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Modal Analysis of Thin Plates with Damage Simply Supported on all Edges

This paper presents the natural frequencies obtained by modal analysis for five types of rectangular thin plates. These plates are simple supported on all edges, and are without damage, with a lateral damage and with a damage placed in the in center, in both cases two types of damage being used. We compared the results and highlighting the way how natural frequencies changes due to damage.

Keywords: *thin plates, natural frequency, vibration, damage*

1. Introduction

A plate is a solid with length and width relatively large compared to the thickness; it is characterized by: the shape, the size of the median area and the thickness [1]. In this paper the frequencies of five plates is analyzed by means of simulation in *SolidWorks* [2, 3], namely: plate without damage, with lateral damage of 250x5 mm, with lateral damage of 5x5 mm, with damage in the center of 250x5 mm and with damage in the center of 5x5 mm. All damages have a depth of 1 mm; the lateral damages are placed at distance 20 mm from the longer plate edge and 10 mm from the shorter beam edge (figure 1). While damages change the natural frequencies of beams or plates [4], we conducted experiments to find out the sensitivity of frequency change due damage for the above presented cases. In this work we determined of the natural frequency [5] for a number of 16 vibration modes.

2. The 3D modeling and simulation

For modeling the plates in the 5 analyzed cases, was considered a thin plate with dimensions of 1000x500x2 mm [5]. The material related from SolidWorks library is the AISI 316 austenitic stainless steel.

This material with numeric symbols: 1.4401 and alphanumeric symbols: X5CrNiMo17-12-2 has the mechanical properties shown in Table 1.

Table 1.

Yield strength	Tensile strength	Mass density	Elastic modulus	Poisson's ratio	Thermal expansion coefficient
[N/mm ²]	[N/mm ²]	[kg/m ³]	[N/mm ²]	[-]	[K ⁻¹]
172,369	580	8.000	193.000	0,27	0,000016

After creating the plate geometry and choosing the material, other four stages are necessary: the application of restrictions, the meshing [6, 7], the modal analysis calculation and the results view [8, 9]. Plates were fixed by simply supporting the four contour edges and the mesh was made by the "Solid mesh" pyramid elements (with 4 characteristic points - Jacobian points), the maximum size of one side being 4 mm. Table 2 present the mesh data for each plate analyzed, in terms of node and element number. Figures 2 to 4 show the first 16 vibration modes for the undamaged plate; for the damaged plates the mode shapes do not change significantly, while the natural frequencies are subjected to important changes.

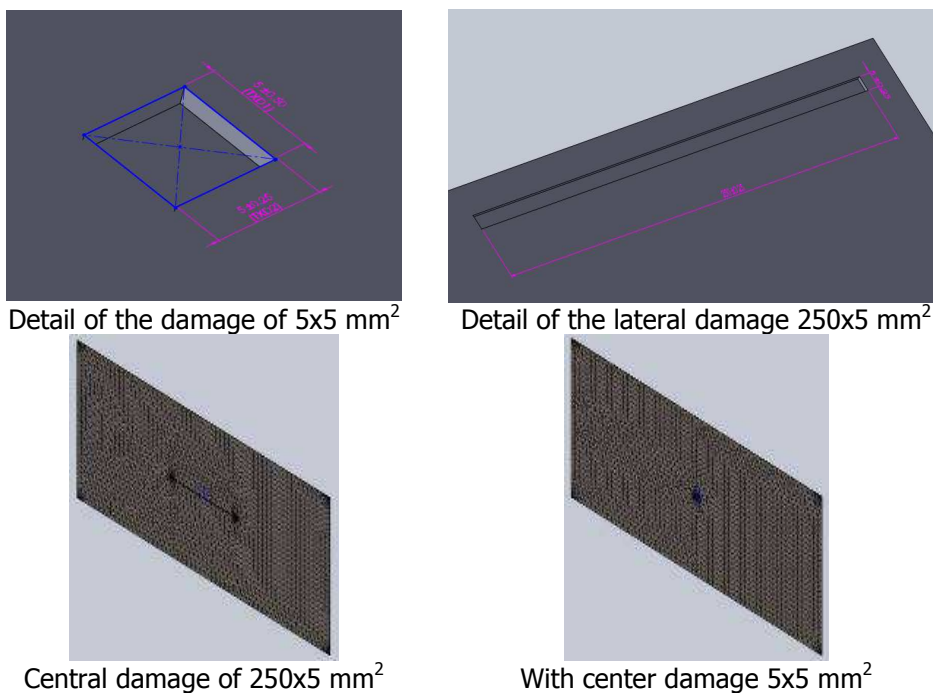
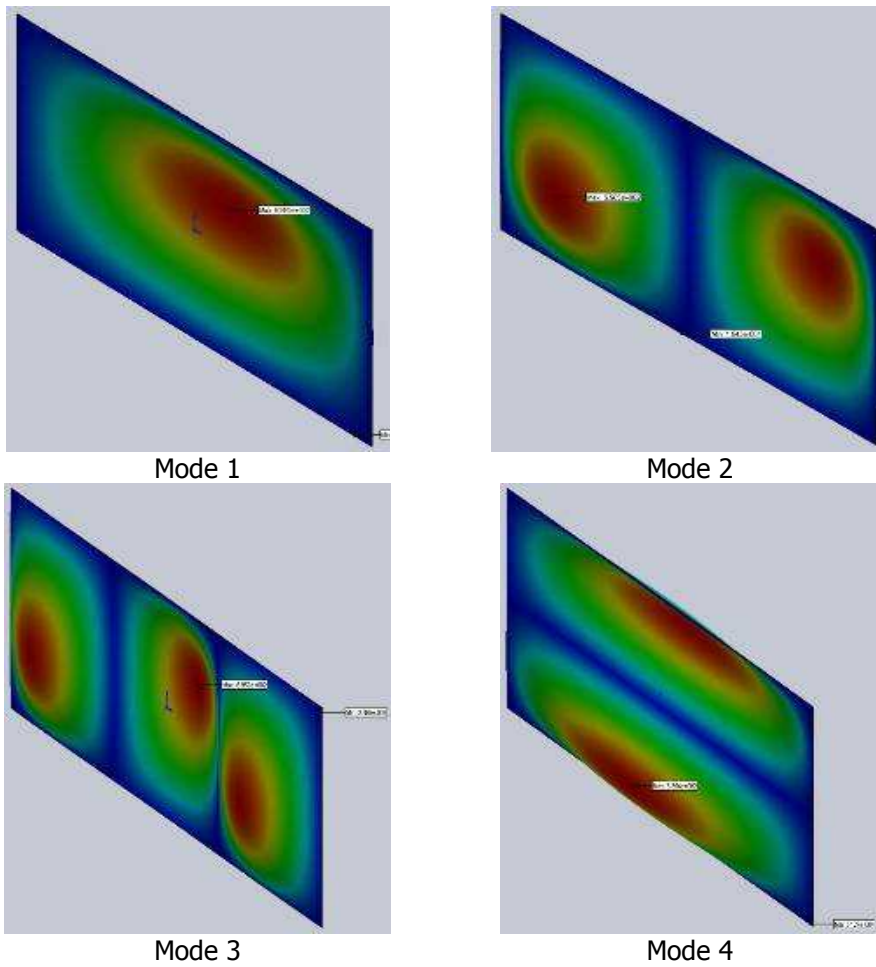
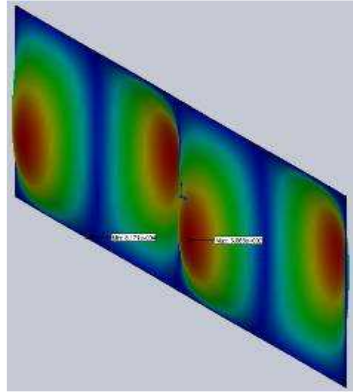


Figure 1. The mesh for analyzed plates.

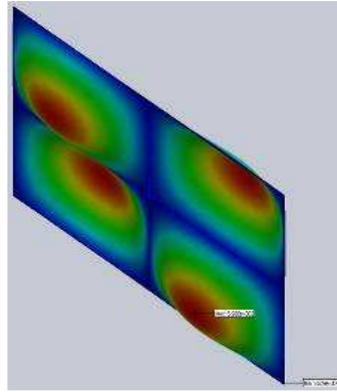
Table 2.

Plate type	Total Nodes	Total Elements	Maximum Aspect Ratio	Time to complete mesh [s]
Without damage	30.531	17.558	83,911	17
Laterally damage 250x5	36.808	19.698	40,821	15
Laterally damage 5x5	38.872	22.120	75,497	19
Center damage 250x5	28.863	15.039	64,127	11
Center damage 5x5	41.505	24.073	78,378	25

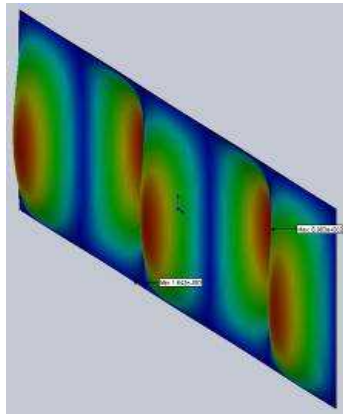
**Figure 2.** First four vibration modes of plate without defect.



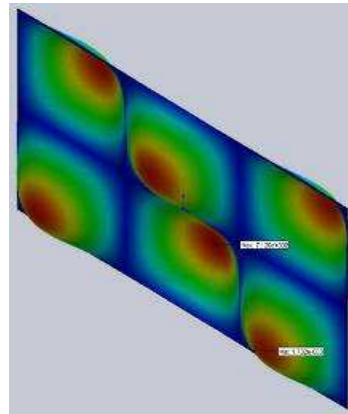
Mode 5



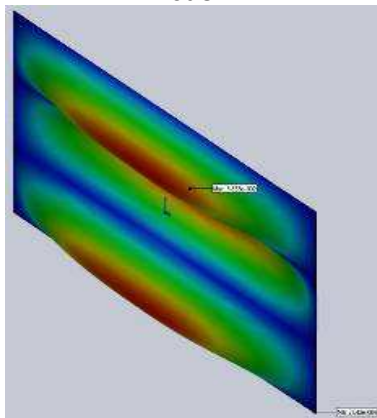
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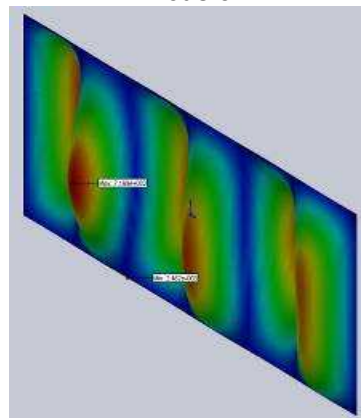
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Mode 8

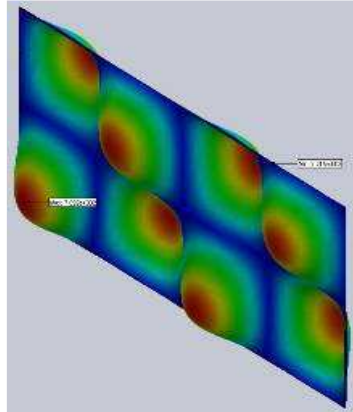


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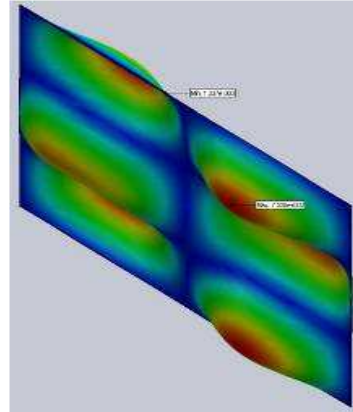


Mode 10

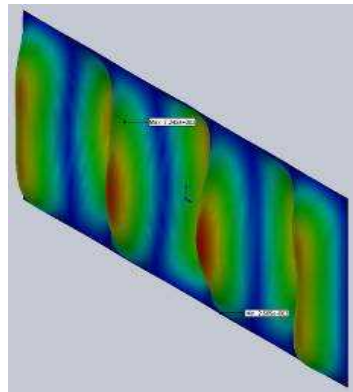
Figure 3. Vibration modes five to ten of the plate without defect.



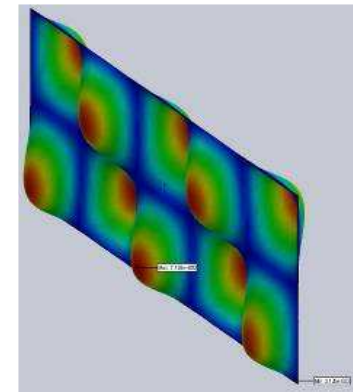
Mode 11



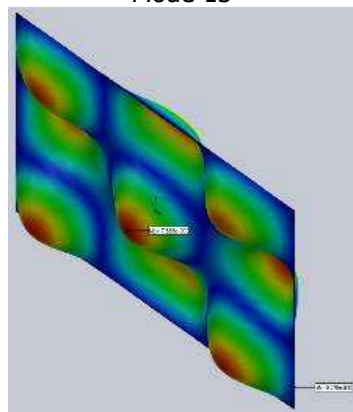
Mode 12



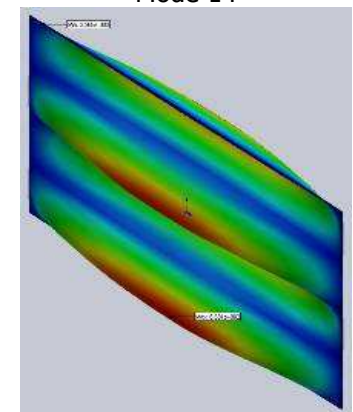
Mode 13



Mode 14



Mode 15



Mode 16

Figure 4. Vibration modes eleven to sixteen of the plate without defect.

3. Simulations and results

After the numerical simulation, for the natural frequencies of the first 16 vibration modes, we obtained results that are presented in Table 3 and represented graphically in Figures 5 and 6.

Table 3.

Mode Number	Calculated frequency values for the 5 plate types				
	Without damage	With lateral damage 250x5	With lateral damage 5x5	With central damage 250x5	With central damage 5x5
M1	4.323998	3.60266	4.03385	3.848479	4.323998
M2	7.781096	6.188377	7.086651	6.868601	7.781096
M3	8.76251	6.78665	7.103074	6.707902	8.76251
M4	4.83763	4.007282	3.650244	3.623231	4.83763
M5	8.787569	6.262213	6.829813	5.95515	8.787569
M6	11.66477	9.1339	10.69271	10.25484	11.66477
M7	14.69737	10.72876	13.69396	11.7579	14.69737
M8	10.50973	7.901203	6.735555	7.182912	10.50973
M9	4.664502	3.506494	2.949134	3.073593	4.664502
M10	17.20279	12.69373	15.6389	15.08639	17.20279
M11	9.128419	6.012981	4.904961	5.24803	9.128419
M12	11.5455	8.555614	10.47333	10.28156	11.5455
M13	18.27824	12.95386	17.02316	15.60191	18.27824
M14	14.57387	9.819618	12.75956	10.72153	14.57387
M15	12.62981	9.752983	7.692825	8.462443	12.62981
M16	8.41607	3.467249	7.896597	5.621984	8.41607

To determine the relative frequency shift Δf_i^* of mode i , the relation below is used [10]:

$$\Delta f_i^* = \frac{\Delta f_i}{f_i^U} = \frac{f_i^U - f_i^D}{f_i^U} \quad (1)$$

where f_i^U and f_i^D are the natural frequencies of mode i for the healthy and damaged case respectively. To be expressed in percents, relation (1) has to be multiplied by 100.

Table 4 presents the values of the relative frequency shift for the four damage cases, while figure 7 presents those values in a graphical manner.

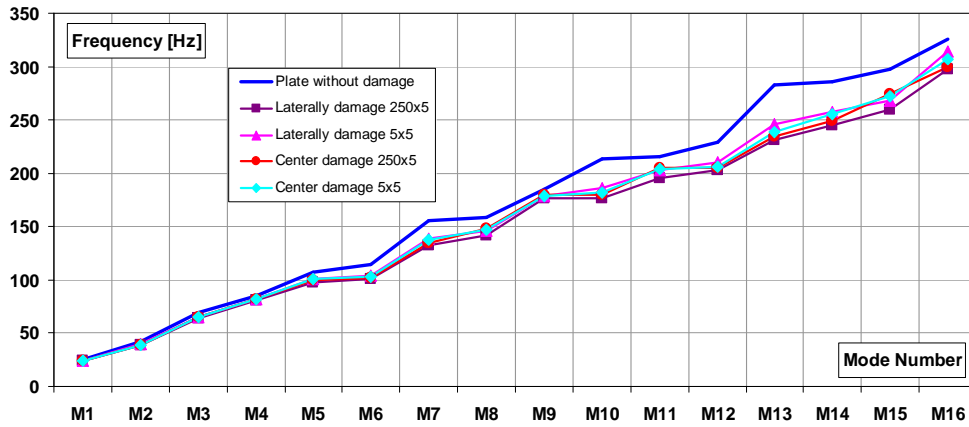


Figure 5. Natural frequencies vs. the mode number for the healthy plate and the four damage scenarios.

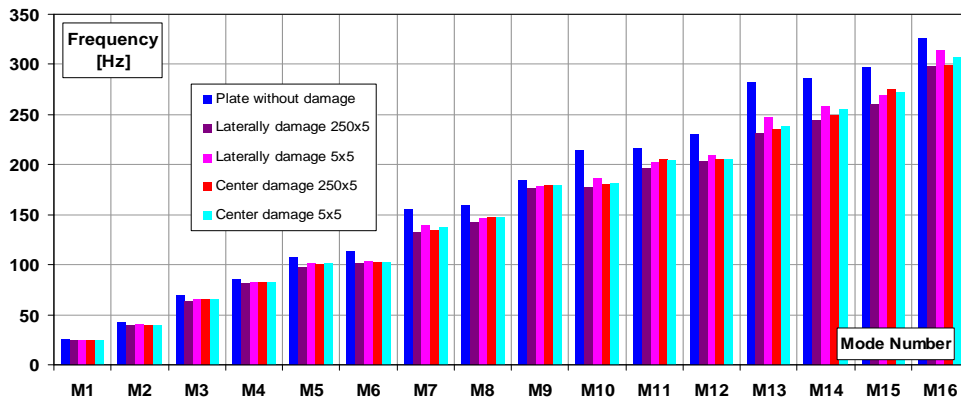


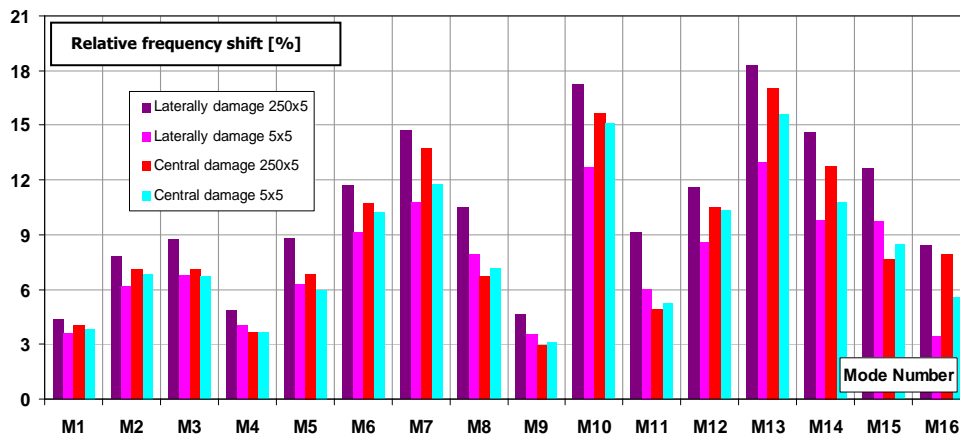
Figure 6. Comparison of calculated frequency variation for each mode number of the 5 types of analyzed plates.

It is obvious that for some modes the frequency shifts are insignificant, while for other modes the frequency changes are significant. The shift is produced by the decrease of stored energy for the given mode, dependent on the damage position on the plate [10]:

- for damages placed on the inflection points of the mode shapes the frequency change is null
- for damages placed on points of local extrema of the mode shapes, the frequency change attend highest local values.

Table 4.

Mode Number	Relative frequency shifts for the 4 damage scenarios [%]			
	With lateral damage 250x5	With lateral damage 5x5	With central damage 250x5	With central damage 5x5
M1	4.323998	3.60266	4.03385	3.848479
M2	7.781096	6.188377	7.086651	6.868601
M3	8.76251	6.78665	7.103074	6.707902
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M13	18.27824	12.95386	17.02316	15.60191
M14	14.57387	9.819618	12.75956	10.72153
M15	12.62981	9.752983	7.692825	8.462443
M16	8.41607	3.467249	7.896597	5.621984

**Figure 7.** Relative frequency shift for the 16 modes of the analyzed damaged plates.

4. Conclusion

The modal analysis, i.e. determination of natural frequencies, leads us to following conclusions:

- the highest frequency values for all vibration modes belong to the undamaged plate;
- the damage influence upon changes in natural frequencies is different for different vibration modes, depending on the strain energy loss due damage in the affected region – it is proportional to the bending moment, i.e. mode shape curvature square, similar to the case of beams;
- not in all cases the bigger damage produced the highest frequency changes, meaning that the mass loss influence is to be considered for accurate analysis;
- for all analyzed bending vibration modes the lateral damage with dimensions 250x5x1 mm³ produced the highest frequency changes.

To improve the results' precision and increase the trust level, a finer mesh is necessary. However, the results obtained show that natural frequency changes of plates are sensitive to the damage, encouraging us to continue developing a reliable damage detection method applicable to plate structures.

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