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On the Robustness of the Minkowski Distance for Histogram Dissimilarity Evaluation with Focus on Damage Location in Beam-Like Structural Elements

This paper analyses the robustness of a histogram dissimilarity estimator, namely the Minkowski Distance, and its sensitivity to input/ measurement errors. As a result of prior researches we developed a method to detect, locate and assess damages in beams, based on frequency shifts. The localization process consists in fact in comparing a histogram derived from vibration measurements, with numerous histograms representing the damage signature for all possible locations along the beam. We tested the stability of Minkowski Distance L_2, for frequencies measured on real beams; other tests were made for results artificially debased by noise. In all cases this metrics proved its robustness and reliability.

Keywords: *beam, frequency, damage detection, histogram, Minkowski Distance*

1. Introduction

Detection of damages in structural elements presents great interest for engineering applications. Global dynamic methods are able to assess damages; they quantify the integrity of structures by examining changes in their dynamic response to excitations. Almost all dynamic methods presented in the specialized literature [1, 2] bases on features like: natural frequencies, mode shapes and its derivatives (e.g. mode shape curvatures), stiffness matrices and flexibility matrices. These methods make use of some damage indicators that are sensitive to structural changes; the idea is adjust features of the model to fit its response to that identified by measurement on the damaged structure [3-6], or to compare the features of healthy state with the damaged one by means of recognition techniques [7-10]. The majority of vibration-based methods need measured responses at numerous locations on the structure, which is not always possible due to technical and operational constrains. However, vibration-based methods do not consider the physical phenomenon in deep, being just oriented to fit features or on cost reduction; thus no scientific feedback regarding their reliability is possible.

Previous researches made by our research group conducted to contriving a relation expressing the natural frequency shifts due to damage in respect to the location and severity of the damage. Based on it, we developed a two-step method to assess damages in beams, first being identified the damage position and afterwards its severity. While the localization process consists mainly in comparing histograms representing the normalized relative frequency shift as a damage signature with damage location indexes, derived for any possible location on the structure, dissimilarity estimators have to be utilized. We found the Minkowski Distance as the most reliable of them presented in literature; this paper presents an analysis performed to find the influence of measurement errors upon the stability of this estimator.

2. Damage identification method based on damage location indexes

This original method covers the first tree levels of damage identification defined by Rytter [11], namely detection, localization and evaluation of damage severity. In previous researches [12,14] we derived the exact solution for frequency changes due to damage in beams, for any transversal vibration mode and beam support type. It makes possible to express the frequency shift for the i-th mode of the damaged beam. We denoted the frequency of the undamaged beam $f_{i\text{-}U}$, the frequency of the damaged beam $f_{i\text{-}D}(x,a)$, and $\delta f_i(x,a)$ the relative frequency shift, that is: in that mode and two terms controlling the depth and location of the damage. This relation is:

$$\delta f_i(x,a) = \frac{\Delta f_i(x,a)}{f_{i\_U}} = \frac{f_{i\_U} - f_{i\_D}(x,a)}{f_{i\_U}} \tag{1}$$

where a is depth of the crack placed at distance x from the fixed end.

We also demonstrated that the frequency of the damaged beam is:

$$f_{i\_D}(x,a) = f_{i\_U}\left[1 - \gamma(x_{B\max},a)\cdot(\bar{\phi}_i''(x))^2\right] \tag{2}$$

where:

- $\gamma(x_{\mathrm{Bmax}},a)$ is a term representing the stiffness reduction calculated on the location where the bending moment attend maxima (for the cantilever beam at the fixed end x_{Bmax} = 0)
- $\bar{\phi}_i''(x)$ is the normalized mode shape curvature, taking values between 0 and 1.

From relation (1) and (2) results:

$$\delta f_i(x,a) = \gamma(0,a)\cdot(\bar{\phi}_i''(x))^2 \tag{3}$$

Imagine now that we obtain the *relative frequency shift* by processing data from measurements for *n* vibration modes. This series of *n* values determined with the left term of relation (3) can be normalized by dividing them to the maximum value of the series, obtaining:

$$\Psi_1 = \frac{\delta f_1}{\max(\delta f_i\ )}, \ldots, \Psi_n = \frac{\delta f_n}{\max(\delta f_i\ )} \tag{4}$$

On the other hand, for any location x_j on the beam we can derive the values of the relative frequency shift for *n* bending vibration modes. Normalizing the values obtained with the right term, by dividing these one by one to the highest value of the series, we obtain the *damage location coefficients*, as:

$$\Phi_i(x_j) = \frac{(\bar{\phi}_1''(x_j))^2}{\max\left\{(\bar{\phi}_i''(x_j))^2\right\}}, i=1..n,\ j=1..k \tag{5}$$

One observe that the damage location coefficients depend only on the mode shape curvature squares $(\bar{\phi}_i''(x))^2$, as the term $\gamma(0,a)$ is independent of location x and thus eliminated by normalization. A series of damage location coefficients specific for a damage location are called damage location index.

To find the damage location and severity, the frequencies of the structure must be measured in-situ, periodically or continuously. By occurrence of significant differences between two consecutive measurements, the appearance of damage is presumed. In this case relative frequency shifts are determined and compared with patterns analytically determined using the mode shape curvatures: both series of values being normalized to have the greatest value equal to 1. By finding the pattern best matching the relative frequency shifts determined by measurement, the location of damage is identified. It follows the evaluation of damage depth, by determining the value of the term $\gamma(0,a)$.

The process implies the performance of following steps [15]:

A. Damage detection

1. The first ten natural frequencies of the weak-axes bending vibration modes for the undamaged beam have to be determined. It results the series

A:{ f_{1_U} ; f_{2_U} ; f_{3_U} ; f_{4_U} ; f_{5_U} ; f_{6_U} ;… f_{n_U} }

In case of older structures, the actual status of the beam can be considered as start point, neglecting the possible existing cracks. Thus, only the evolution of new or developing cracks can be assessed.

2. For the same vibration modes the frequencies have to be measured periodically. For each chosen moment, a series

$$S:\{ f_{1\_D} ;\ f_{2\_D} ; f_{3\_D} ; f_{4\_D} ; f_{5\_D} ; f_{6\_D} ;\dots f_{n\_D} \}$$

is obtained.

3. Comparison with the initial estate has to be performed. The series

$$D:\{ \Delta f_1 ;\ \Delta f_2 ; \Delta f_3 ; \Delta f_4 ; \Delta f_5 ; \Delta f_6 ;\dots \Delta f_n \}$$

ensue. Differences representing frequency shifts Δf_i indicate damage appearance.

B. Damage localization

4. The relative frequency shift is determined by dividing the values of D series to the one of A (see relation (1)). Results the series R

$$R:\{ \delta f_1 ;\ \delta f_2 ; \delta f_3 ; \delta f_4 ; \delta f_5 ; \delta f_6 ;\dots \delta f_n \}$$

5. The values of the R series have to be normalized, by dividing all values of the series by the highest one. It results

$$\Psi=\{\Psi_1 ; \Psi_2 ; \Psi_3 ; \Psi_4 ; \Psi_5 ; \Psi_6 ;\dots\Psi_n\}$$

The series' elements take positive subunit values. Normally, only one element takes the unit value.

6. Series representing square of the mode shape curvature $\left(\overline{\phi_i''}(x)\right)^2$ for various locations on the beam are determined, resulting

$$C:\{ \left(\overline{\phi_1''}(x)\right)^2 ; \left(\overline{\phi_2''}(x)\right)^2 ; \left(\overline{\phi_3''}(x)\right)^2 ; \left(\overline{\phi_4''}(x)\right)^2 ; \left(\overline{\phi_5''}(x)\right)^2 ; \left(\overline{\phi_6''}(x)\right)^2 ;\ \dots ; \left(\overline{\phi_{10}''}(x)\right)^2 \}$$

7. The values of the C series have to be normalized, by dividing all values of the series by the highest one (see relation (5)). It results:

$$\Phi=\{\Phi_1 ; \Phi_2 ; \Phi_3 ; \Phi_4 ; \Phi_5 ; \Phi_6 ;\dots\Phi_i\}$$

8. Resulted Ψ series is compared with the determined Φ series by means of dissimilarity estimators. The x coordinate of the mode shape curvature which provides the best fit to frequency change series, indicates the damage location.

C. Severity evaluation

9. Ratio between values of N and C series has to be determined to obtain the value of coefficient $\gamma(0, a)$ presented in relation (3).

10. Damage depth a can be now extracted as the single unknown in this relation.

11. Two values $a_{\inf}$ and $a_{\sup}$ around damage depth are picked. For those values, considering the x coordinate, the relative frequency shift chart is plotted, together with the one of the measurements.

12. It is searched if the measured frequency shifts are well framed by the analytically determined ones.

In case of satisfactory results, a fine tuning can be made, by relocating coordinate x around already found location. If unsatisfactory results, a new coordinate x is searched again and steps 8 to 12 has to be reload.

3. Recognition of damage location based on the Minkowski metrics

Having the damage location coefficients for numerous locations along the beam (we work wit a step between 0.5% and 1%), and the measurement result, the localization problem becomes a pattern recognition problem. In fact $\Psi=\{\Psi_i\}$ and $\Phi_j(x_j)=\{\Phi_{ij}\}$ are histograms, i.e. representations of non-negative data corresponding to n bins ($i = 1 \ldots n$). Among numerous measures proposed for the dissimilarity between two histograms $\Psi=\{\Psi_i\}$ and $\Phi=\{\Phi_i\}$ we found the Minkowski Distance as the most appropriate for our applications. It is a bin-by-bin dissimilarity measure, only comparing contents of corresponding histogram bins, i.e. they compare Ψ_i and Φ_i for all i, but not Ψ_i and Φ_k for $i \neq k$. The dissimilarity between the two histograms is a combination of all the pair-wise comparisons [16]. The Minkowski Distance L_r is given by:

$$d_{L_r}(\Psi,\Phi) = \left(\sum_i \left| \Psi_i - \Phi_i \right|^r \right)^{\frac{1}{r}} \tag{6}$$

where $r \geq 1$. The Minkowski distances for $r = 1$ is the so-called Manhattan distance, while the Euclidian distance stay for $r = 2$. In our study the bins are represented by vibration modes and the content represents the normalized relative frequency shift and damage location index respectively. In fact the continuity condition is not reached, so that the term "histogram" is not proper, rather the term "bar diagram" should be used. However, the estimator is appropriate to compare the series $\Psi=\{\Psi_i\}$ and $\Phi_j(x_j)=\{\Phi_{ij}\}$.

To prove the robustness of the Minkowski distance, experiments were done on cantilever steel beams having the length L = 1400 mm and a quadratic cross-section of dimension B = 12 mm. The steel physical/mechanical characteristics are: ρ = 7850 kg/mm^3, the Young's modulus $E = 2 \cdot 10^{11}$ N/m^2 and the Poisson's ratio υ = 0.3. For the damaged case, saw cuts of width w = 2 mm were produced. The experimental stand with the analyzed specimen is presented in figure 1.

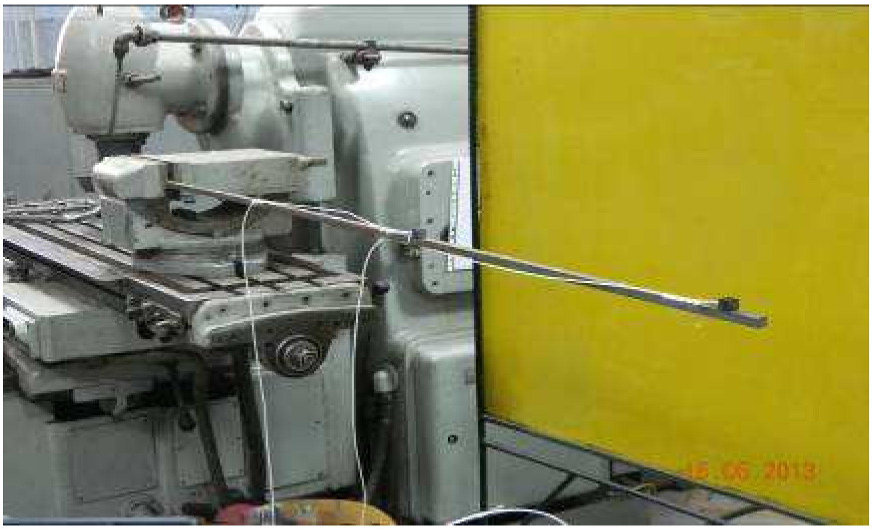

Figure 1. Experimental stand and specimen B1.7

The measurement results are presented in table 1. The natural frequencies of the damaged beam are measured for cracks located at x/L = 0.25, x/L = 0.35 and x/L = 0.55 respectively.

Table 1. Natural frequencies of the beam and the resulting relative frequency shifts

Mode i	1	2	3	4	5	6	7	8
$f_{i\text{-}U}$	4.6888	29.6013	83.6565	165.2375	275.8033	411.0190	573.2848	747.5973
$f_{i\text{-}D0.25}$	4.4890	29.5477	80.8725	158.9544	274.0221	407.9146	548.9728	716.0426
$f_{i\text{-}D0.35}$	4.5596	28.9477	80.4641	165.2037	265.6707	396.0327	573.1114	715.7496
$f_{i\text{-}D0.55}$	4.6577	28.3402	83.0678	160.2967	270.9955	403.8232	556.0161	743.0494
$\delta f_{i\text{-}0.25}$	0.0426	0.0018	0.0333	0.0380	0.0065	0.0076	0.0424	0.0422
$\delta f_{i\text{-}0.35}$	0.0276	0.0221	0.0382	0.0002	0.0367	0.0365	0.0003	0.0426
$\delta f_{i\text{-}0.55}$	0.0066	0.0426	0.0070	0.0299	0.0174	0.0175	0.0301	0.0061
$\Psi_{i\text{-}0.25}$	1.0000	0.0425	0.7812	0.8926	0.1516	0.1773	0.9955	0.9908
$\Psi_{i\text{-}0.35}$	0.6468	0.5183	0.8958	0.0048	0.8624	0.8559	0.0071	1.0000
$\Psi_{i\text{-}0.55}$	0.1557	1.0000	0.1652	0.7019	0.4092	0.4110	0.7071	0.1428

The relative frequency shifts of all measured data for the above analyzed cases are calculated with relation (1); to find the damage location index $\Phi(x_j)=\{\Phi_i\}$ best fitting to the normalized relative frequency shifts $\Psi=\{\Psi_i\}$ obtained from measurements, the d_{L2} is applied. Figure 2 present the Minkowski distances for the three considered damages.

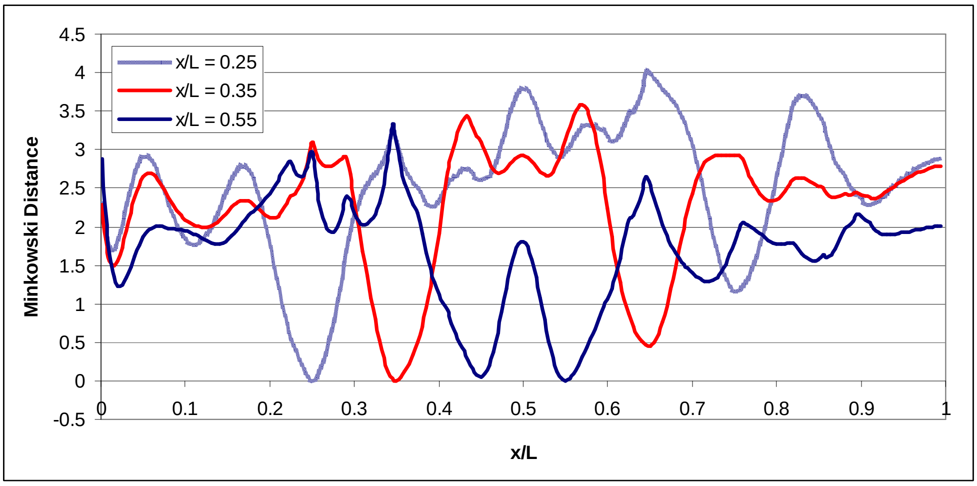

Figure 2. Dissimilarity chart for Minkowski Distance d_{L2} for damages placed at distances x/L = 0.25, x/L = 0.35 and x/L = 0.55 from the clamped end

In a second step, the measurement results for the damage located at the dimensionless distance x/L = 0.25 from the clamped end have been altered artificially, by considering three cases of contaminated measurements. As shown in Table 2, in the first case only 2 frequencies are affected by noise (case C1), in the second and third cases all frequencies are affected differently by noise. The second case (C2) is affected by errors in the positive and negative domain as well, while in the third case (C3) only negative errors occur.

Table 2. Errors affecting the measured frequencies

Mode i	$Err\ f_{i\text{-}D}$ (C1)	$Err\ f_{i\text{-}D}$ (C2)	$Err\ f_{i\text{-}D}$ (C3)
1	9.51%	-9.53%	-1.93%
2	0	3.34%	-6.9%
3	7.27%	-1.37%	-3.1%
4	0	5.82%	-4.55%
5	0	2.35%	-7.07%
6	0	-6.16%	-2.55%
7	0	5.95%	-5.97%
8	0	1.17%	-3.2%

One observes from figure 2 that this method permits obtaining univocally results, always the minimum value of the Minkowski distance is obtained for the location where the damage is placed. Thus, for some particular locations there are very low values for d_{L2} in other locations too (e.g. x/L = 0.55 in figure 2).

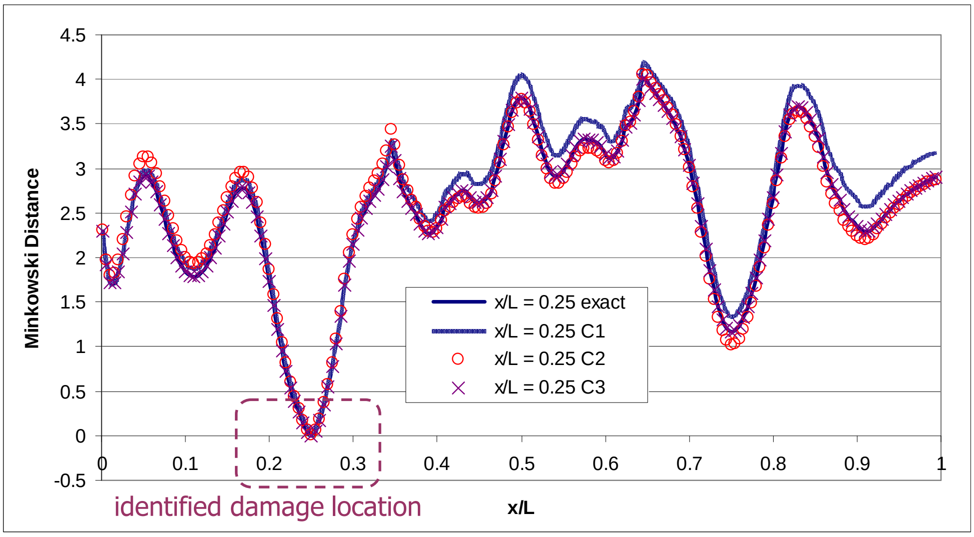

Figure 3. Dissimilarity chart for the Minkowski Distance d_{L2} for the damage placed at distances x/L = 0.25 from the clamped end, using exact end contaminated measurement results

Regarding the localization obtained made with strongly contaminated measurement results, the d_{L2} curves are quite similar, which proves the method's robustness, which qualify it for involvement in damage detection applications even for industrial use.

4. Conclusion

This paper analyzes the robustness of the Minkowski distance, for the particular case r = 2. It is used to compare histograms elaborated in the process of structural health monitoring, aiming to detect the occurrence of damage and to locate it. The researcher revealed that the Minkowski distance is stabile and permit accurate assessment of damages even for some errors affecting the measurement results.

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