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Dynamic Response of a System under Anti-Vibration Forced Regime Depending on the Settlement Angle of the Elastomeric Isolators

For a number N of elastomeric elements grouped in parallel, but mounted at an angle alpha between the compression axis and the vertical axis, the total stiffness should be assessed accordingly. The vibration transmissibility will be determined for two borderline cases, so that intermediate values can be evaluated for an anti-vibration isolation techniques solution

Keywords: *elastomeric elements, transmissibility, isolation degree*

1. Introduction

In case of a number N of elastomeric elements grouped in parallel, but mounted at an angle alpha between the compression axis and the vertical axis, then the total stiffness should be assessed accordingly. For this reason, for an isolation system we take into account the inclined assembly of the elastomeric elements which correspond to an equivalent stiffness coefficient $k=Nk_{\alpha}$.

2. Dynamic Schema

Figure 1 gives a schema of a system of two masses m_1 and m_2 connected by a linear visco-elastic system with vertical stiffness k and damping c.

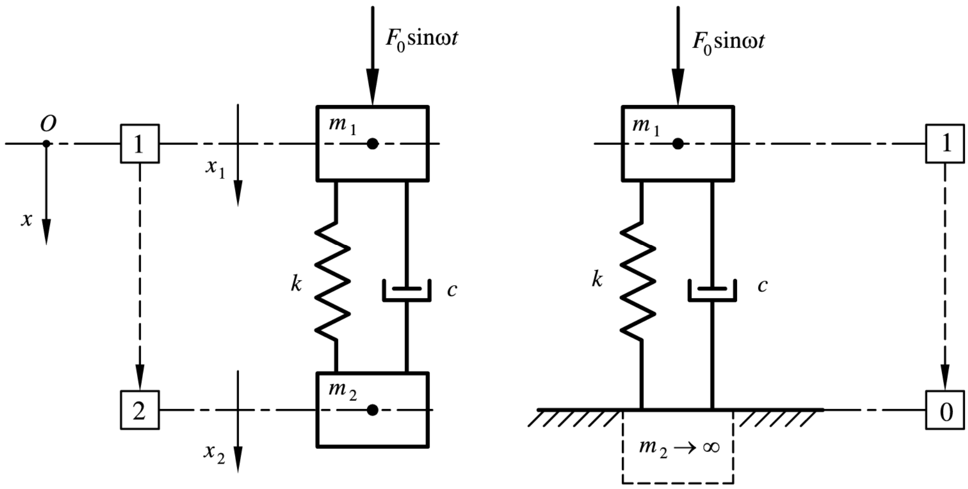


Figure 1. Dynamic System

Figure 2 shows the assembly of a group of two elements, placed symmetrically at an angle α.

For N elements arranged symmetrically at an angle α, the condition is the same, that is, all elements to be same physico-mechanical and geometrical.

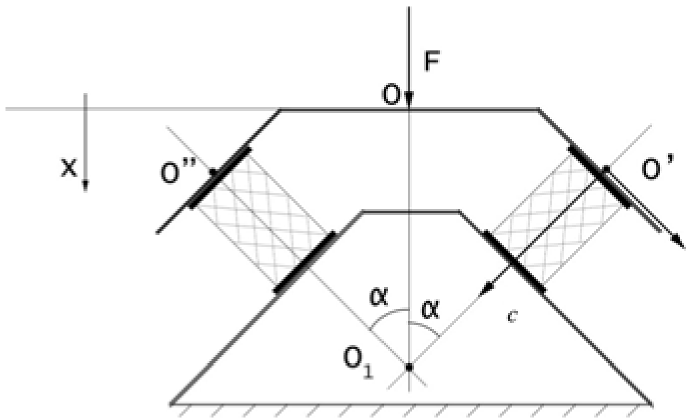


Figure 2. Assembly of a group of two elements

For one elastomeric isolator stiffness on the x axis is given by:

$$k_{1x} = k_{1\alpha} = k_c \cos^2 \alpha + k_f \sin^2 \alpha \quad (1)$$

where k_c is compression stiffness and k_f shear stiffness.

For N elastomeric isolators symmetrically positioned in relation to the vertical axis OO1, under the angle α, the total stiffness is [1]:

$$k = Nk_{1,x} = Nk_c \cos^2 \propto + Nk_f \sin^2 \propto. \tag{2}$$

If all the isolators are mounted to compression, ie α=0, results:

$$k_1 = k_{max} = Nk_c. \tag{3}$$

In this case, vibration transmissibility will be determined for the two borderline cases, so that intermediate values can be evaluated for an anti-vibration isolation techniques solution.

3. Analysis of the dynamic response and anti-vibration isolating capacity

Dynamic harmonic $F(t) = F = F_0 \sin \omega t$ is applied in point 1 to the mass m_1, and the flow of forced vibrations is transmitted through the spring k and the viscous damper c to the receiving point for 2 on mass m_2.

For the unbound system from figure 1, the differential equations of motion have the form [2]:

$$\begin{cases} m_1\ddot{x}_1 + c(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_2) = F \\ m_2\ddot{x}_2 - c(\dot{x}_1 - \dot{x}_2) - k(x_1 - x_2) = 0 \end{cases} \tag{4}$$

We divide the first relation with m_1 and second with m_2, subtract member by member and obtain differential equation in the relative y-coordinate or or instantaneous deformation:

$$\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m}y = \frac{F}{m_1} \tag{5}$$

We will use notations:

$y = x_1 - x_2 = A\sin(\omega t - \varphi),$ where A is the amplitude of the instantaneous deformation;

$\Omega = \dfrac{\omega}{p}$ – relative pulsation;

$\dfrac{c}{m} = 2\zeta p$ – for the given system;

$\dfrac{c\omega}{k} = 2\zeta\Omega;$

$p^2 = \dfrac{k}{m} = \dfrac{k(m_1 + m_2)}{m_1 m_2}$ – quadratic own pulsation;

$\mu = \dfrac{m_1}{m_2}$ – mass ratio;

$H_0 = \dfrac{F_0}{1+\mu}$ – amplitude of reduced disruptive force;

$H = H_0 \sin \omega t$ – reduced disruptive force;

ω –phase difference between the force $F = F(t)$ and instantaneous deformation $y = y(t)$.

The system response is given by expression of relative amplitude A as [3]:

$$A = \frac{H_0}{k} \cdot \frac{1}{\sqrt{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \tag{6}$$

$$\tan\varphi = \frac{2\zeta\Omega}{1-\Omega^2}. \tag{7}$$

The force transmitted through the spring and damper is of the form [4]:

$$F_T = ky + c\dot{y}, \tag{8}$$

where

$$\dot{y} = A\omega\cos(\omega t - \varphi).$$

So, we have

$$F_T = F_T(t) = kA\sin(\omega t - \varphi) + cA\omega\cos(\omega t - \varphi),$$

but $F(t) = F_T^{\max}\sin(\omega t + \theta)$,

where $F_T^{\max}$ and θ is determined from the condition of existence of trigonometric identities of the form [5]:

$$F_t^{\max}\sin(\omega t + \theta) \equiv kA\sin(\omega t - \varphi) + cA\omega\cos(\omega t - \varphi),$$

So we have

$$F_t^{max} sin\omega t cos\theta + F_T^{max} sin\theta cos\omega t = kA sin\omega t cos\varphi - kA sin cos\omega t + cA\omega cos\omega t cos\varphi + cA\omega sin\theta sin\varphi$$

By identification we obtain:

$$\begin{cases} F_T^{\max}\cos\theta = kA\cos\varphi + c\omega A\sin\varphi \\ F_T^{\max}\sin\theta = -kA\sin\varphi + c\omega A\cos\varphi, \end{cases} \tag{9}$$

So results:

$$F_T^{\max} = A\sqrt{k^2 + c^2\omega^2} = Ak\sqrt{1 + (2\zeta\Omega)^2} \tag{10}$$

$$\tan\theta = \frac{-k\tan\varphi + c\omega}{k + c\omega\tan\varphi} = \frac{2\zeta\Omega^3}{1-\Omega^2 + (2\zeta\Omega)^2}\,. \qquad (11)$$

The maximum transmitted force can be achieved under final form replacing *A* from (6) in (10):

$$F_T^{\max} = H_0\sqrt{\frac{1+(2\zeta\Omega)^2}{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \qquad (12)$$

and $F_T(t) = F_T^{\max}\sin(t-\theta)$,
where θ is phase deference between forces $F = F(t)$ and $F_T = F_T(t)$ and

$f(\Omega, \varsigma) = \sqrt{\dfrac{1+(2\zeta\Omega)^2}{(1-\Omega^2)^2 + (2\zeta\Omega)^2}}$ is amplitude of transmitted force.

Absolute transmissibility for dynamic excitation $T_{abs}^F = T_{12}^F$ is

$$T_{12}^F = T_{abs}^F = \frac{F_T^{\max}}{F_0} = \frac{H_0}{F_0} f(\Omega, \zeta),$$

$$\text{or } T_{12}^F = \frac{1}{1+\mu}\sqrt{\frac{1+(2\zeta\Omega)^2}{(1-\Omega^2)^2 + (2\zeta\Omega)^2}}\,. \qquad (13)$$

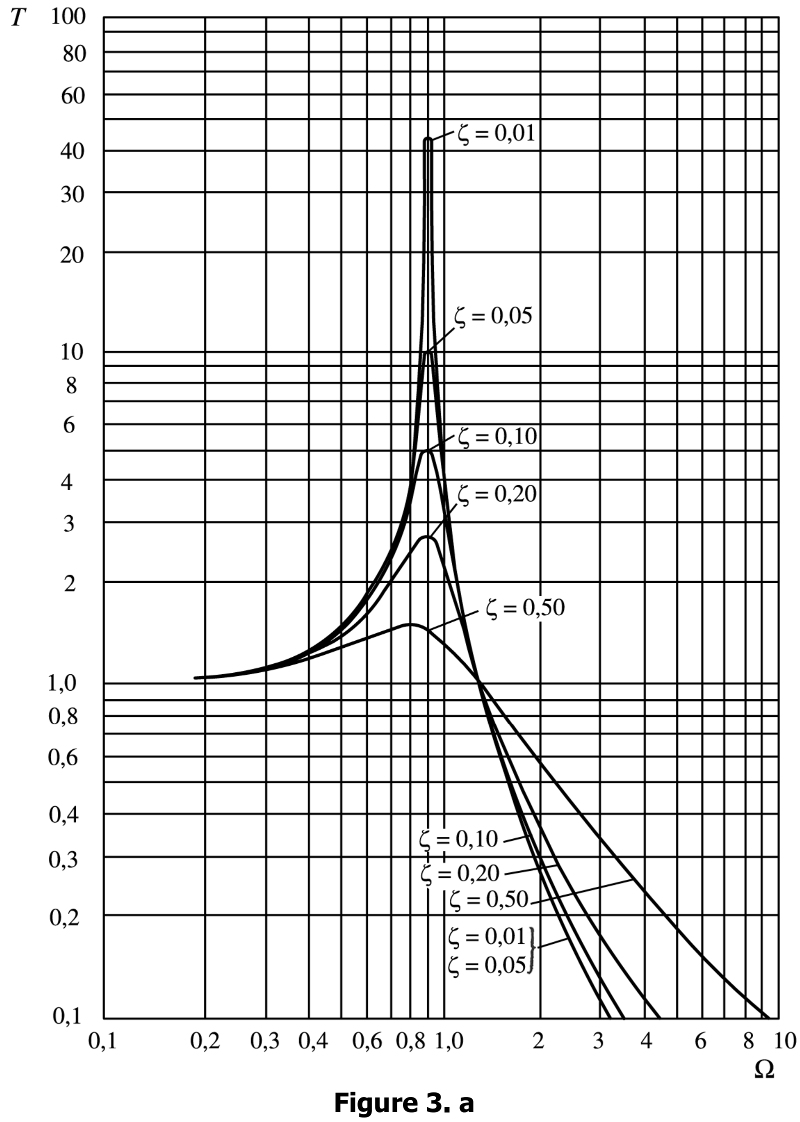
For case $m_2 \to \infty$, ie mass m_2 face is part of foundation, the system is connected to a fixed part, we have μ = 1 so T_{abs}^F became $T_{abs}^F = T_{10}^F(\Omega, \zeta) = T$, ie we have

$$T = \sqrt{\frac{1+(2\zeta\Omega)^2}{(1-\Omega^2)^2 + (2\zeta\Omega)^2}} \qquad (14)$$

and the phase difference θ between $F(t)$ and $F_T(t)$ is

$$\theta = \arctan\frac{2\zeta\Omega^2}{1-\Omega^2 + 4\zeta^2\Omega^2}\,, \qquad (15)$$

which are represented in figure 3, a and 3 b.


T
100
80
60
40
30
20
10
8
6
4
3
2
1,0
0,8
0,6
0,4
0,3
0,2
0,1
ζ = 0,01
ζ = 0,05
ζ = 0,10
ζ = 0,20
ζ = 0,50
ζ = 0,10
ζ = 0,20
ζ = 0,50
ζ = 0,01
ζ = 0,05
0,1
0,2
0,3
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1,0
2
3
4
6
8
10
Ω

Figure 3. a

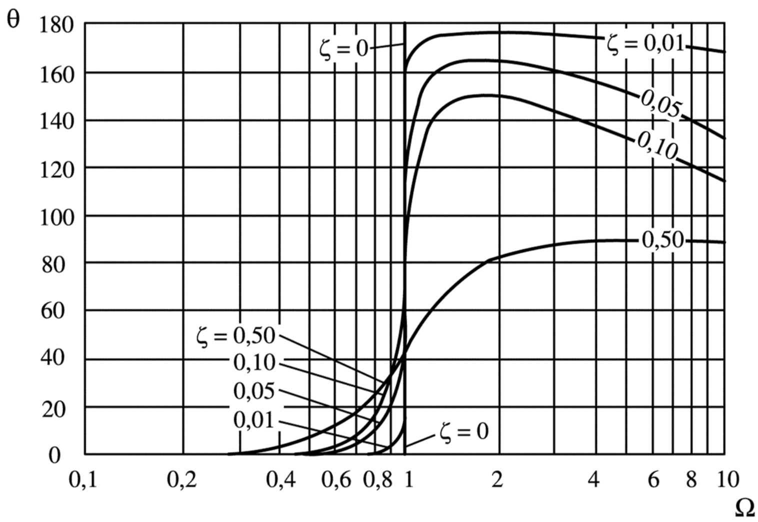

Figure 3. b

Izolation degree of vibration is given by:

$$I = (1 - T)\,100 \quad [\%]. \tag{16}$$

Variation of parameters T and I is given by the pulsations ratio Ω and parameter ζ, in figure 4.

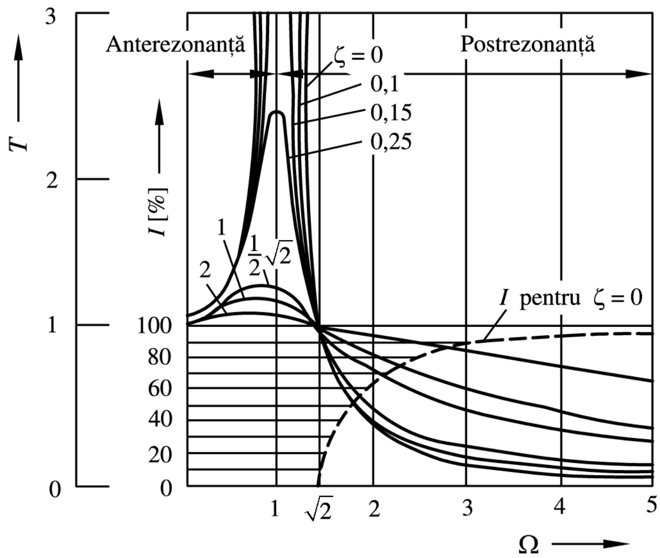

Figure 4. Transmissibility T variation and the isolation degree I depending on pulsations ratio Ω=ω/p and fraction of critical damping ζ

4. Conclusions

The paper presents the case of a viscoelastic isolation system formed by an even number of elastomeric elements positioned at an angle with respect to the direction of action. Thus, in essence, the following are observed:

a) The equivalent stiffness coefficient on the direction of action;

b) The influence of the compression and shear forces on the behavior of the system, on the principal direction;

c) The determination of the kinematic and dynamic parameters of the isolation system at exterior harmonic dynamic actions;

d) The transmissibility curves are represented by adimentional parameters and take into consideration the inclination of the elastomeric elements.

References

[1] Bratu P., *Analiza structurilor elastice*, Editura Impuls, 2011.

[2] Bratu P., *Statica şi dinamica structurilor elastice*. Universitatea Dunărea de Jos, Galaţi, 1996.

[3] Bratu P., *Analyze insulator rubber elements subjected to actual dynamic regime.* Proc. of the 9th ICSU, University Orlando, Florida, USA, 2002.

[4] Clough R.W., Penzien J., *Dynamics of structures.* McGraw Hill, 1975.

[5] Bratu P., *Elastical characteristics of antivibrating rubber elements.* Symposion „Dynamics of machine foundation", Bucharest, Romania Academy, 1985.

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