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Frequency Changes in Thin Rectangular Plates due to Geometrical Discontinuities. Part II: Frequency Shift Interpretation

This paper, as part of a series dedicated to highlight the way how natural frequencies of plates change due to damage, presents results of numerical and experimental modal analysis made on thin rectangular plates clamped at all four lateral surfaces. These results where processed and the so-called relative frequency shift for thirty vibration modes determined. It was revealed the rules how the frequencies change by varying the damage geometry, and proved that these data ca be used for damage identification. Finally, the results are compared with measurements, in order to demonstrate the validity of the theoretical considerations.

***Keywords**: defect, plate, natural frequency, vibration mode shapes, experimental analysis*

1. Introduction

Modal testing is the process of experimentally determining all the modal parameters. There are two types of testing – Classical and Operational Modal Analysis. In Classical Modal Analysis the structure under test is stimulated with measurable forces. The vibration response/force ratio is studied using FFT techniques. It ranges from a simple mobility test with an impact hammer to multi-shaker testing of large and complex structures with hundreds of response accelerometers.

In the paper, finite element analysis results obtained by means of SolidWorks simulations are processed and interpreted. The specimens for numerical and experimental analysis were thin rectangular plates, without defect and with a central defect of 5x50 mm, 10x50 mm, 20x50 mm, 30x50 mm, 40x50 mm and 50x50 mm considered one by one. Irrespective to its dimensions, the damage was placed always symmetrical to the plates ends.

The defective plates used for real-scale experiments are presented in figure 1. While defects change the natural frequencies of beams or plates, see [1] to [4], we conducted experiments to find out the sensitivity of frequency change due defect for the above presented cases.

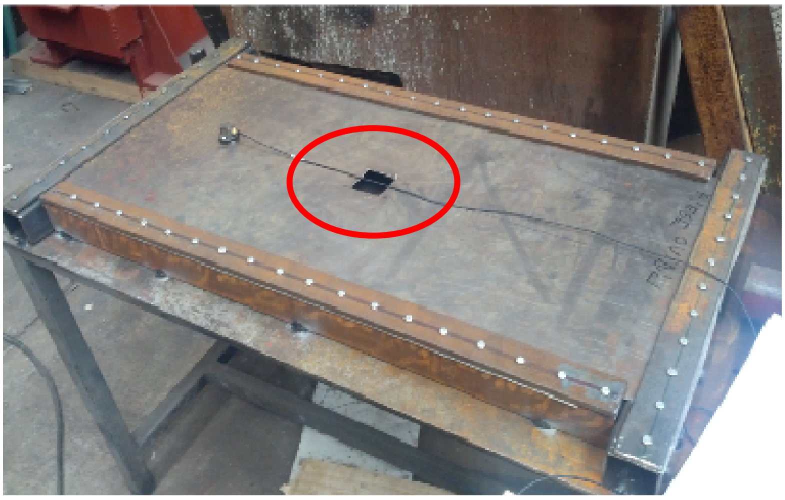

Figure 1. Experimental stand for modal analysis experiments and the plate with central damage of 50x50 mm.

In this work we determined of the natural frequency for a number of 30 vibration modes in the numerical analysis and 15 vibration modes in the experimental analysis.

2. Interpretation of simulations results

After the numerical simulation with FEA, for the natural frequencies of the first 30 vibration modes, we obtained results that are presented in Table 1, the frequencies being grouped after the *m* index of the modes. To determine the relative frequency shift Δf_i^* of mode *i*, a relation contrived by our research group is used, see [5] to [7]. This relation can be written in the form presented below:

$$\Delta f_i^* = \frac{\Delta f_i}{f_i^H} = \frac{f_i^H - f_i^D}{f_i^H} \tag{1}$$

In the above relation, f_i^H and f_i^D are the natural frequencies of mode *i* for the healthy plate and damaged plate case respectively. To express the relative frequency shift in percents, relation (1) wave to be multiplied by 100.

Table 1.

Mode m - n	Calculated frequency values for the seven plate types grouped into m-n						
	Without defect	With central defect					
		5x50 [mm]	10x50 [mm]	20x50 [mm]	30x50 [mm]	40x50 [mm]	50x50 [mm]
1-1	56.494	56.422	56.427	56.416	56.387	56.343	56.281
1-2	71.611	71.596	71.596	71.589	71.582	71.575	71.567
1-3	98.771	98.203	98.181	98.151	98.139	98.137	98.139
1-4	137.96	137.9	137.89	137.84	137.78	137.71	137.62
1-5	188.64	188.5	188.49	188.44	188.36	188.23	188.03
1-6	250.42	250.26	250.19	250	249.76	249.47	249.16
1-7	323	319.97	320.01	320.3	320.78	321.31	321.81
1-8	406.41	405.99	405.8	405.28	404.64	403.95	403.57
1-9	500.39	496.14	496.27	496.9	497.74	498.65	499.49
2-1	147.91	147.88	147.88	147.82	147.73	147.59	147.4
2-2	162.85	162.78	162.74	162.63	162.52	162.42	162.32
2-3	188.53	187.15	187.11	187.11	187.2	187.34	187.5
2-4	225.46	225.31	225.2	224.94	224.67	224.42	224.17
2-5	273.81	273.77	273.77	273.72	273.62	273.47	273.22
2-6	333.5	333.3	333.15	332.79	332.43	332.1	331.79
2-7	404.36	404.31	404.31	404.24	404.12	403.92	403.25
2-8	486.28	486.05	485.85	485.42	485.05	484.75	484.54
2-9	578.99	578.91	578.9	578.81	578.63	578.29	577.69
3-1	285.31	284.95	284.85	284.56	284.24	283.89	283.51
3-2	300.32	300.24	300.25	300.22	300.2	300.18	300.16
3-3	325.76	325.56	325.47	325.24	325.02	324.85	324.76
3-4	361.82	361.63	361.6	361.5	361.39	361.3	361.19
3-5	408.94	407.89	407.96	408.11	408.34	408.61	408.9
3-6	467.16	466.75	466.66	466.41	466.15	465.45	463.53
3-7	537.36	536.64	536.52	535.96	535.38	534.94	534.77
4-1	468.32	468.17	468.05	467.56	466.72	465.92	465.7
4-2	483.4	483.3	483.3	483.2	483.06	482.9	482.66
4-3	508.81	508.65	508.55	508.11	507.39	506.37	505
4-4	544.68	544.47	544.35	544.04	543.72	543.42	543.09
4-5	591.27	591.11	591.04	590.65	590.01	589.13	587.97

The relative frequency shifts are calculated in respect to relation (1); the results, grouped after the value of the *m* component describing the vibration mode, are presented in table 2.

Table 2.

Mode m - n	Relative frequency shifts for the six defect scenarios [%]					
	Central defect 5x50 [mm]	Central defect 10x50 [mm]	Central defect 20x50 [mm]	Central defect 30x50 [mm]	Central defect 40x50 [mm]	Central defect 50x50 [mm]
1-1	0.127	0.119	0.138	0.189	0.267	0.377
1-2	0.021	0.021	0.031	0.040	0.050	0.061
1-3	0.575	0.597	0.628	0.640	0.642	0.640
1-4	0.043	0.051	0.087	0.130	0.181	0.246
1-5	0.074	0.080	0.106	0.148	0.217	0.323
1-6	0.064	0.092	0.168	0.264	0.379	0.503
1-7	0.938	0.926	0.836	0.687	0.523	0.368
1-8	0.103	0.150	0.278	0.436	0.605	0.699
1-9	0.849	0.823	0.697	0.530	0.348	0.180
2-1	0.020	0.020	0.061	0.122	0.216	0.345
2-2	0.043	0.068	0.135	0.203	0.264	0.325
2-3	0.732	0.753	0.753	0.705	0.631	0.546
2-4	0.067	0.115	0.231	0.350	0.461	0.572
2-5	0.015	0.015	0.033	0.069	0.124	0.215
2-6	0.060	0.105	0.213	0.321	0.420	0.513
2-7	0.012	0.012	0.030	0.059	0.109	0.275
2-8	0.047	0.088	0.177	0.253	0.315	0.358
2-9	0.014	0.016	0.031	0.062	0.121	0.225
3-1	0.126	0.161	0.263	0.375	0.498	0.631
3-2	0.027	0.023	0.033	0.040	0.047	0.053
3-3	0.061	0.089	0.160	0.227	0.279	0.307
3-4	0.053	0.061	0.088	0.119	0.144	0.174
3-5	0.257	0.240	0.203	0.147	0.081	0.010
3-6	0.088	0.107	0.161	0.216	0.366	0.777
3-7	0.134	0.156	0.261	0.368	0.450	0.482
4-1	0.032	0.058	0.162	0.342	0.512	0.559
4-2	0.021	0.021	0.041	0.070	0.103	0.153
4-3	0.031	0.051	0.138	0.279	0.480	0.749
4-4	0.039	0.061	0.118	0.176	0.231	0.292
4-5	0.027	0.039	0.105	0.213	0.362	0.558

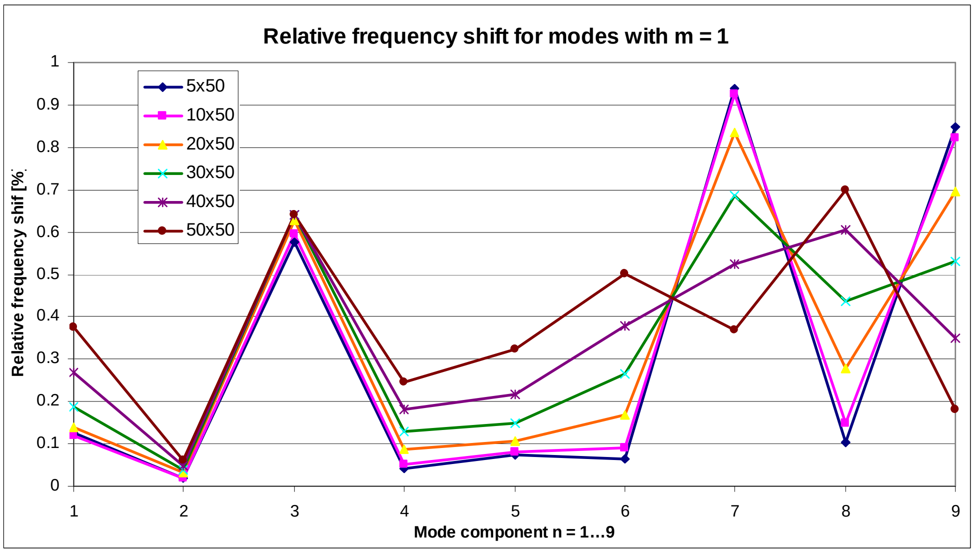

Figure 2. Relative frequency shift for the modes having m = 1 and n = 1…9

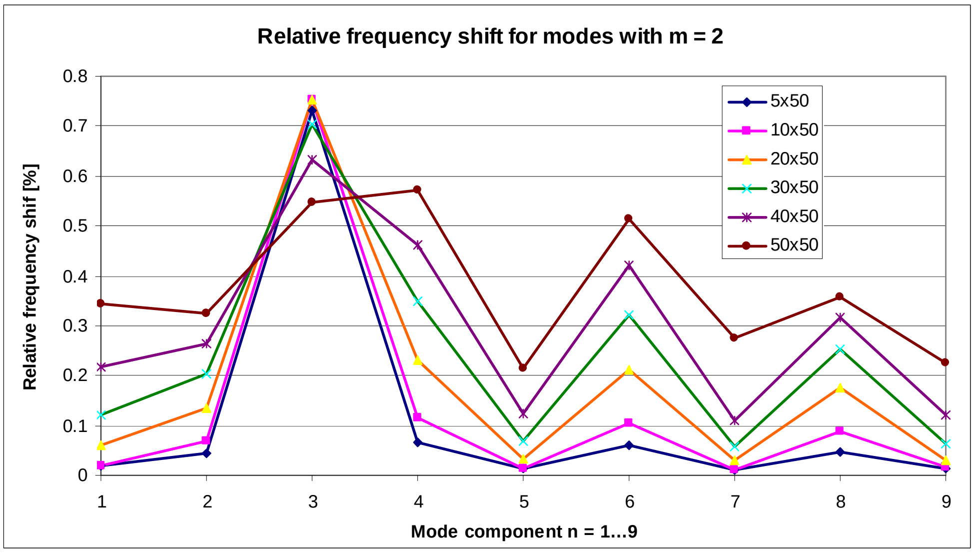

Figure 3. Relative frequency shift for the modes having m = 2 and n = 1…9

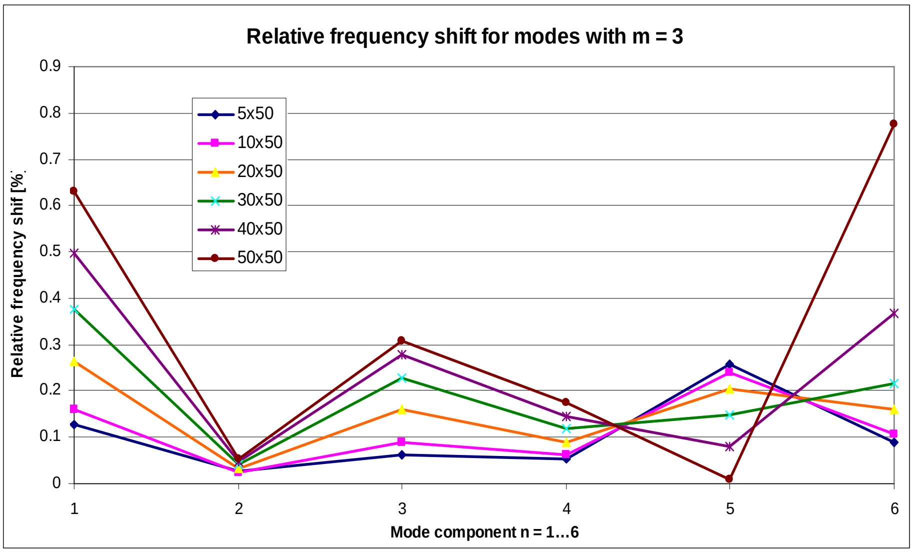

Figure 4. Relative frequency shift for the modes having m = 3 and n = 1…6

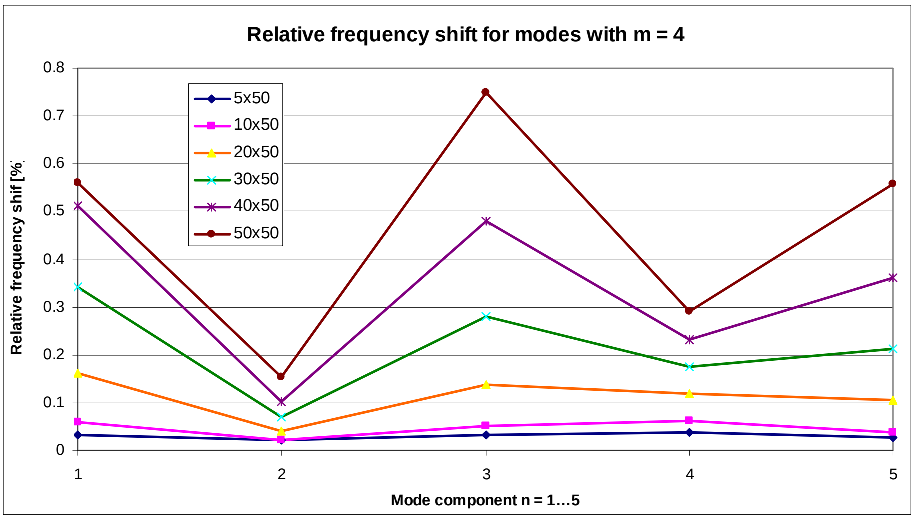

Figure 5. Relative frequency shift for the modes having m = 3 and n = 1…6

It is obvious that for some modes the frequency shifts are insignificant, while for other modes the frequency changes are significant. The shift is produced by the decrease of stored energy for the given mode, dependent on the damage position on the plate [8]:

- for the defect placed on the inflection points of the mode shapes the frequency change is null;

- for the defect placed on points of local extreme of the mode shapes, the frequency change attend highest local values.

If the loss of mass brings an important contribution, this phenomenon is somehow distorted, being known that the loss of mass lead to the increase of frequency. This makes necessary the use of a variable reference value for the frequency of the undamaged plate.

On the other hand, an increased missing area on one location on the plate, can supplement the effect of stiffness reduction, increasing the relative frequency shift of the plate.

A study made for some typical cases (in terms of locations) is presented below. For instance, for vibration mode m = 1 and n = 6, the frequency decrease is continuous and not dramatically affected by loss of mass or change supplementary stiffness decrease. The evolution of the relative frequency shift, for the six considered damage widths, is presented in figure 6.

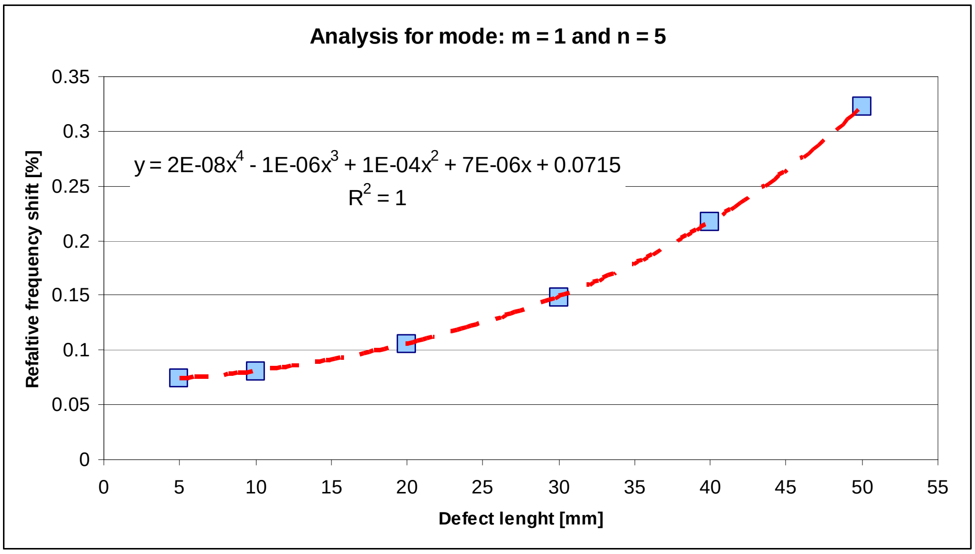

Figure 6. The relative frequency shift for vibration mode m = 1 and n = 5 for the six analyzed damage scenarios

From figure 7, presenting the analysis for vibration mode m = 2 and n = 8, one may remark that the effect of loss of mass increase by increasing the damage width, so that the frequency shift is reduced.

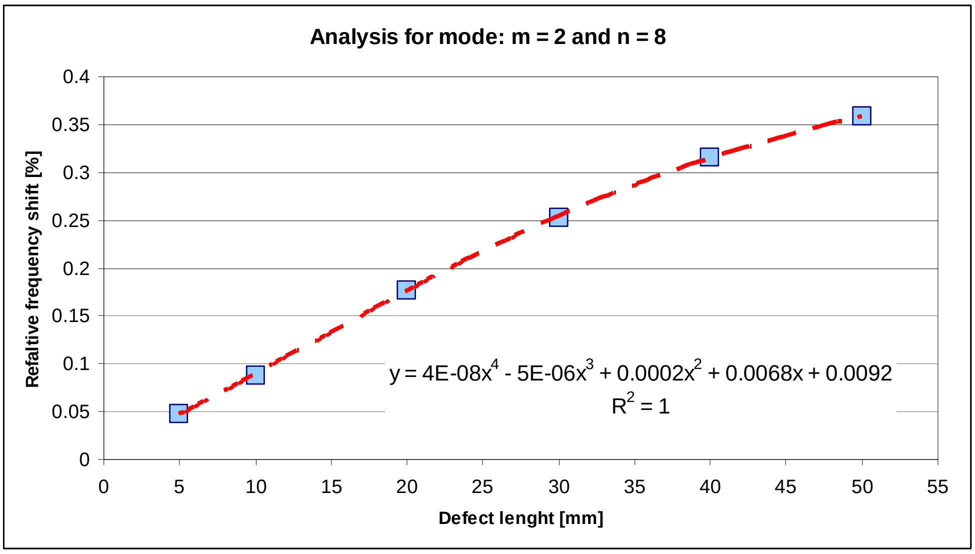

Figure 7. The relative frequency shift for vibration mode m = 2 and n = 8 for the six analyzed damage scenarios

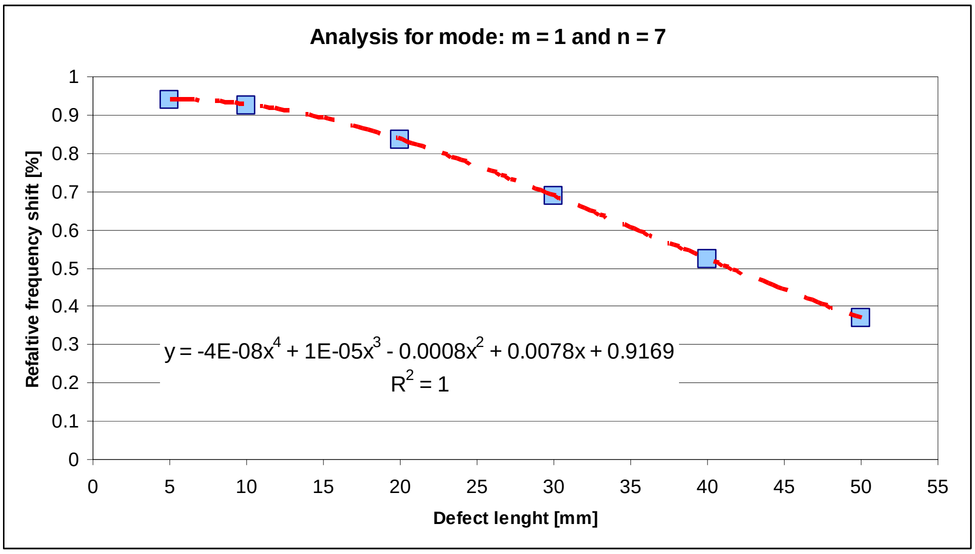

Figure 8. The relative frequency shift for vibration mode m = 1 and n = 7 for the six analyzed damage scenarios

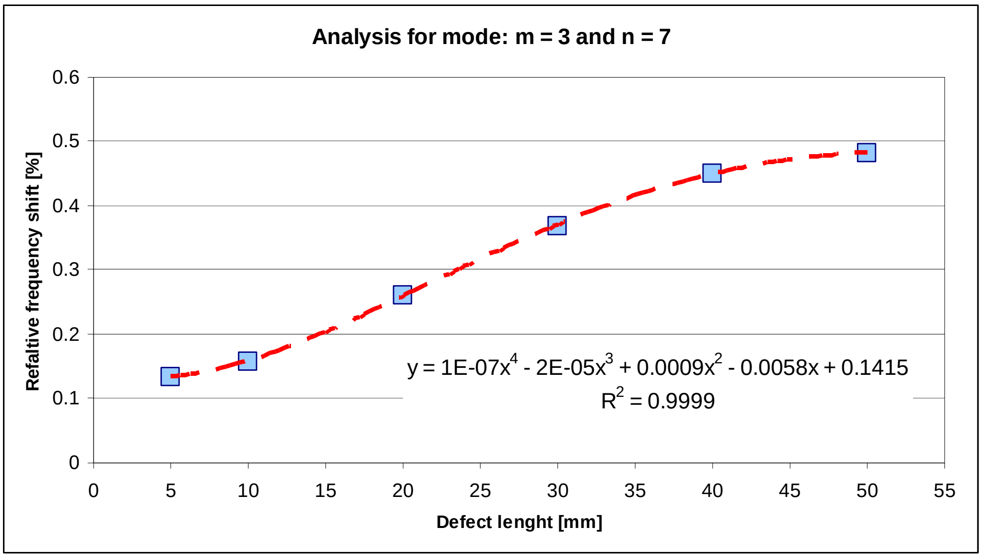

Figure 9. The relative frequency shift for vibration mode m = 3 and n = 7 for the six analyzed damage scenarios

Figure 8 present a case where the effect of loss of mass in much more important then the influence of stiffness reduction. Consequently, the relative frequency shift decrease continuously. The last presented case, figure 9 for vibration mode m = 3 and n = 7, is a mix of the above presented case. For small widths the effect of stiffness reduction is relevant, while for higher values of the damage width the loss of mass is more relevant.

4. Experimental determinations and results

To validate the theoretical and numerical models, experiments were made on plates similar to that presented above. The plates were tested on a stand (see figure 1) which involved instrumental and information systems. The acquisition system is composed by a laptop, a four-channel NI USB-9233 module, and tri-axial accelerometers (figure 10). To acquire and process the data, a Virtual Instrument in graphic language LabVIEW was developed.

After the experimental analysis for determination the natural frequencies of the first 15 vibration modes, we obtained results that are presented in Table 3, in ascending order of obtained frequency.

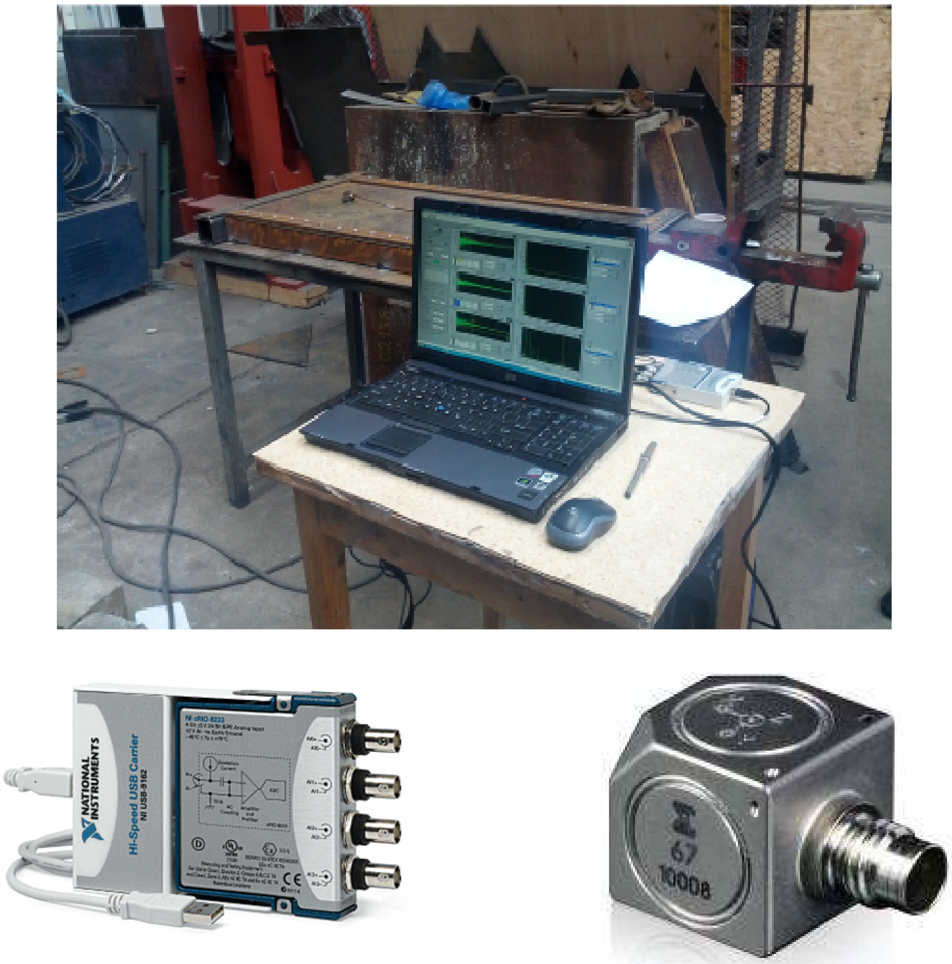

Figure 10. Experimental stand for modal analysis of damaged plate with NI USB-9233 acquisition system and tri-axial accelerometer

By comparing experimental results with those determined numerically, we determine the measurement errors by the relation

$$\varepsilon = \left[\frac{f_{FEM} - f_E}{f_{FEM}} \right] \cdot 100[\%] \tag{2}$$

and the obtained average value of the error $\varepsilon_{AVG} = 2.98\%$, considered quite good for the experimental testing conditions.

Table 3.

Mode	Measured frequency values by experimental stand for the seven considered plates						
	Without defect	With central defect					
		5x50	10x50	20x50	30x50	40x50	50x50
1	56.114	53.122	53.741	52.132	53.255	51.251	55.211
2	69.211	68.11	73.211	69.221	68.523	69.511	70.985
3	96.587	97.32	101.252	97.452	101.2	96.214	95.122
4	136.325	133.542	140	134.23	133.11	132.11	133.412
5	146.895	143.65	143.56	149.533	149.333	144.14	142.362
6	160.693	159.451	158.63	159.123	160.41	159.2	162.32
7	185.211	183.25	184.2	183.963	185.5	187.34	189.12
8	184.472	184.4	185.55	185.653	185.65	190.123	191.541
9	220.124	222.32	221.012	220.45	220.1	219.542	221.114
10	254.987	2547.34	244.52	247.56	244.562	244.526	245.012
11	277.012	269.312	270.22	275.55	270.21	270.147	273.22
12	281.12	281.14	285.311	280.1	283.11	279.621	280.75
13	308.421	302.61	297.87	295.65	302.141	287.2	303.2
14	315.41	315.45	317.11	316.333	315.2	324.21	318.253
15	322.74	321.3	320.512	319.511	321.311	327.1	320.1

5. Conclusion

The shifts of natural frequencies of plates due to damages constitute a reliable indicator, signalizing the appearance of damage and information about the position and severity. Further work will focus on finding mathematical relations to describe the dynamic parameter changes in respect to a given geometric discontinuity.

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