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Reliability Study Regarding the Use of Histogram Similarity Methods for Damage Detection

The paper analyses the reliability of three dissimilarity estimators to compare histograms, as support for a frequency-based damage detection method, able to identify structural changes in beam-like structures. First a brief presentation of the own developed damage detection method is made, with focus on damage localization. It consists actually in comparing a histogram derived from measurement results, with a large series of histograms, namely the damage location indexes for all locations along the beam, obtained by calculus. We tested some dissimilarity estimators like the Minkowski-form Distances, the Kullback-Leibler Divergence and the Histogram Intersection and found the Minkowski Distance as the method providing best results. It was tested for numerous locations, using real measurement results and with results artificially debased by noise, proving its reliability.

Keywords*: beam, frequency, damage detection, histogram, Minkowski Distance, Kullback-Leibler Divergence, Histogram Intersection*

1. Introduction

Structural health monitoring of structures present great interest for engineering practitioners. By using global dynamic methods to assess damages, they quantify the integrity of structures by examining changes in their dynamic response to excitations. The majority of dynamic methods presented in literature [1, 2] bases on parameters like: the natural frequencies, the mode shapes, the mode shape curvatures, flexibility matrices and stiffness matrices. The main idea of this type of methods is to find some damage indicators that are sensitive to structural changes and to use these data either to compare the features of healthy state with the damaged one by means of recognition techniques [3-6], or to change features of the model to fit its response to that identified by measurement on the damaged structure [7-10].

Almost all vibration based methods require measuring responses at several locations on the structure, not always possible due to operational and technical constrains. However, these methods dose not consider the physical phenomenon in deep, being just oriented to fitting features or cost reduction; consequently no scientific feedback regarding their reliability is possible.

Our prior researches lead to a mathematical relation expressing the frequency changes due to damage in respect to the position and severity of that damage. This finding was used to develop a two-step method to assess damages in beams, first being found the position and afterwards the severity of damage. This paper presents an analysis of some dissimilarity estimators involved in automatic damage localization, from which the Minkowski metrics was find the most appropriate.

2. Method to detect damage locations based on frequency shifts

In previous researches [11,12] we derived the exact solution for frequency changes due to damage in beam-like structures, for any transversal vibration mode and beam support type. It makes possible to express the frequency for mode i of the damaged beam with a crack of depth a placed at distance x from one end, denoted $f_{i\_D}(x,a)$, considering the frequency of the undamaged beam f_{i_U} in that mode and two terms controlling the depth and location of the damage. This relation is:

$$f_{i\_D}(x,a) = f_{i\_U}\left[1-\gamma(x_{B\max},a)\cdot(\bar{\phi}_i''(x))^2\right] \tag{1}$$

where $\gamma(x_{\mathrm{Bmax}},a)$ is the term representing the stiffness reduction calculated on the location where the bending moment attend maxima (for the cantilever beam at the fixed end x_{Bmax} = 0) and $\bar{\phi}_i''(x)$ the normalized mode shape curvature, weighting the influence of stiffness reduction according to the damage position and mode shape. Obviously, this last squared tem takes values between 0 and 1. From relation (1) one can derive the **relative frequency shift** as:

$$\delta f_i(x,a) = \frac{\Delta f_i(x,a)}{f_{i\_U}} = \frac{f_{i\_U} - f_{i\_D}(x,a)}{f_{i\_U}} \quad \text{or:} \quad \delta f_i(x,a) = \gamma(0,a)\cdot(\bar{\phi}_i''(x))^2 \tag{2}$$

For any location x_j on the beam we can derive the values of the relative frequency shift for n bending vibration modes. Normalizing the values obtained with the right term, by dividing these one by one to the highest value of the series, we obtain the **damage location coefficients** DLC, as:

$$\Phi_i(x_j) = \frac{(\bar{\phi}_i''(x_j))^2}{\max\left\{(\bar{\phi}_i''(x_j))^2\right\}}, i=1..n,\ \ j=1..k \tag{3}$$

One observe that the DLC depend only on the mode shape curvature squares $(\bar{\phi}_i''(x))^2$, as the term $\gamma(0,a)$ is independent of location x and thus eliminated by normalization. A series of DCL specific for a damage location are called **damage location index** DLI; it can be represented as a histogram (e.g. that presented in figure 1) and characterize uniquely the dimensionless damage position x/L on any cantilever.

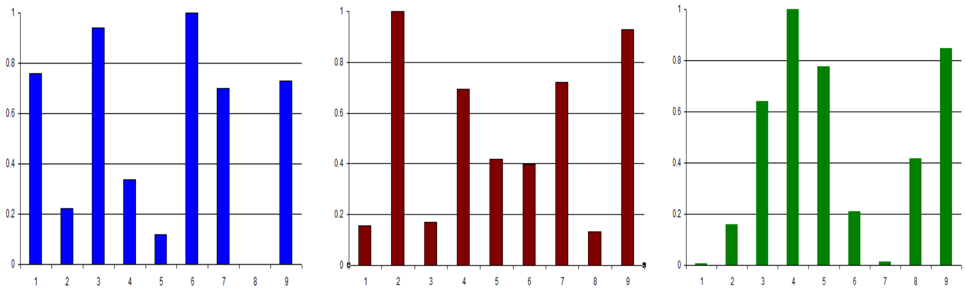

Figure 1. Examples of resulting histograms for damages in a cantilever beams placed at: x/L = 0.3; x/L = 0.55; x/L = 0.8 respectively.

Imagine now that we obtain the **relative frequency shift** by processing data from measurements for n vibration modes. This series of *n* values determined with the left term of relation (2) can be normalized by dividing them to the maximum value of the series, obtaining:

$$\Psi_1 = \frac{\delta f_1}{\max(\delta f_i)}, \dots, \Psi_n = \frac{\delta f_n}{\max(\delta f_i)} \tag{4}$$

Comparing now the resulted series from relation (4) with each DLI from relation (3), the x_j coordinate indicating the damage location is found.

3. Comparative study for damage location recognition base on histogram dissimilarity evaluation

A histogram $\Psi=\{\Psi_i\}$ is a representation of non-negative data Ψ_i corresponding to n bins ($i = 1\dots n$). Numerous measures are proposed for the dissimilarity between two histograms $\Psi=\{\Psi_i\}$ and $\Phi=\{\Phi_i\}$. The bin-by-bin dissimilarity measures only compare contents of corresponding histogram bins, i.e. they compare Ψ_i and Φ_i for all i, but not Ψ_i and Φ_k for $i \neq k$. The dissimilarity between the two histograms is a combination of all the pair-wise comparisons [15]. Some available estimators are presented in table 1.

Table 1. Dissimilarity estimators

Minkowski-Form Distance	Kullback-Leibler Divergence	Histogram Intersection
$d_{L_r}(\Psi,\Phi)=\left(\sum_i \left\|\Psi_i-\Phi_i\right\|^r\right)^{\frac{1}{r}}$	$d_{\log}(\Psi,\Phi)=\sum_i \Psi_i \log\frac{\Psi_i}{\Phi_i}$	$d_{\text{int}}(\Psi,\Phi)=1-\frac{\sum_i \min(\Psi_i,\Phi_i)}{\sum_i \Psi_i}$

First we tested a metrics similar to the bin-by-bin approach involving the Minkowski distances r = 2, r = 3 and r = 4. The bins are represented by vibration modes and the content represents the normalized relative frequency shift and damage location index respectively. Additionally, due to the possibility that the frequencies of some modes cannot be read accurately, we introduced a weighting factor w_i. Thus, the estimator becomes:

$$d_{L_2}(\Psi,\Phi)=\sqrt{\sum_i w_i \left|\Psi_i-\Phi_i\right|^2} \tag{5}$$

The simulation was made for all weighting factors w_i =1, for a damage location situated at x_j/L=0.38. The measured frequencies for the healthy and damaged beam are presented in table 2, for the first nine weak-axis bending vibration modes.

Table 2. Frequencies of the healthy and damaged beam and the resulting relative frequency shifts

Mode ***i***	1	2	3	4	5	6	7	8	9
f_{i_D}	4.06	25.411	71.17	140.48	229.33	346.44	482.56	639.06	830.76
f_{i_U}	4.09	25.626	71.75	140.62	232.52	347.45	485.45	646.56	830.78
δf_i	0.7182	0.8388	0.8058	0.1036	1.3713	0.2885	0.5956	1.1590	0.0027
Ψ_i	0.48959	0.5955	0.6185	0.0555	1	0.2777	0.3637	0.9526	0.0216

Calculating the Minkowski Distance for locations placed consequently from 0.05 to 0.05 mm along the beam, we obtained values depicted in the chart presented in figure 3. The values best fitting the normalized relative frequency shift Φ_i correspond to location x/L = 0.38 where the damage is placed; the lowest value of the Minkowski Distance is obtained by using r = 4. However, one can remark that this estimator, whatever the rage of r is, correctly indicate the damage position.

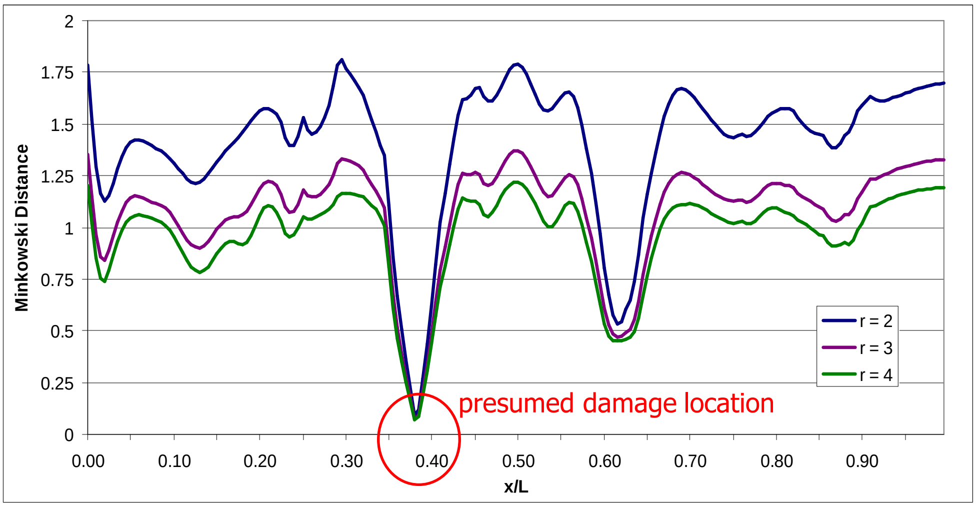

Figure 3. Dissimilarity chart for Minkowski Distance: r = 2; r =3 and r = 4

Afterwards, we tested the other two dissimilarity estimators, obtaining the charts presented in figures 4 and 5. One observe that for the cantilever beam, the Kullback-Leibler Divergence indicate the damage position, but predict false damage locations around the fixed end and introduce confusion in some other locations.

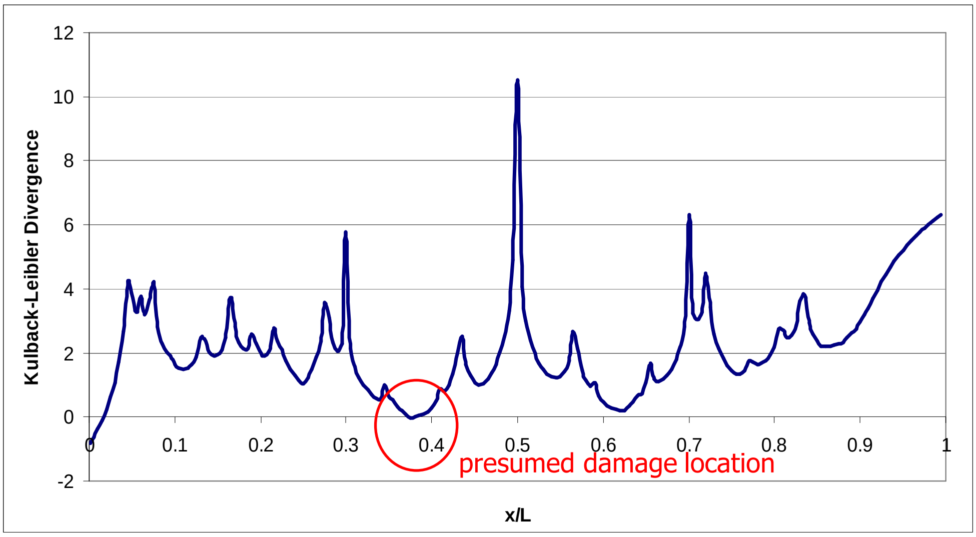

Figure 4. Dissimilarity chart for the Kullback-Leibler Divergence

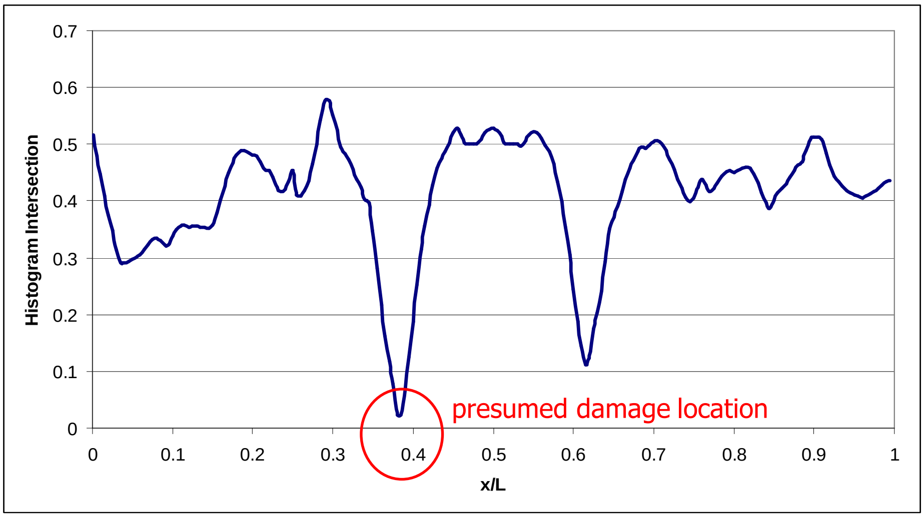

Figure 5. Dissimilarity chart for the Histogram Intersection

On the other hand, the Histogram Intersection is for this case somehow similar with the Minkowski Distance, but for other cases in the region close to the clamped end it can take negative values like the Kullback-Leibler Divergence. This can lead to misunderstandings and make the automation of the recognition process impossible or at least more complicated. Concluding, from these estimators the Minkowski distance seems to be the most reliable.

4. Conclusion

This paper proposes a method to assess damages in beams, based on the changes occurring in the natural frequencies due to damages. It consists in comparing the measured relative frequency shifts with calculated damage location indicators; this can be made using more dissimilarity estimators. The researches revealed that the Minkowski distance permit accurate assessment of damages, while the Histogram Intersection and the Kullback-Leibler Divergence provide ambiguous predictions even in case of for accurate measurements.

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