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Some Theoretical Observations Concerning the Reverberation Time for the Case of a Harmonic Emitting Source

In the paper, the reverberation phenomenon produced by the harmonic emission of a sound source is theoretically examined, taking into consideration the absorption of the sound in the air. We obtain a formula which contains, as particular cases, the similar formulae presented in the literature.

***Keywords**: reverberation time; absorption of sound; acoustic energy*

1. Introduction

Reverberation (the word comes from the Latin "reverberare" meaning to beat back) has long been observed although only in 1900 was a quantitative method achieved for measuring and predicting this.

The reverberation phenomenon takes place in a close room when a source is suddenly turned off but the sound persists, so this phenomenon must to be considered at the projection activity in the buildings.

A sound is made, the sound stops, one listens to the sound until it died away. The time this takes is defined as the *reverberation time* T_r of the space- the time taken by mean squared pressure to decay by 60 dB or the time taken by acoustic energy density to decay of 10^6 times.

Within the framework of a simple model, for that it supposes that the sound source emits with a constant power, the expression of reverberation time by means of Sabine's formula is [1], [2]:

$$T_r = \frac{24 \ln 10}{u} \frac{V}{\sum_i \overline{\alpha}_i S_i} = 0{,}16 \frac{V}{\sum_i \overline{\alpha}_i S_i} , \tag{1}$$

where: $\overline{\alpha}_i$ and S_i are the absorption coefficients, respectively the surface of the different zone of the room's walls with volume V , and u is the sound velocity in air.

In general the reverberation time of a room is proportional to the volume of the room and inversely proportional to the sum of all absorptions.

In reality every place within a room has its own specific reverberation and this is determined by the size and geometry of the space's boundaries and the sound being made as well as the acoustic qualities of the materials of enclosure.

Other formula from literature, for reverberation time which considers the absorption of sound waves in air, is the formula of Eyring [3], [4]:

$$T_r = 0{,}16 \frac{V}{4\alpha V + \overline{\alpha} S} \quad , \tag{2}$$

where α is the absorption coefficient of sound wave in air and $\overline{\alpha}$ is the average absorption coefficient of the different absorbent surfaces of the walls, of surfaces S_i and the absorption coefficient $\overline{\alpha}_i$, the total surface of the walls is S :

$$\overline{\alpha} = \frac{1}{S} \sum_i \overline{\alpha}_i S_i \ . \tag{3}$$

The real sound sources don't emit usually with the constant sound power, because their power depends of the time. In the present paper we will consider a sound source with an emission power that has a harmonic variation with the time:

$$P(t) = P_m \cos^2 \omega t \tag{4}$$

where the amplitude P_m and the pulsation ω are known.

2. The density sound energy

Because at the frequencies more then 4000 Hz the absorption in air is the same order or superior at the absorption of the delimited surfaces, this can't be neglect, so the acoustic energy balances from the room, for an elementary interval of the emission times dt writes in the form:

$$dW_{em} = dW + dW_{abs,walls} + dW_{abs,air} \tag{5}$$

where:

$$dW_{em} = P \cdot dt \tag{6}$$

is the emission energy from the source,

$$dW = V \cdot dw \tag{7}$$

is the energy from the interiour of room in the times interval $(t, t+dt)$,

$$dW_{abs,wall} = \frac{u\overline{\alpha}S}{4} w \cdot dt \tag{8}$$

is the absorption energy from the walls, and

$$dW_{abs,air} = V \cdot dw_{abs,air} \tag{9}$$

is the absorption energy from the air; in last three relations the symbol w represent the volume density energy.

For the deduction of the expression of the absorption energy in air, we consider the Beer's law of absorption, it determines the intensity of the sound wave after the traversing in absorbent medium at the distance $u \cdot t$ [2], [4]:

$$I = I_0 e^{-\alpha u t} \tag{10}$$

I_0 is the wave intensity in the vicinity of the source that is in the origin and the relation between the sound wave intensity and density of the acoustic energy in a diffusion sound field [4]:

$$I = \frac{uw}{4} . \tag{11}$$

The density of the absorption energy in air, equal and the opposite sign with the variation of the density acoustic energy in the interval dt, is:

$$dw_{abs,air} = -\frac{4}{u} dI = 4\alpha I_0 e^{-\alpha u t} dt \tag{12}$$

Using the relations (4), (6), (7), (8), (9) and (12), the balances equation (5) leads to the differential equation for the density acoustic energy in the room, in the case of the sound source has a harmonic emission:

$$\frac{dw}{dt} + Cw = \frac{P_m}{V} \cos^2 \omega t - 4\alpha I_0 e^{-\alpha u t} \tag{13}$$

where we use the notation:

$$C = \frac{u}{4V} \overline{\alpha} S \quad . \tag{14}$$

The solution of the differential nonhomogeneous equation (13) is [6]:

$$w(t) = e^{-\int C\,dt} \left[\int \left(\frac{P_m}{V} \cos^2 \omega t - 4\alpha I_0 e^{-\alpha u t} \right) e^{\int C\,dt} dt + C_0 \right] \tag{15}$$

where C_0 is an integral constant that will be determined.

After some calculations we obtain:

$$w(t) = \frac{P_m}{V} \left[\frac{1}{C^2 + 4\omega^2} \left(\frac{C}{2} \cos 2\omega t + \omega \sin 2\omega t \right) + \frac{1}{2C} \right] - \frac{4\alpha I_0}{C - \alpha u} e^{-\alpha u t} + C_0 e^{-C t} \tag{16}$$

The integral constant is determined using that at t=0 (the begin of emission source) the density energy is null:

$$w(0) = 0 = \frac{P_m}{V} \left[\frac{1}{C^2 + 4\omega^2} \frac{C}{2} + \frac{1}{2C} \right] - \frac{4\alpha I_0}{C - \alpha u} + C_0 \tag{17}$$

We obtain:

$$C_0 = -\frac{P_m}{V}\frac{1}{C}\frac{C^2+2\omega^2}{C^2+4\omega^2}+\frac{4\alpha I_0}{C-\alpha u} \tag{18}$$

So, the expression of the density volume energy is:

$$w(t)=\frac{P_m}{V}\left[\frac{1}{C^2+4\omega^2}\left(\frac{C}{2}\cos 2\omega t+\omega\sin 2\omega t\right)+\frac{1}{2C}\right]-$$
$$-\frac{P_m}{V}\frac{1}{C}\frac{C^2+2\omega^2}{C^2+4\omega^2}e^{-Ct}-\frac{4\alpha I_0}{C-\alpha u}\left(e^{-\alpha u t}-e^{-Ct}\right) \quad 19)$$

Using the notation:

$$\tan\varphi=\frac{C}{2\omega} \tag{20}$$

The expression of the density energy has the form:

$$w(t)=\frac{P_m}{V}\frac{1}{2\sqrt{C^2+4\omega^2}}\sin(2\omega t+\varphi)+\frac{P_m}{2CV}-$$
$$-\frac{P_m}{V}\frac{1}{C}\frac{C^2+2\omega^2}{C^2+4\omega^2}e^{-Ct}-\frac{4\alpha I_0}{C-\alpha u}\left(e^{-\alpha u t}-e^{-Ct}\right) \tag{21}$$

From expression (21) of the density sound energy, we observe that the emission sound source with the power expressed by a harmonic function, so and the density energy must to have a harmonic component, dephased from the power.

The diphase φ depends of the emission frequency and the absorption average coefficient of the walls from the room. The diphase begins null for very high values of the sources frequency or for the case that the walls of room are perfectly reflected.

We can observe that the average value of the oscillate term of density energy is equal with zero, so, in average, the density energy increases exponentially to a value of the stationary regime.

It is very easy to see that for a constant emission power: $P_m/2$ (effective power), so for $\omega = 0$, and in absent of the absorption in air ($\alpha = 0$), we obtain the know expression of the density acoustic energy from the room [3]:

$$w(t)=\frac{P}{CV}\left(1-e^{-Ct}\right) \tag{22}$$

We can observe from analyses of the behaviour in time of the function $w(t)$ that after a certain interval of time, corresponding to a transitory regime (theoretical infinite, but with a finite duration in practice), the exponentials from this expression tend to zero, and the solution becomes oscillating around the stationary value.

$$w_s = \frac{P_m}{2CV} = \frac{2P_m}{u\overline{\alpha}S} \tag{23}$$

At this value, the acoustic energy from the room, begins to decrease after the emission source (P=0) and the differential equation of the density energy has the form:

$$\frac{dw}{dt} + Cw = -4\alpha I_0 e^{-\alpha u t} \tag{24}$$

This equation is the same type with the equation (13), so we obtain the solution:

$$w(t) = \frac{4aI_0}{C-\alpha u}\left(e^{-Ct} - e^{-\alpha u t}\right) + C_0 e^{-Ct} \tag{25}$$

where the integral constant is noted by C_0. For determination of this constant we consider that at $t = 0$ (the moment of stopped emission source), the density energy is $w(0) = w_s$, so that it results:

$$C_0 = w_s = \frac{P_m}{2CV} \tag{26}$$

and the expression for density energy has the form:

$$w(t) = \left(w_s + \frac{4\alpha I_0}{C-\alpha u}\right)e^{-Ct} - \frac{4\alpha I_0}{C-\alpha u}e^{-\alpha u t} \tag{27}$$

3. The reverberation time

The definition of the reverberation time leads to the equation:

$$\frac{w(T_r)}{w_s} = \frac{1}{10^6} \quad , \tag{28}$$

that, using the equation (26), it becomes:

$$\left(1 + \frac{1}{w_s}\frac{4\alpha I_0}{C-\alpha u}\right)e^{-CT_r} - \frac{1}{w_s}\frac{4\alpha I_0}{C-\alpha u}e^{-\alpha u T_r} = 10^{-6} \tag{29}$$

So, from this equation we can obtain, in principle, the reverberation time. The equation (28) is a transcendent equation, but it can be resolve using a numerical or graphic method.

In the next, we propose other method for resolve this transcendent equation, an analytical method. We note:

$$B=\frac{1}{w_s}\frac{4\alpha I_0}{C-\alpha u} \tag{30}$$

and multiply the equation (28) with $\exp(-\alpha u T_r)$, writing the number 10^{-6} in the exponential form, we obtain:

$$(1+B)e^{-(C-\alpha u)T_r}-B=e^{-6\ln 10+\alpha u T_r} \tag{31}$$

From development in power series of equation (31) and keeping only the terms of first order we has:

$$(1+B)\left[1-(C-\alpha u)T_r\right]-B=1-6\ln 10+\alpha u T_r \tag{32}$$

Using the equations (14) and (29), it obtains the next expression for the reverberation time:

$$T_r=\frac{24\ln 10}{u}\frac{V}{4\alpha V+\overline{\alpha}S+\alpha\dfrac{8I_0V}{P_m}\dfrac{\left(\overline{\alpha}S\right)^2}{\overline{\alpha}S-4\alpha V}} \tag{33}$$

This expression for the reverberation time T_r is more comprehensive than the expression from literature (Sabine's and Eyring's formulas), in the sense that expression (33) includes them as the particular cases. For a large interval of temperatures, the velocity of sound waves in air doesn't change very much, so the velocity u can be considered equals with 340m/s. So, the expression for reverberation time can write in the form:

$$T_r=0{,}16\frac{V}{4\alpha V+\overline{\alpha}S+\alpha\dfrac{8I_0V}{P_m}\dfrac{\left(\overline{\alpha}S\right)^2}{\overline{\alpha}S-4\alpha V}} \tag{34}$$

In the particular case, for the high emission powers, the third term from denominator becomes null and we obtain the Eyring's formula. On the other hand, if we don't consider the absorption of sound waves in air, so we consider $\alpha=0$, we obtain the Sabine's formula (1).

4. Conclusion

In the present paper we consider the case of a sound source with the harmonic variation in time of the power which emits in the closes room with knew characteristics. In the solving of the differential equation for the density energy from the room, we taking into consideration the absorption of the sound in the air from the interior, in the time of emission from source and after the emission stopped.

This model offers a comprehensive expression for the reverberation time.

For the transcendent equation of the reverberation time, we obtain an analytical solution (approximately), it permits the evidence of the dependence measurements of the reverberation time, but it contains, as particular cases, the similar formulae presented in the literature.

The reverberation time depends on:

- geometrical characteristics of the room, through the *C* constant, namely by the room volume, the average absorption coefficient $\overline{\alpha}$ and by the total surface of he room;
- medium properties by which the sound waves propagate, namely the absorption coefficient of the air α which depends from the frequency of the sounds and the relative humidity from the room [1];
- source characteristics, namely the maxim emission power and the intensity of the sound waves in the next of the source.

The study of the dependence of the reverberation time with the mentioned factors can to contribute to the efficiently of the using activity from the rooms with the determined geometrical characteristic, without that to necessity other constructive adaptation and our formula obtained permits an analytical evaluation more precise for the reverberation time.

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