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Theoretical Assessments on Dynamics of Helical Flexible Coupling within Vibratory Equipments

This study deals with vibratory equipments dynamic behaviour and especially reveals some critical aspects regarding the flexible coupling within and those influences on entire system evolution. Changing of dynamic characteristics, additional dynamic loadings and variances reducing of vibration parameters frame the main hypothesis of the research. This paper briefly presents a few theoretical assessments in terms of linear and nonlinear approaches of driving chain elements. Realistic expanded, also computational simplified models have been provided. Simulated behaviors denote that many of the reported observation regarding the serviceable dynamic distortions are correctly, and impose a set of additional instrumental tests in order to obtain a good accuracy for mathematical models.

Keywords: *vibratory equipment, computational dynamics, helical flexible coupling, functional optimization*

1. Introduction

Experimental investigations of systems with vibratory equipments driven by rotary electric motors had revealed that helical flexible coupling although provides lot of advantages in regular utilization, but can also provide a major source for transitory dynamics into the driving system. Various distorting signals acts such an inputs and perturb the technical system thus that it is quite difficult to identify, estimate and evaluate them just based on instrumental tests [2-5]. Hereby, appears the necessity of a suitable mathematical model that had to be implemented into a computational environment and used for analysis and estimations of dynamic behaviour of elastic coupling [4].

The basic approaches proposed in this study are based on the next main hypothesis as follows:

i) The dynamic nonlinearity of torsional spring zone within the helical flexible coupling, assuming that enlarging or reducing of coil diameter can induces modifications of the dynamic characteristics because of the continuous changing of dynamic masses distribution during a complete rotation cycle [2].

ii) The variances of rotational speeds at the coupling input and output respectively, induce certain abnormal dynamic evolution in terms of functional deformation, globally or locally arising, taking into account that positive or negative applied bending moment means changing of the diameter onwards or backwards with respect in direction of coil winding [3].

iii) Inside the torsional spring area, can appears resonances, locally into a certain coil curl or a group of, or globally into the spring on a whole. Effective conditions leading to a resonant phenomenon in coupling coil cannot be strictly evaluated because of continuous changing both of the internal system parameters (dynamic mass distribution and stiffness), and of the rotational speed (actually the variances of the velocity through the coupling spring, between input and output clamping bushes).

2. The models for flexible coupling within driving system

A simple model for a *motor - coupling device - load* scheme has depicted in Fig.1. In respect with the option that the motor will have been simulate such a real or an ideal mechanical power supply element, results the system of two dynamic equations or a single one respectively (the model have two degree of freedom for the real case, and a single degree of freedom for ideal case).

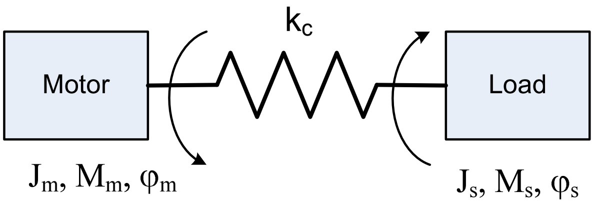

Figure 1. Simple lumped mass model for flexible coupling driving system

Obviously, this simple model includes a single way to analyze the flexible torsional coupling dynamics that is the stiffness expression

$$k = \frac{\pi}{180}\frac{E\,d^4}{64\,n\,D}\,[\text{deg}], \tag{1}$$

for the round wire torsional springs or, for the rectangular wire torsional springs

$$k = \frac{1}{180}\frac{E\,b\,t^3}{12\,n\,D}\,[\text{deg}], \tag{2}$$

where *E* denotes the modulus of elasticity, *d* is the spring wire diameter, *D* denotes mean coil diameter, *n* is the number of the active coils, *b* is the wire width, and *t* is the wire thickness. First terms in both Eqn. (1) and (2) help to obtain the stiffness value in deg units.

An extended approach of the model in Fig. 1 must be providing by nonlinear laws for torsional rigidity. This kind of model even it will be able to accomplish a realistic simulation, also requires difficult evaluations for many parameters and assumes higher order computations capabilities [1,6].

Next model supposes a lumped mass approach for the coupling device thus that it will be able to dignify the way to behave for each elementary part of flexible coupling. In Fig. 2 was depicted a complex lumped mass model for a helical flexible coupling.

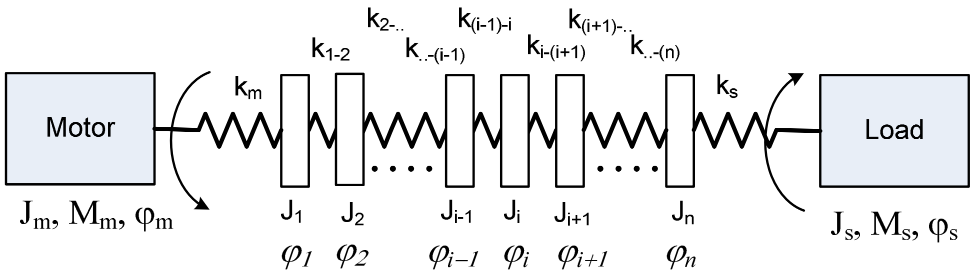

Figure 2. Complete lumped mass model for flexible coupling driving system

The last model (see Fig. 2) can acquires (n+1) or (n+2) degrees of freedom (DoF) according with the initial option for the motor device simulation complexity (with ideal or real power supply characteristics). The basic nDoF corresponds to the number of the elements assumed for helical device sampling, and the load provides the additional DoF. A matrix formulation of dynamic equations is

$$\mathbf{J}\,\ddot{\varphi} + \mathbf{K}\,\varphi = \mathbf{F} \tag{3}$$

where **J** denote the matrix of inertia, **K** is the matrix of rigidity, **F** is the external forces, and φ denotes the angular displacement.

The matrix of inertia **J** is a diagonal array containing the moment of inertia for each element of the system, including additional inertia due to the dynamic effects in any coil of coupling spring.

External forces column vector denoted by **F** contains the external resistant or supplying moments of forces for the entire systems. In case of hypothesis of the resistant moment of forces developed by the instant dynamic eccentricity of a single or a group of coils, those values have to written also into the vector **F**.

Matrix of rigidity **K** is a real symmetrical array contains the stiffness value of every linkage inside the model

$$K_{i,j}\Big|_{\substack{i=1\ldots n\\ j=1\ldots n}} = \begin{cases} k_{i-1}+k_i & for \quad i=j \\ -k_i & for \quad \left\langle \begin{matrix} j=i+1 \\ i=j+1 \end{matrix} \right. \\ k_0, k_{n+1}=0 \end{cases} . \tag{4}$$

Damping matrix is missing from the Eqn. (3) because even if it assumes Rayleigh-type proportional dissipative component approximation, or it supposes loss factor-type damping, both hypotheses does not provide any additional information for system response in terms of the output signal spectral composition.

Nonlinear characteristics for stiffness can be assigned for any of the linkages but it had to taken into account the remarks at beginning of this paragraph regarding the nonlinearity of torsional rigidity and the additional aspects that involved.

Results evaluation for this model assumes very large systems with big number of parameters to accomplish, to analyze and to present. For usual cases of analysis it can be simulated the flexible helical coupling like two separate linear linkage elements, with half stiffness, mounted between the motor/load components and a central virtual rotational body included full inertia characteristics of coupling.

3. Simulation, analysis and discussions

Diagrams depicted in Fig. 3 and Fig. 4 shows the evolution of the load and coupling angular velocity and the spectral composition of each signal. Actually, it has drawn the deviation velocity between the variable outputs and the input fixed value of angular speed at motor shaft.

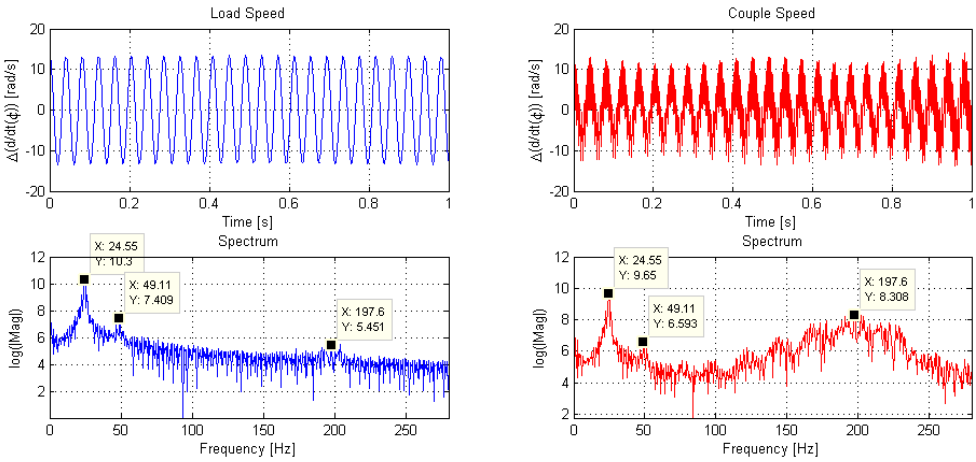

Figure 3. Steady state evolutions of driving system parameter

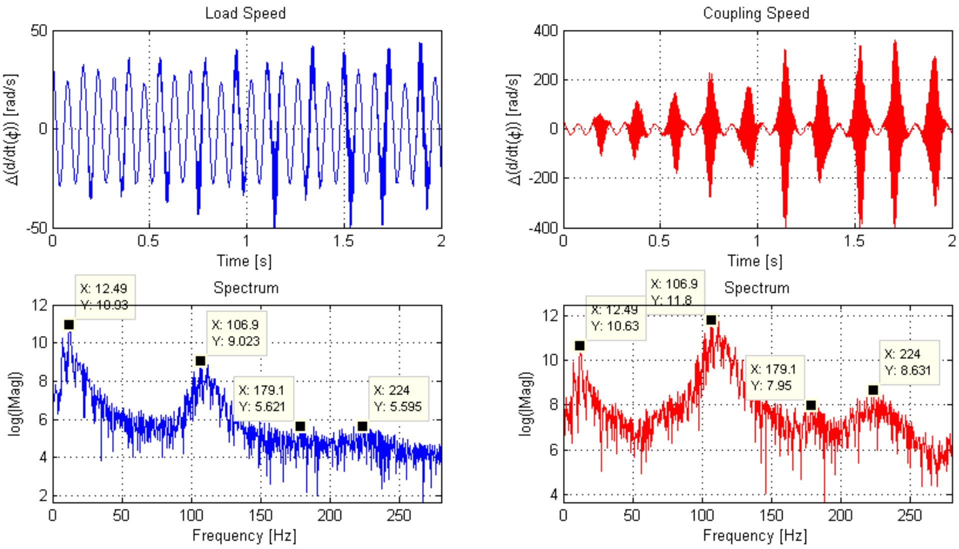

Figure 4. Transitory state evolutions of driving system parameter

Graphs in Fig. 3 denote a few seconds timed evolution of the system supposing a regular dynamic behaviour. This means a very rigid coupling with relative low mass and no additional eccentric effects both at load and inside the coupling spring. Missing of the additional inertial effect at the load can be assigned to the situation while the vibratory equipment that works into the horizontal plan.

The graphs depicted in Fig 4 presents the same parameters as in Fig. 3 but for a strong transitory case meaning considerable mass and additional inertial effects in the entire coupling element, and the presence of the load eccentricity effect. Time length for the second test has enlarged to dignify that the coupling evolution is not at the resonance and its velocity amplitude even if acquiring a periodically great values its variance has finite limits.

Comparative analysis between the two sets of diagrams has to taken into account different excitation for each case. Although, it easily results a "silenced" evolution in the first case and in the same time a relative "noisy" behavior for the second one. Additional dynamic effects induce a frequency modulation of general movement with very strong impact into the coupling coils.

Most significant peaks were marked on the spectral composition diagrams. Each pairs of spectrum graphs shows that the peaks has preserved for the same case (acquire the same frequency values). However, it also preserved the global trend, which reveals that the loads acquire maximum magnitudes for low frequencies and, in the same time, the flexible coupling slides to the higher values of frequency for relevant magnitudes.

4. Conclusion

The theoretical basics presented in this paper enable the future developments regarding the helical flexible coupling dynamics and its influences on the global evolution of the entire driving systems. The briefly presented results have shown the differences between spectral compositions of the load movement for two extreme cases and have revealed the opportunity of such analysis. The research will be continuing with a set of instrumental tests performed on a special laboratory stand with the purpose of evaluation for different cases of dynamic regimes and their influences on the load evolution.

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