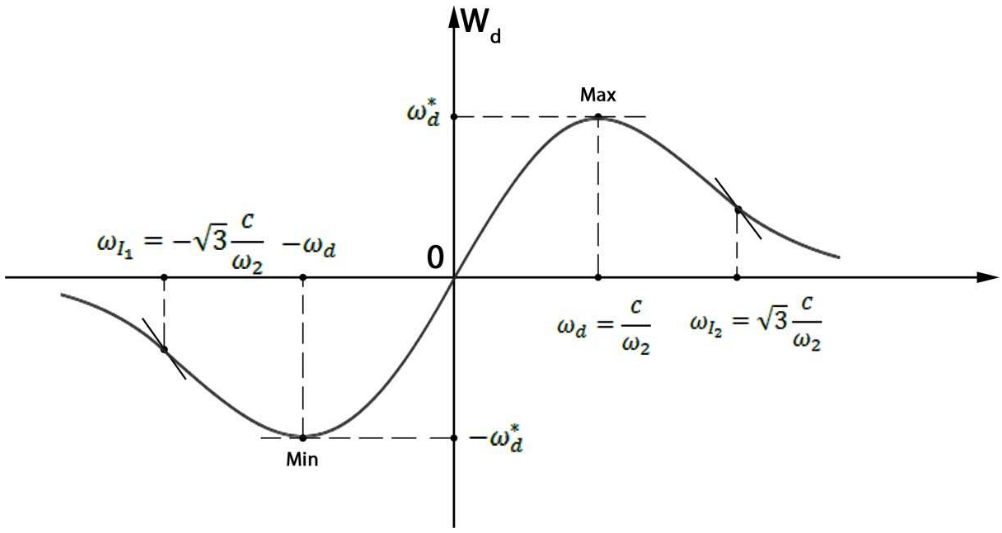

Figure 2. Representation of W_d

The maximum value of the dissipated energy, W_d^{max}, results from the condition $\omega_d = \frac{c}{m_2} = \omega$, in which $c^* = c = m_2\omega = 60*100\pi = 6\pi*10^3\frac{Ns}{m}$ and $\Omega = 100\pi$, Ω representing the pulsation of the disturbing force so that $\nu^* = \frac{m_2\omega}{c^*} = 1$.

In this case $\lambda = \sqrt{1+1} = \sqrt{2}$, that is $\frac{A_1}{A_2} = \sqrt{2}$

where $A_2 = \frac{A_1}{\sqrt{2}}$, and the energy is $W(\omega) = \pi c^* \omega_d A_2^2$ or

$$W^*(\omega) = \frac{\pi}{2}\frac{c^2}{m_2} A_1^2 \qquad (10)$$

For $A_1 = 0{,}5*10^{-3} m$, $m_2 = 60 kg$, $c^* = 6\pi * 10^3 \frac{Ns}{m}$ and

$$\omega_1 = \frac{1}{2}\omega = 50\pi,\ \omega_2 = \omega_d = \omega = 100\pi,\ \omega_3 = \frac{3}{2}\omega = 150\pi,$$

$$\omega_4 = \omega_I = \sqrt{3}\omega = \sqrt{3}*100\pi,\ \omega_5 = 2\omega = 200\pi,$$

the following values for the dissipated energy given by formula (5) are obtained:

$$W_1(\omega_1) = W_1\left(\frac{1}{2}\omega\right) = 1{,}85\,\bar{j}$$

$$W_2(\omega) = W_d^*(\omega_d) = 2{,}35\,\bar{j}$$

$$W_3\left(\frac{3}{2}\omega\right) = 2{,}17\,\bar{j}$$

$$W_4(\sqrt{3}\omega) = W_I(\omega_I) = 1{,}98\,\bar{j}$$

$$W_5(2\omega) = 1{,}857\,\bar{j}$$

and which satisfy the condition $W_I = \frac{\sqrt{3}}{2} W_d^* = \frac{\sqrt{2}}{2} * 2{,}35 = 1{,}98$.

The function $W_d(\omega) = \pi A_1^2 \frac{c^3 \omega}{c^2 + \omega^2 m_2^2}$ can be represented numerically with the parameter c, the current variable ω=0.....500 rad/s considering a step of 0,1rad/s.

4. Hysteretic Loop

The transmitted force is $F_T = Q(t) = -c(\dot{x}_1 + \dot{x}_2)$,
where $x_1(t) = A_1 \cos\omega t$ with $\dot{x}_1(t) = -\omega A_1 \sin\omega t$
and $x_2(t) = A_2 \cos(\omega t - \varphi)$ with $\dot{x}_2(t) = -\omega A_2 \sin(\omega t - \varphi)$, given by:

$$Q(t) = -c\omega\left[-\left(A_1 + A_2\cos\varphi\right)\sin\omega t + A_2\sin\varphi\cos\omega t\right] \quad (11)$$

or

$$Q(t) = c\omega\left[\left(A_1 + A_2\cos\varphi\right)\sin\omega t - A_2\sin\varphi\cos\omega t\right] \quad (11')$$

The time variable, t, is eliminated from equation (11) or (11′), taking into account that $x_1 = A_1\cos\omega t$

so that $\cos\omega t = \frac{x_1}{A_1}$ and $\sin\omega t = \pm\sqrt{1-\frac{x_1^2}{A_1^2}}$.

One finally obtains:

$$Q(x_1) = c\omega\left[\pm\left(A_1 + A_2\cos\varphi\right)\sqrt{1-\frac{x_1^2}{A_1^2}} - A_2\frac{x_1}{A_1}\sin\varphi\right] \quad (12)$$

Where: $\sin\varphi = \frac{m_2\omega}{\sqrt{c^2 + m_2^2\omega^2}}$, $\cos\varphi = \frac{c}{\sqrt{c^2 + m_2^2\omega^2}}$

and in which $tg\varphi = \frac{m_2\omega}{c}$ was taken into consideration.

Replacing in (12) the functions sinφ and cosφ it results:

$$Q(x_1) = c\omega\left\{\left[\pm A_1\left(1+\frac{A_2}{A_1}\frac{c}{\sqrt{c^2 + m_2^2\omega^2}}\right)\frac{\sqrt{A_1^2 - x_1^2}}{A_1}\right] - \frac{A_2}{A_1}x_1\frac{m_2\omega}{\sqrt{c^2 + m_2^2\omega^2}}\right\} \quad (13)$$

where, introducing $\frac{A_2}{A_1} = \frac{c}{\sqrt{c^2 + m_2^2\omega^2}}$, one has

$$Q(x_1) = \pm\frac{2c^3\omega + m_2^2\omega c}{c^2 + m_2^2\omega^2}\sqrt{A_1^2 - x_1^2} - \frac{c^2 m_2\omega^2}{c^2 + m_2^2\omega^2}x_1 \quad (14)$$

Equation (14) represents a family of ellipses in the coordinates $Q(x_1) - x_1$ with the parameter ω or c. The area of the ellipse, that is, the hysteresis loop, represents the dissipated energy correlated with the degree of concrete compaction.

5. Conclusions

Based on the schematization of the visco-linear model for the compacting of fresh concrete under vibration, the following can be determined:

a) the variation law of the dissipated energy in steady-state vibration regime for the compaction process;
b) the maximum value of the dissipated energy through the correlation of the excitation pulsation with the concrete mass and the viscous constant of the dissipation energy;
c) determination of the hysteresis loops using the vibration parameters A_1, ω and the mass m_2 and damping constant c of the fresh concrete.

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