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## **On the Modeling of Contact Interfaces with Frictional Slips**

*The paper analyses the contact interfaces between the scatterers and the matrix into the sonic composites, in the presence of the frictional slips. The sonic composite is a sonic liner designed in order to provide suppression of unwanted noise for jet engines, with emphases on the nacelle of turbofan engines for commercial aircraft.*

**Keywords:** *sonic composite, contact interface, friction, slip*

### **1. Introduction**

The control of the acoustical properties of sonic composites needs the study not only of the distribution of the scatterers in the matrix but also the contact interfaces between the scatterers and the matrix. Different models have been postulated to represent the interaction between waves at the interfaces, the wave front transformation, scattering and focusing. The boundary conditions for the acoustic field, which are to be enforced at the imperfect interfaces, are important topics for behavior of the sonic composites.

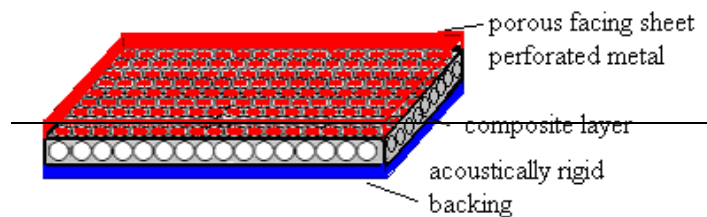
The boundary conditions that relate stresses and displacements on both sides of the interface were studied in [1]. They obtained asymptotic representations of 3D solutions for an interface layer in the limit of small wavelength to thickness ratio. Gulyayev and Ivanchenko [2] were interested in the problem about dynamic interaction of discontinuous waves with interfaces between anisotropic elastic media. They investigated the formation of reflected and refracted quasi-longitudinal and quasi-shear discontinuous waves, by using a technique based on joint usage of the zero approximation of the ray theory and method of stereomechanical impact. The nonlinear interaction between an acoustic plane wave and an interface formed by two rough, nonconforming surfaces in partial contact is analysed in [3]. The macroscopic elastic properties of such nonlinear interface were derived from micromechanical models accounting for the elastic interaction that is characteristic of spherical bodies in contact.

A conventional sonic composite is a finite size periodic array composed of scatterers embedded in a homogeneous material [4-6]. The existence of a large sound attenuation band is due to the superposition of multiple reflected waves within the array according to the Bragg's theory, for which the band-gaps occur at different frequencies inverse proportional to the central distance between two scatterers. If the band-gaps are not wide enough, their frequency ranges do not overlap. These band-gaps can overlap due to reflections on the surface of the scatterers, as well as due to wave propagation inside them. Then, any wave is reflected completely from this periodic array in the frequency range where all the band-gaps for the different periodical directions overlap. This is the fundamental mechanism for the formation of a full band-gap which is required for sonic composites.

The primary goal of this paper is to analyse the contact interfaces between the scatterers and the matrix in the presence of the frictional slips, in a sonic liner designed to provide suppression of the noise for jet engines.

## 2. Contact and friction forces

Let us consider a liner consisted of a porous or perforated facing sheet forming the interior duct wall, bonded on a composite layer and is terminated with a rigid back wall (Fig.1) [5]. The composite layer is a thin plate consisting of an array of acoustic scatterers embedded in an epoxy matrix. The acoustic scatterers are spheres made from conventional foam and the matrix is made from an epoxy resin (Fig.2). The plate consists of 144 local spherical resonators of diameter  $a$ . The length of the plate is  $L$ , its width is  $d$ , while the diameter of the sphere is  $a$  and its thickness is  $e > a$ . The purpose of this sonic liner is to suppress the noise generated by the fan before it radiates out of the fan inlet and the fan exhaust ducts and in some instances, to reduce the combustion and turbine noise in the exhaust duct of the core engine [7,8].

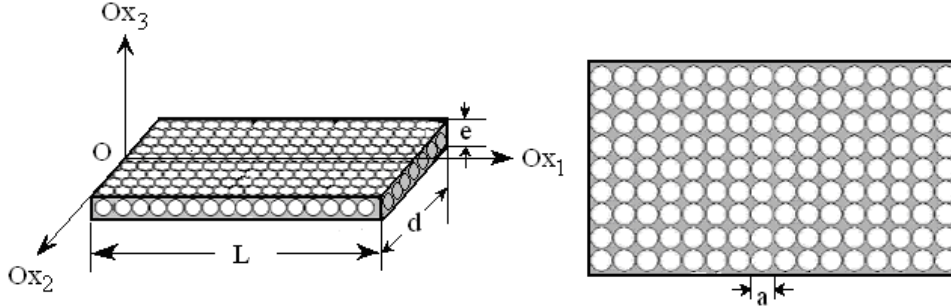


**Figure 1.** The plate with spherical resonators [5]

The general boundary conditions for an elastic-elastic interface are continuity of the displacement vector and the continuity of the normal component of the stress tensor, respectively

$$u^- = u^+, \sum_i \sigma_{ni}^- n_i = \sum_i \sigma_{ni}^+ n_i, \quad (1)$$

where the signs  $\pm$  indicates that the quantities have to be evaluated at both media.



**Figure 2.** Sketch of the composite layer with spherical scatterers [5]

The possibility of having the matching of impedance with the embedded matrix is related to the anti-reflective effects that occur in acoustics, where the transmittance at the interface between the scatterers and matrix will be equal to one, although sound propagates with different speed in each medium.

Let us consider now the contact between a sphere and the matrix. We suppose that the contact interface undergoes the frictional slip. The indentation  $\delta$  is the principal factor in defining the contact force [9]

$$F_c = f(\delta, \dot{\delta}). \quad (2)$$

A particular form of (2) is

$$F_c = k\delta + b\dot{\delta}, \quad (3)$$

where  $k$  and  $b$  are constants depending on the material and geometry [10]. We must say that the indentation  $\delta$  depends on the the length scale  $R$  which is the radius of the sphere. Another particular form of (3) is the Hertz model

$$F_c = k\delta^n, \quad (4)$$

with  $k$  and  $n$  constants depending on the material and geometry.

The friction  $F_t$  occurring at the contact point during sticking can be defined as [11]

$$F_t = k_t \delta_t, \quad (5)$$

where  $\delta_t$  is the tangential component of displacement at the contact point and  $k_t$  is the tangential stiffness. As before, the indentation  $\delta_t$  depends on the length scale  $R$ . In the following we suppose that the contact and friction forces at the interface between the scatterers and matrix are given by (3) and (5), respectively.

### 3. Analysis

At the interfaces between the spheres and the matrix, sharp periodic boundary conditions for the displacement and traction vectors are added in the presence of the (3) and (5).

In order to avoid unphysical reflections from the boundaries of the plate, two simple porous absorbing layers with the flow resistivity  $\sigma_e$  are considered at the ends of the plate. Constructing a non-reflecting boundary condition is still a challenging problem. The flow resistivity of the absorbing layers has a significant role in the modeling and stability of the computational scheme [4]. Depending on the value  $\sigma_e$ , the absorbing boundary conditions may coexist with different patterns of dynamics, including chaos.

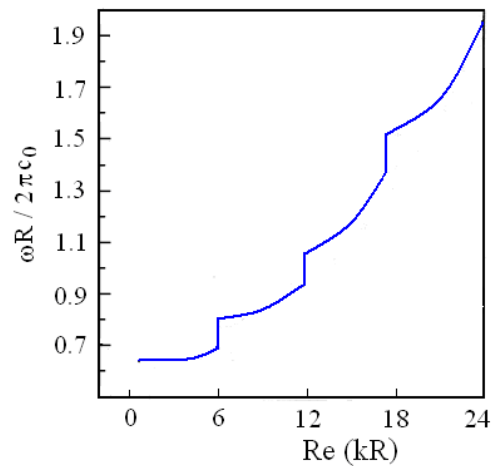
It is easy to observe that reflection is reduced when resistivity is high, but the thickness of the absorbing layer can be small since the wave is quickly damped inside a high resistivity layer. When the resistivity is low, the reflection can also be reduced, but the thickness of the absorbing layer has to be large in order to damp the wave inside the absorbing layer. Otherwise, the remaining wave can reflect at the end of the absorbing layer and still propagates back to the medium domain.

The selection of the values  $\sigma_e$  is made in order to accommodate the requirement of reducing unphysical reflections from the boundaries of the plate with a reasonable absorbing-layer thickness [4]. According to our results, for  $1 < \sigma_e \leq 3.5$ , the problem exhibits stable solutions. The chaotic motion is predicted when  $\sigma_e$  belongs to the intervals  $0 \leq \sigma_e \leq 1$  and  $3.5 < \sigma_e \leq 4.9$ .

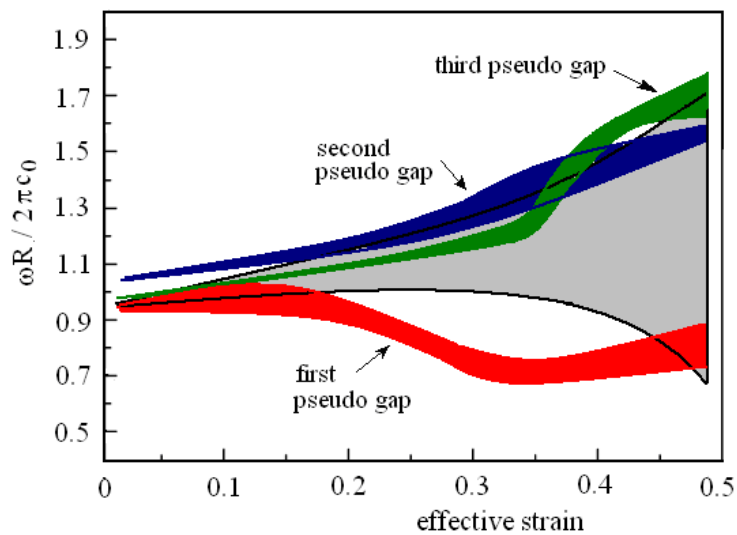
In the acoustic liner the sound attenuation is based on the superposition of multiply reflected waves, and it is essential to calculate a large number of periods of the transmitted wave, in order to obtain the correct transmission coefficient. We consider Featurig of the length scale  $R$  and the structure of the full band-gap can be better understood obtained via representing the linear band structure (dispersion curve). Fig. 3 plots for example the dispersion curve, which includes the first pseudo band-gaps. The reduced units for the frequency is  $\omega R / 2\pi c_0$ , with  $c_0$  the speed of sound in air.

The overlapping of all pseudo gaps obtained from reflections on the scatterers as well as due to wave propagation in the scatterers, generates the full band-gap. Any wave is reflected completely in the frequency range where all the pseudo band-gaps for the different directions overlap. This is the fundamental mechanism

for the formation of a full band-gap. Fig. 4 shows the first, second and third pseudo gaps (red, green and blue zones) delimited by the attenuation peaks lines calculated along four symmetric directions, and the full band-gap (grey zone), with respect to the effective strain.



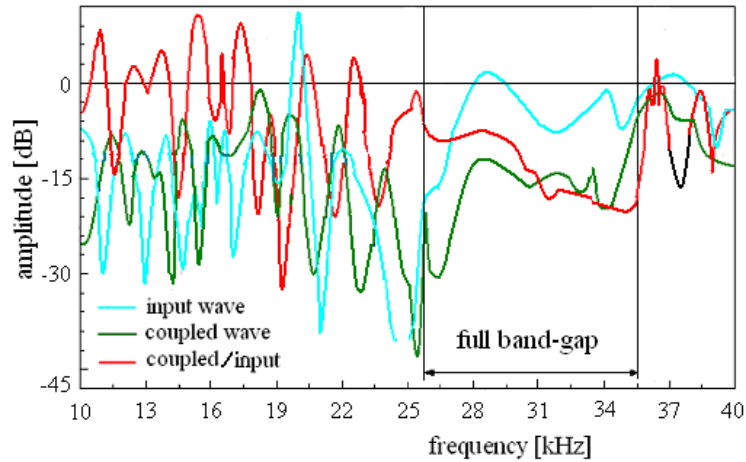
**Figure 3.** Linear dispersion curve.



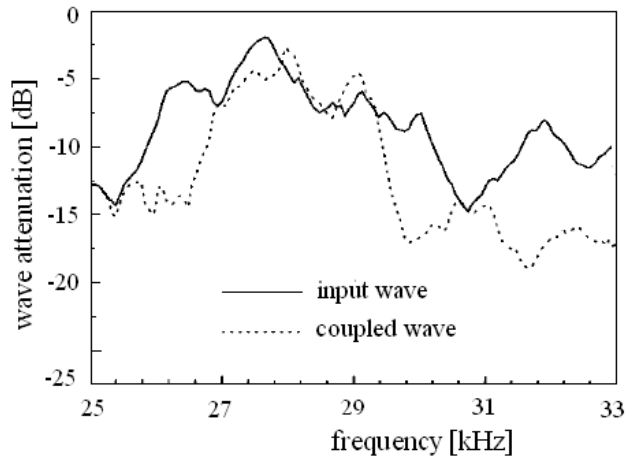
**Figure 4.** The full-band gap structure

The wave simulation in the acoustic liner shows that the guided waves are accompanied by evanescent waves which extend to the periodic array of the

scatterers surrounding the wave-guide. The mode coupling waves is a significant phenomenon which arises between adjacent wave-guides.



**Figure 5.** The input, the coupled waves and the ratio between the coupled and input waves for  $\sigma_e = 2.2\text{kPasm}^{-2}$ .

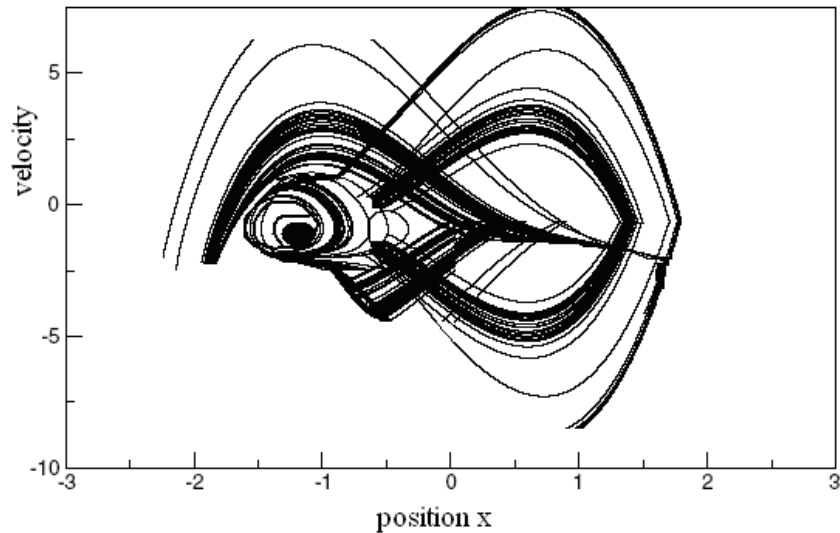


**Figure 6.** The input and coupled waves for  $\sigma_e = 3.5\text{kPasm}^{-2}$ .

The guided waves are accompanied by evanescent waves which extend to the periodic array of the scatterers surrounding the wave-guide. It is strongly expected that mode coupling waves arise between adjacent wave-guides. The output of the coupled modes is compared with the input waves, as shown in Fig. 5, in the case of  $\sigma_e = 2.2\text{kPasm}^{-2}$ . Fig. 5 also display the variation of the ratio between the

coupled and input waves, with respect to frequency. This ratio is important because it is reflecting the way in which the modes of the sound waves excited in the coupled wave-guide grow cumulatively traveling in the composite.

Fig. 6 shows the input and coupled waves at the stability limit value of  $\sigma_e = 3.5 \text{ kPa s m}^{-2}$ . We see in Fig. 7. that, depending on the value of  $\sigma_e$ , the absorbing boundary conditions may coexist with different patterns of dynamics, including chaos.



**Figure 7.** Chaotic phase portrait of sonic composite for  $3.5 < \sigma_e \leq 4.5$ .

#### 4. Conclusion

An important problem in the design of the acoustic liners for jet engines is that the characteristic noise sources and flow field in the engine's duct vary widely over the operating range of the engine. The purpose of these liners is to how more can absorb the noise generated by the fan before it radiates out of the fan inlet and the fan exhaust ducts. This composite consists of an array of acoustic scatterers having the shape of spherical shells and made of conventional foam embedded into the epoxy matrix. The paper analyses the contact interfaces between the scatterers and the matrix in the presence of the frictional slips.

The result is the understanding of the influence of the boundary conditions on the the wave motion into the sonic composite and a clear improvement of the noise suppression for a wide full band-gap of frequencies. The above results on the mode coupling between the wave-guides in a sonic liner showed that this composite is a promising platform for new acoustic integrated devices.

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