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## **Particular Approaches about Symmetrical Structures Dynamics**

*The paper presents some aspects about dynamic behaviour of a rigid structure building insulated on anti-seismic elastic devices. The structure presents symmetries in terms of the geometrical and insulation configurations, and this allows decoupling of the eigenmodes. Thereby, is simpler to evaluate the impact of the dynamic forces transmitted through the terrain-structure path during the earthquake. Based on the vibration isolation theory, it can be evaluated the isolation degree for the considered structure. It has considered as an excitation factor the complex signal of an earthquake defined through the ground motion acceleration. The analysis that was made in the paper reveals the need to identification and evaluation requirements for the functional correlations between the reference parameters of the considered structure and the characteristics of the isolating and insulating devices.*

**Keywords:** *anti-seismic elastic devices, dynamic behaviour, vibration isolation theory, isolation degree, eigenmodes.*

### **1. Introduction**

The values of the vibration parameters there are transmitted - amplitude, frequency, transmissibility - determine the performance level that characterizes the use of anti-seismic elastic devices [1], [2], [4]. To determine these parameters it is analysed a practical situation of use of these elastic systems, namely the vibration and seismic waves isolation for buildings or parts thereof - was considered a building P+0, insulated on special elastic systems [16], [17], [18], [20]. We mention that for the analyzed situation it was adopted a physical model of computation based on the rigid solid hypothesis. The isolated structure is considered a concentrated mass elastic orthogonal insulated on terrain [3], [5], [6], [7].

## 2. Physical and mathematical model of rigid structures

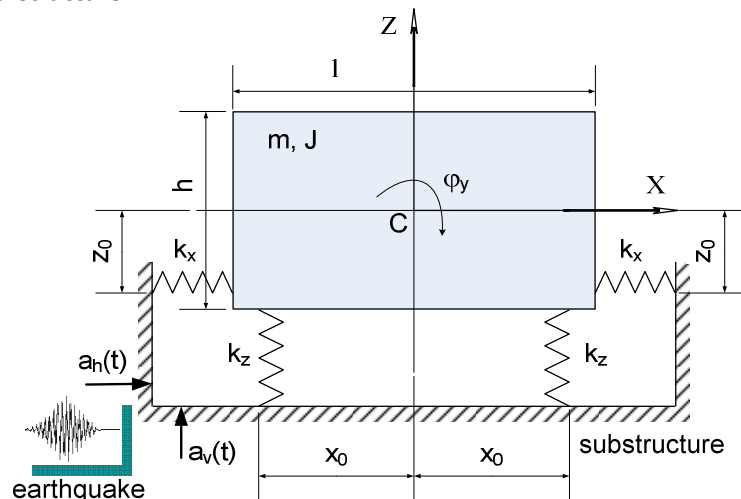
The analysis of the performance level in the case of a anti-seismic isolation for a building P +0, insulated on elastic special systems [15], [19], [21], [24]. The characteristic parameters for the building are:

- ❖ the number of levels = 1
- ❖ the built area =  $40 \times 40 = 1600 \text{ m}^2$
- ❖ the total building height = 4 m
- ❖ the total mass of the construction = 2912 tons

The physical adopted model is shown in figure 1 and is characterized by the motion equations system (1). The adopted version, characterized by a single concentrated mass (the rigid solid model), elastic orthogonal insulated to the extremities, is justified by the following hypothesis [8], [9], [10], [11], [23]:

➤ the characterization of the performance level regarding the vibration and seismic waves isolation is based on the evaluation of the values of the specific parameters for the transmitted vibrations to the foundation;

➤ the installing of an additional elastic element offers to the isolated structure a very low value of the fundamental frequency, compared with the situation in which the structure is rigid mounted on the foundation and, also, compared with the frequencies domain specify for the terrain movement (the foundation movement). The first dynamic mode of the isolated structure involves deformations only in the isolated system. The structure has a specific behavior of a rigid building. The superior dynamic modes don't participate to the movement, so, if there is a higher energy contribution on these high frequencies, this energy is not transmitted to the isolated structure.



**Figure 1.** The dynamic model of computation for a P+0 structure subjected to seismic actions

We consider the fact that the seismic excitation - which is an acceleration - is acting by the two orthogonal directions X and Z. The system of differential equations of motion, corresponding to the schematized model from Figure 1, is the following:

$$\begin{cases} \ddot{x} + 4 \frac{k_x x}{m} + 4 \frac{z_0 k_x \varphi_y}{m} = 4a_h \\ \ddot{\varphi}_y + 4 \frac{\varphi_y (k_z x_0^2 + k_x z_0^2)}{J_y} + 4 \frac{z_0 k_x x}{J_y} = 4 \frac{m z_0 a_h}{J_y} \\ \ddot{z} + 4 \frac{k_z z}{m} = 4a_v \end{cases} \quad (1)$$

where  $a_h, a_v$  are the seismic waves accelerations measured on the two orthogonal directions. For  $a_h$  is chosen the horizontal NS or EV direction, depending on the placement mode of the building and on the prevailing direction of the seismic motion.

$$A_x = \frac{ma_h}{4k_z} \frac{\sqrt{\left[ \left( \frac{x_0}{\rho_y} \right)^2 - \left( \frac{\omega}{\omega_z} \right)^2 \right]^2 + \left[ \frac{k_x x_0 z_0}{k_z \rho_y \rho_y} \right]^2}}{\left( \frac{\omega}{\omega_z} \right)^4 - \left[ \frac{k_x}{k_z} + \frac{k_x}{k_z} \left( \frac{z_0}{\rho_y} \right)^2 + \left( \frac{x_0}{\rho_y} \right)^2 \right] \left[ \left( \frac{\omega}{\omega_z} \right)^2 + \frac{k_x}{k_z} \left( \frac{x_0}{\rho_y} \right)^2 \right]} \quad (2)$$

$$A_\varphi = \frac{ma_h}{4k_z \rho_y} \frac{\sqrt{\left[ \frac{z_0}{\rho_y} \left( \frac{\omega}{\omega_z} \right)^2 \right]^2 + \left[ \frac{x_0}{\rho_y} \left( \frac{k_x}{k_z} - \frac{\omega^2}{\omega_z^2} \right) \right]^2}}{\left( \frac{\omega}{\omega_z} \right)^4 - \left[ \frac{k_x}{k_z} + \frac{k_x}{k_z} \left( \frac{z_0}{\rho_y} \right)^2 + \left( \frac{x_0}{\rho_y} \right)^2 \right] \left[ \left( \frac{\omega}{\omega_z} \right)^2 + \frac{k_x}{k_z} \left( \frac{x_0}{\rho_y} \right)^2 \right]} \quad (3)$$

$$A_z = \frac{ma_h}{4k_z} \frac{1}{1 - \left( \frac{\omega}{\omega_z} \right)^2} \quad (4)$$

A comparative analysis between the three parameters variations goes to the following issues:

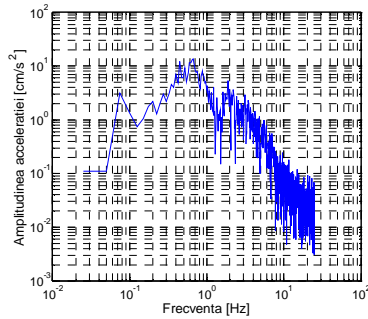
- for the vertical displacement  $Z$ , we can see that once with the improvement of the insulation solution, the maximum level of the signal significantly decreases.
- regarding the maximum earthquake magnitude on vertical direction, for the direct insulation solution on elastic elements, the system response in this direction is about two times lower versus the response to the horizontal direction.
- on the NS direction, the system response analysis should take into account the corresponding values of the equivalent rigidities of the special insulating systems. Thus, for any insulating solutions that were analyzed, the maximum amplitude of the system response is lower than that of the excitation on the same direction (NS).
- regarding the rotation motion around the Y axis, it has very low values (less than 0.008 degrees), whose influence on the system motion can be neglected - the displacement on the horizontal direction at the upper end of the building is approximately 0.002 m.
- even that the spectral composition of the seismic signals indicate a fairly wide frequency range, it is observed that the eigen frequencies for the analyzed SES ensure a stable operation of the whole structure - support - foundation system, in post-resonance regime, with a high degree of the vibration isolation.

### 3. Computational analysis and results

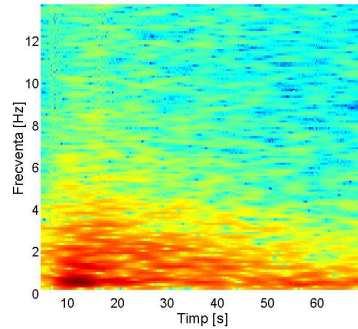
Based on the transmissibility and on the vibration isolation can be done an assessment for the isolation degree. For this, is analyzed the significant spectral component of the earthquake complex signal as an excitation factor defined through the terrain motion acceleration. The case used for analysis is the Vrancea earthquake of 1977 [12], [13], [14]. The earthquake had as a distinct characteristic a multishock behaviour and the movement propagation was made on NE-SW direction. According to INCERC recordings, the peak acceleration of the terrain on NS direction had the value  $194,93\text{ cm/s}^2$ , the peak speed had the value  $71,94\text{ cm/s}$  and the peak displacement of the terrain had the value  $16,31\text{ cm}$ . Considering the peak acceleration value, on the NS direction were plotted and analyzed some physical parameters such as:

- ❖ the acceleration response spectrum;
- ❖ the acceleration signal spectrogram;

In the figure below is represented the acceleration of the seismic action on the North - South direction.



**Figure 2.a** The acceleration response spectrum [12]



**Figure 2.b** The 2D acceleration spectrogram [12]

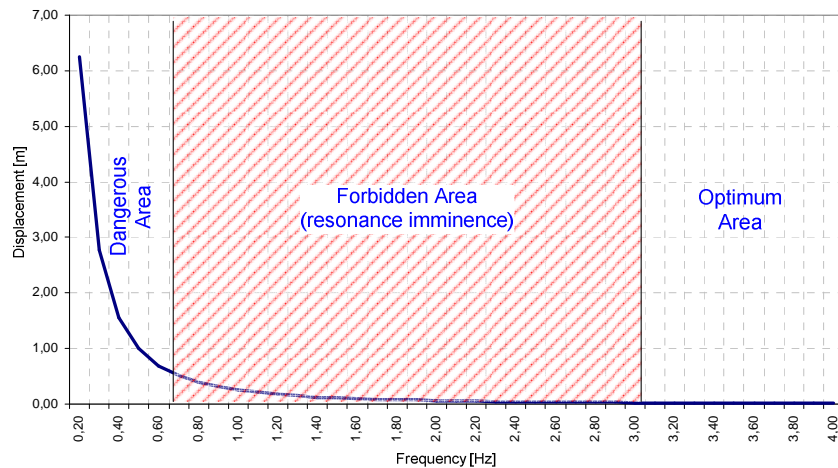
I have represented the proposed parameters on the NS direction in the horizontal plane, because in this direction it is observed that the acceleration has the maximum value and the domain of the dominant frequencies has significant values. The acceleration spectrogram shows the dependence of the frequency in respect with the time, but for this study is only with a qualitative character. The table below summarizes the main numerical values of the analyzed parameters [12].

**Table 1.** Reference values for 1977 Vrancea earthquake

Crt. no.	1977 Vrancea earthquake	Maximum value of the acceleration [m/s <sup>2</sup> ]	Domain of the dominant frequencies [Hz]	Domain of the significant frequencies for the power distribution [Hz]
1	The EW direction	1.62	0.3-3	0.4-0.31
2	The NS direction	1.94	0.65-3.12	0.65-3.12
3	The vertical direction	1.05	0.51-5.64	0.51-5.64

Considering at reference the parameter values from the table corresponding to the NS direction in the horizontal plane, I wanted to analyze the eigen frequencies values of the structure relative to the pre- and post-resonance, so as to avoid the resonance danger [22]. It must be also satisfied the essential structural and functional requirements for static and dynamic solicitations. The calculation showed that, for small values of the eigen frequencies, are obtained unacceptable results regarding the large static deformations of the insulating system. In these circumstances the positioning in the post-resonance area of the working feature is not possible (feasible). For high values of the eigen frequencies, which corresponds to an operating mode in the pre-resonance area, are obtained high stiffness coefficients, respectively static small deformations, acceptable, of the insulating systems.

Based on this first phase of the study was made the diagram from Figure 3. In that diagram was marked the restricted area, corresponding to the spectral domain of the excitation signal, in the analysed case of the seismic wave. Analysing the dependence of the movement in respect with the frequency, it is observed that the optimal working area of the elastic insulating systems is in the right part of the dominant excitation frequencies area, which is pre-resonance.



**Figure 3.** The static displacement versus frequency dependence

On the vertical direction, (as shown in Fig. 1) there is a static load, which is the weight of the considered structure, and a dynamic overload, which is the seismic excitation on this direction. Thus, the total arrow of the insulating devices has on this direction two components: a static one and a dynamic one. On the vertical direction, as shown in the preceding paragraphs, the insulating elements must have high rigidity. In these conditions is ensured the working condition in pre-resonance regime and increase the bearing capacity of these elements, so that there is a reserve for static and dynamic loads to take over.

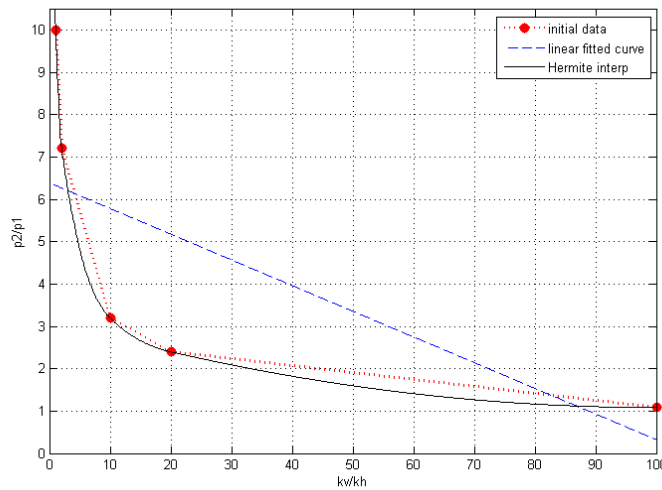
On the horizontal direction, the insulating elements take over only a dynamic load. According to the classical theory of the vibration isolation, the dimensioning of the elements it will be made to ensure a working regime in post-resonance, so the rigidity of these elements must be small.

For the functional reasons in static regime will be considered as known the stiffness coefficient in the vertical direction  $k_v$ , as a reference value against which it will examine the influence of the stiffness coefficient in the horizontal direction  $k_h$  on the global isolation characteristic of the structure-insulator-terrain system. Thus, we will consider a set of values of the  $k_v/k_h$  ratio. According to this ratio

was realized a qualitative study regarding the spectrum amplitudes for the two coupled degrees of freedom of the structure-insulator-terrain system ( $x$  and  $\varphi$ ).

From the comparative analysis of the spectral characteristics results that the influence of the rigidities ratio on the frequencies corresponding to the coupled modes can be evaluated considering the diagram from Fig. 4. This diagram shows the dependence between the stiffness ratio and the natural frequencies ratio for the coupled modes, ratios considered for the five analysed cases.

From Fig. 4 results that the increase of the stiffness ratio (which means the increasingly lower values of the stiffness in the horizontal direction compared with the reference value of the stiffness in the vertical direction), involves the reducing of the value of the eigen frequencies ratio (for  $x$  and for  $\varphi$ ).



**Figure 4.** The dependence between the stiffness ratio and the natural frequencies ratio

The previous observation mean, practically, an approach of the eigen frequencies values, in other words, a decrease of the global domain of the dominant eigen frequencies of the analysed system. Making a analysis, we can observe that the reducing of the eigen frequencies domain come together with the maintaining of a constant lower limit and with the shifting of the upper limit to the low frequency values. This is based on the initial hypothesis, according to all structural and functional parameters of the system structure-insulator-terrain which don't covered by this analysis are constant. The reduction of the eigen frequency domain together with its shifting to low values of the spectrum is particularly beneficial because it reduces or even eliminates the risk that the dominant seismic action spectrum to overlap or interfere with the eigen frequencies domain of the analysed system (thus avoiding the resonance).

#### 4. Conclusions

Although from the functional and static/dynamic stability considerations the working regime in the vertical direction must be in the pre-resonance, according to the presented analysis, result that in the horizontal direction is assured the functioning in post-resonance regime with major advantages regarding the isolation degree to dynamic disturbing actions, which can be intense and varied, such as seismic waves.

This analysis shows the need to identify and evaluate the functional correlations between the reference parameters of the structure and the characteristics of the isolating and insulating devices. So can be avoided the resonance phenomena, which occur due to the overlap of the two essential spectra, the excitation signal one and the perturbed system one.

One future research direction is to identify an additional set of essential parameters which can be able to provide the shifting of the lower limit of the dominant eigen frequencies spectrum, corresponding to the two coupled modes of the structure-insulator-terrain system to low values. In this way it will be possible to grant very precise the working regime in the post-resonance with the reference spectrum of the disturbing dynamic action of seismic wave type.

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