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A Continuous Approach of the Contact Dynamics

Ther paper is devoted to the analysis of a sonic composite under dynamic contact with friction loading, by using LISA (local interaction simulation approach). LISA is an an efficient tool for the numerical simulation of the acoustic wave propagation in heterogeneous material specimens, in particular those with sharp boundaries between different materials, like in sonic composites. Boundary conditions are introduced to contain contact interfaces with frictional slips.

Keywords: sonic composite, auxetic foams, contact, friction, LISA.

1. Introduction

The sonic composites subjected to dynamic contact with friction loading is a multiscale problem because of the particular structure of the material and the multiple frictional contacts. The dynamics of contact depends on many properties of contacting bodies such as material, geometry and velocity.

The continuous approach of the contact dynamics has several advantages over the discrete formulation, because it does not require differentiating between impact and contact situations and permits the use of solution methods employed for non-impact dynamics problems [1].

Sonic composites are artificial structures consisting of a periodic array of acoustic scatterers embedded in a homogeneous matrix material, with a usually large impedance mismatch between the two materials. They exhibit strong sound attenuation at selective frequency bands due to the interference of multiply reflected waves [2].

The behavior of the sonic composites under dynamic contact with friction loading is analysed in this paper by using LISA (local interaction simulation approach). LISA is based on the Finite Difference Equations (FDE) with sufficiently small spatial and temporal discretization steps in order to obtain numerical stability of the algorithm and to reproduce reasonably the shape of the scatterers.

The sonic plate is composed of an array of acoustic scatterers which are piezoceramic hollow spheres embedded in an epoxy matrix [3]. The scatterers are made from functionally graded materials with radial polarization, which support the Reddy and cosine laws [4-6]. The proposed approach is based on the theory of piezoelectrics. For a single sphere made from a functionally graded material, the free vibration problem was analyzed in [7, 8].

We suppose that the interfaces in the composite simultaneously undergo the frictional slip and vibro mechanisms. The indentation δ is the principal factor in defining the contact force [9]

$$F_c = f(\delta, \dot{\delta}) . \tag{1}$$

An explicit form of (1) is

$$F_c = k\delta + b\dot{\delta} , \tag{2}$$

with k and b constants depending on the material and geometry [1]. This model has some limitations. Firstly, the contact force at the start of the impact is discontinuous, due to the damping term. When the contacting bodies are separating when the indentation is tending to zero, their relative velocity tends to be negative. As a result, a negative force holding the objects together is present.

Another form of (1) is the Hertz model

$$F_c = k\delta^n , \tag{3}$$

with k and n constants depending on the material and geometry. In this model, $e=1$, because the dissipation energy is not present. However, this model can be used only for low impact speeds and hard materials.

Another version for (1) is reported in [10]

$$F_c = k\delta^n + b\delta^p\dot{\delta}^q , \tag{4}$$

where n, p, q are constants, coefficient k depends on the material and the geometric properties of the bodies in contact, and b is defined with respect to the coefficient of restitution $0 \le e \le 1$. These coefficients are calculated based on the viscoelastic theory. For example, $n = 3/2$ in the case of two spheres in central impact and k is defined in terms of Poisson's ratios, Young's moduli and the radii of the two spheres. The standard values are $p = n$ and $q = 1$. In the case of central impact between two bodies, the coefficient of restitution is $e = 1 - 2b\dot{\delta}_0 / 3k$ [1].

The friction F_t occurring at the contact point during sticking can be defined as [11]

$$F_t = k_t\delta_t , \tag{5}$$

where δ_t is the tangential component of displacement at the contact point and k_t is the tangential stiffness. As before, the indentation δ_t depends on the the length scale R.

In the following we suppose that the contact and friction forces at the interface between the scatterers and matrix are given by (4) and (5), respectively.

2. Formulation of the problem

The sonic composite consists of an array of acoustic scatterers embedded in an epoxy matrix. The acoustic scatterers are hollow spheres made from piezoelectric ceramic, while the matrix is made from the epoxy resin (Fig. 1). the transversal view of a scatterer is presented in Fig.2. The sonic plate consists of 72 scatterers of diameter a. The length of the plate is l, its width is d, while the diameter of the hollow sphere is a and its thickness is $e > a$.

A sonic composite exhibits the full band-gaps, where the sound is not allowed to propagate due to complete reflections. The band-gaps or the well-known Bragg reflections occur at different frequencies inverse proportional to the central distance between two scatterers. If the band-gaps are not wide enough, their frequency ranges do not overlap. These band-gaps can overlap due to reflections on the surface of thick scatterers, as well as due to wave propagation inside them [12, 13].

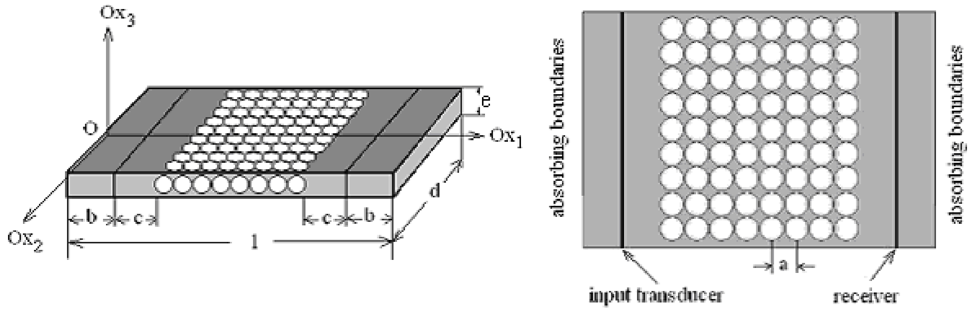

Figure 1. Sketch of the sonic plate [3]

Consider now two piezoceramic hollow spheres with the ratio of the inner and outer radii ξ_0. Two laws represent the functionally graded property of the material. The first one is the Reddy law [4-6] given by

$$M = M_p \mu^{\lambda} + M_z (1 - \mu^{\lambda}), \tag{6}$$

where μ is the gradient index [8], M_p and M_z are material constants of two materials, namely PZT-4 and ZnO. The case $\mu = 0$ corresponds to a homogeneous

PZT-4 hollow sphere and $\mu \to \infty$, to a homogeneous ZnO hollow sphere. The second law is expressed as

$$M = M_p \cos\mu + M_z (1 - \cos\mu) . \quad (7)$$

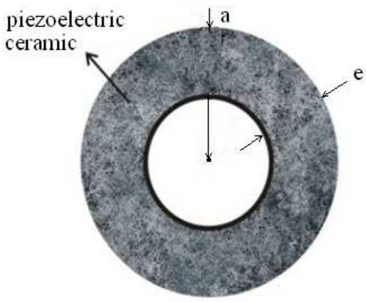

Figure 2. Transversal view of a scatterer.

The constitutive equations for the piezoelectric hollow sphere are given by [3]

$$\Sigma_{\theta\theta} = r\sigma_{\theta\theta} = C_{11}S_{\theta\theta} + C_{12}S_{\varphi\varphi} + C_{13}S_{rr} + f_{31}r\phi_{r} ,$$

$$\Sigma_{\varphi\varphi} = r\sigma_{\varphi\varphi} = C_{12}S_{\theta\theta} + C_{11}S_{\varphi\varphi} + C_{13}S_{rr} + f_{31}r\phi_{,r} ,$$

$$\Sigma_{rr} = r\sigma_{rr} = C_{13}S_{\theta\theta} + C_{13}S_{\varphi\varphi} + C_{33}S_{rr} + f_{33}r\phi_{,r} ,$$

$$\Sigma_{r\theta} = r\sigma_{r\theta} = 2C_{44}S_{r\theta} + f_{15}\phi_{,\theta} , r\sigma_{r\varphi} = 2C_{44}S_{r\varphi} + f_{15}\csc\theta\phi_{,\varphi} , \quad (8)$$

$$\Sigma_{\theta\varphi} = r\sigma_{\theta\varphi} = 2C_{66}S_{\theta\varphi}, \;\; \Lambda_{\theta} = rD_{\theta} = 2C_{15}S_{r\theta} - \zeta_{11}\phi_{,\theta} ,$$

$$\Lambda_{\varphi} = rD_{\varphi} = 2 f_{15}S_{r\varphi} - \zeta_{11}\csc\theta\phi_{,\varphi}, \;\; \Lambda_{r} = rD_{r} = f_{31}S_{\theta\theta} + f_{31}S_{\varphi\varphi} + f_{33}S_{rr} - \zeta_{33}r\phi_{,r} ,$$

where σ_{ij} is the stress tensor, ϕ is the electric potential, D_i is the electric displacement vector, C_{ij} are the elastic constants, f_{ij} are the piezoelectric constants f_{ij}, ζ_{ij} are the dielectric constants, and $i = r, \theta, \varphi$. The elastic, piezoelectric and dielectric constants are arbitrary functions of the radial coordinate r. On denoting the components of the strain tensor and displacement vector by ε_{ij} and u_i, $i = r, \theta, \varphi$, respectively, the quantities S_{ij} related to the strain tensor ε_{ij} are defined as [3]

$$S_{rr} = r\varepsilon_{rr} = ru_{r,r} , \quad S_{\theta\theta} = r\varepsilon_{\theta\theta} = u_{\theta,\theta} + u_r ,$$

$$S_{\varphi\varphi} = r\varepsilon_{\varphi\varphi} = \csc\theta u_{\varphi,\varphi} + u_r + u_{\theta}\cot\theta , 2S_{r\theta} = 2r\varepsilon_{r\theta} = u_{r,\theta} + ru_{\theta,r} - u_{\theta} , \quad (9)$$

$$2S_{r\varphi} = 2r\varepsilon_{r\varphi} = \csc\theta u_{r,\varphi} + ru_{\varphi,r} - u_{\varphi}, \quad 2S_{\theta\varphi} = 2r\varepsilon_{\theta\varphi} = \csc\theta u_{\theta,\varphi} + u_{\varphi,\theta} - u_{\varphi}\cot\theta .$$

Denoting the density of the material by ρ, which is assumed to be an arbitrary function of r, the equations of motion become

$$r\Sigma_{r\theta,r} + \csc\theta\Sigma_{\varphi\theta,\varphi} + \Sigma_{\theta\theta,\theta} + 2\Sigma_{r\theta} + (\Sigma_{\theta\theta} - \Sigma_{\varphi\varphi})\cot\theta = \rho r^2 \ddot{u}_{\theta},$$

$$r\Sigma_{r\varphi,r} + \csc\theta\Sigma_{\varphi\varphi,\varphi} + \Sigma_{\theta\varphi,\theta} + 2\Sigma_{r\varphi} + 2\Sigma_{\theta\varphi}\cot\theta = \rho r^2 \ddot{u}_{\varphi}, \tag{10}$$

$$r\Sigma_{rr,r} + \csc\theta\Sigma_{r\varphi,\varphi} + \Sigma_{r\theta,\theta} + \Sigma_{rr} - \Sigma_{\theta\theta} - \Sigma_{\varphi\varphi} + \Sigma_{r\theta}\cot\theta = \rho r^2 \ddot{u}_{r} .$$

The charge equation of electrostatics is given by

$$r\Lambda_{r,r} + \Lambda_r + \csc\theta(\Lambda_{\theta}\sin\theta)_{,\theta} + \csc\theta\Lambda_{\varphi,\varphi} = 0 . \tag{11}$$

The boundary conditions are given for the radial and tangential stresses at the interface between the scatterer and the matrix

$$\sigma_r = F_c(t), \ \sigma_t = F_t(t). \tag{12}$$

3. Results

Consider a plate with the length $l = 18\text{cm}$ and width $d = 11\text{cm}$, while the diameter of the hollow sphere and its thickness are $a = 10.5\text{mm}$ and $e = 12\text{mm}$, respectively, and $\xi_0 = 0.3$. The numerical results are carried out for the following:

for PZT-4: $C_{11} = 13.9\times10^{10}\,\text{N/m}^2$, $C_{12} = 7.8\times10^{10}\,\text{N/m}^2$,
$C_{13} = 7.4\times10^{10}\,\text{N/m}^2$, $C_{33} = 11.5\times10^{10}\,\text{N/m}^2$, $C_{44} = 2.56\times10^{10}\,\text{N/m}^2$,
$f_{15} = 12.7\text{C/m}^2$, $f_{31} = -5.2\text{C/m}^2$,
$f_{33} = 15.1\text{C/m}^2$, $\zeta_{11} = 650\times10^{-11}\,\text{F/m}$, $\zeta_{33} = 560\times10^{-11}\,\text{F/m}$, $\rho = 7500\text{kg/m}^3$,
for ZnO: $C_{11} = 20.97\times10^{10}\,\text{N/m}^2$, $C_{12} = 12.11\times10^{10}\,\text{N/m}^2$,
$C_{13} = 10.51\times10^{10}\,\text{N/m}^2$, $C_{33} = 21.09\times10^{10}\,\text{N/m}^2$, $C_{44} = 4.25\times10^{10}\,\text{N/m}^2$,
$f_{15} = -0.59\text{C/m}^2$, $f_{31} = -0.61\text{C/m}^2$, $f_{33} = 1.14\text{C/m}^2$, $\zeta_{11} = 7.38\times10^{-11}\,\text{F/m}$,
$\zeta_{33} = 7.83\times10^{-11}\,\text{F/m}$, $\rho = 5676\text{kg/m}^3$,
and for epoxy-resin: $\lambda^e = 42.31\times10^9\,\text{N/m}^2$, $\mu^e = 3.76\times10^9\,\text{N/m}^2$,
$A^e = 2.8\times10^9\,\text{N/m}^2$, $B^e = 9.7\times10^9\,\text{N/m}^2$, $C^e = -5.7\times10^9\,\text{N/m}^2$, and
$\rho^e = 1170\,\text{kg/m}^3$.

The motion equations (10) and (11) yield two independent classes of free vibrations. The first class does not involve the piezoelectric or dielectric parameters, being identical to the one for the corresponding spherically isotropic elastic sphere.

The second class depends on the piezoelectric or dielectric parameters. With the increase of the gradient index μ, the natural frequencies increase for all modes and functionally graded laws, the variation being more significant when $\mu \leq 10$.

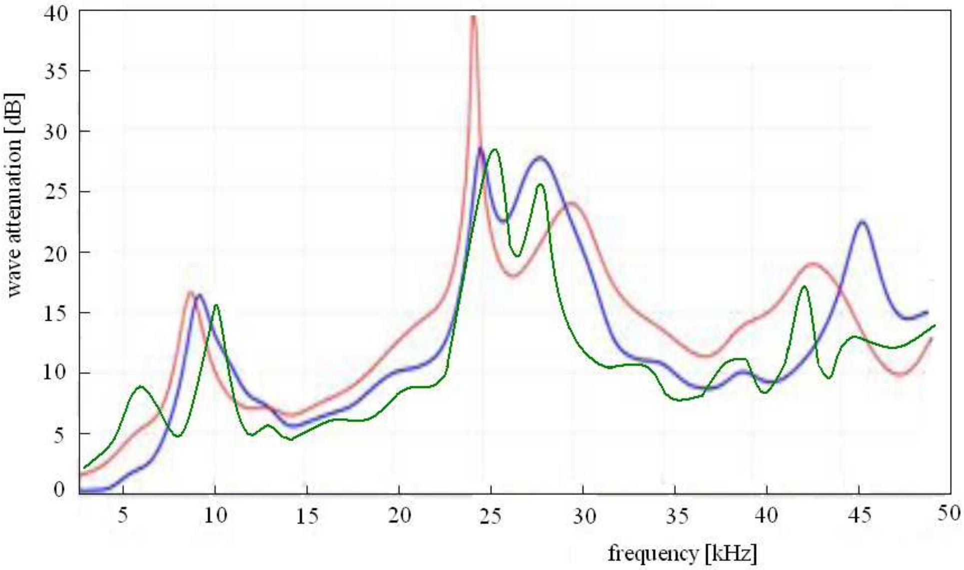

Figure 3. Sound transmission through the sonic composite for the Reddy and cosine graded laws.

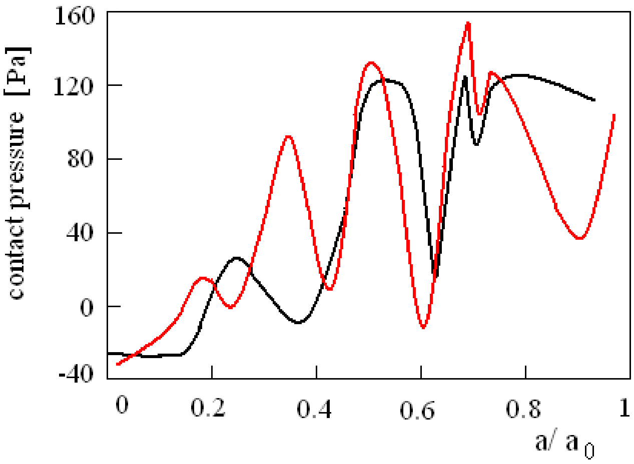

Figure 4. The maximum contact pressure at the interface.

For $\mu \to \infty$ the variation of natural frequencies is not significant with respect to those of $\mu = 10$. It is seen that for a piezoceramic hollow sphere, the piezoelectric effect consists of increasing the values for the natural frequencies in both classes of vibrations. If $\xi = 2r/a$ increases the natural frequencies increase for the first class of vibrations and decrease for the second class.

Fig. 3 shows the wave attenuation variation with respect to frequency for Reddy (red line) and cosine (blue line) laws, in the presence of the boundary conditions (12). The green line represents the Reddy law without absorbing boundary conditions (12).

The maximum contact pressure at the interface is presented in Fig.4 for Reddy (red line) and cosine (black line) laws.

4. Conclusion

Ther paper analyses a sonic composite under dynamic contact with friction loading, by using LISA. The identification of the boundary conditions to take into consideration the deformations in normal and tangential directions is important because the micro-vibrations and frictional slip at the interfaces between the scatterers and the matrix are real observations put into evidence by the experience.

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