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Over-And Understeer Behaviour Evaluation by Modelling Steady-State Cornering

This paper gives an overview of a vehicle cornering in a constant cornering radius, with constant speed. These constant values indicate steady state cornering or not changing its motion character over time. Even if the steering angle δ starting from '0' (straight line driving) is reaching the constant value or the constant cornering radius, the vehicle is not following the desired path or evident is the under-steer or over-steer condition. As presented we can see the vehicle cornering, and it's offset from the neutral driven vehicle. The vehicle is driving trough corner with greater radius. This means that the vehicle is understeered. Neutral driven vehicle is the desired arc which can be followed by vehicle with neutral steer which means that when accelerating the driver doesn't have to change the angle of the steering wheel.

Keywords: *lateral forces, side slip angle, under-steer, over-steer, cornering stiffness.*

1. Introduction

With the growing demand for motor vehicle transportation today's car industry has aims that have to be fulfilled. As competition raises one of the parameters making the vehicle value is knowing the ratio of what is offered to the customer and the economical price of the car. Better and safer transportation, environmental protection, sustainable development of the vehicles has stimulated new interest in the resarch and development of the car's production.

Vehicle safety is one of most important tasks, and needs to be examined in detail. Main task of this paper is to see the respond of the car while cornering as well as the parameters that have influence on it. Handling of the car will be evaluated by the yaw angle (under- and oversteer behaviour) which can be the main reason for numerous accidents. That can lead up to automotive systems that can prevent the collision or reduce the consequences for the driver and the passengers, those systems are nowadays in a widespread use.

The vehicle used is modeled as two wheeled vehicle (bicycle model). The cornering is steady - state, doesn't change its characteristics over time like the vehicles longitudinal speed and cornering radius for the time duration.

1. 1. Analytical approach

a) Modeling of a steady-state cornering

Demonstrating the vehicle as a bicycle model, to study steady-state cornering properties of a vehicle (vx = const.)

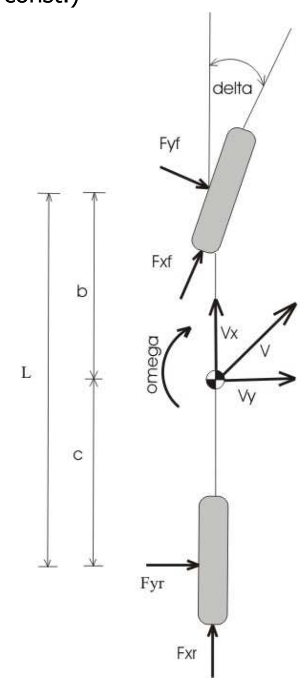

Figure 1. Free body diagram of a bicycle model

- Equations of motion

The centrifugal force acting at the vehicles COG during cornering is moving it away from the centre of rotation and it has to be balanced by the tire/ground lateral forces [1]:

$$Fc = m \cdot R \cdot \Omega^2 = m \cdot Vx^2 / R,$$

Equations of equilibrium: $F_{yf} + F_{yr} = Fc = m \cdot Vx^2 / R,$ and

$$F_{yf} \cdot b - F_{yr} \cdot c = 0,$$

Constitutive equations: $F_{yf} = C_{\alpha f} \cdot \alpha_f$ and $F_{yr} = C_{\alpha r} \cdot \alpha_r,$

Compatibility: $\tan(\delta - \alpha_f) = (b \cdot \Omega + Vy)/Vx,$ and

$$\tan(\alpha_r) = (c \cdot \Omega - Vy)/Vx,$$

Eliminating Vy for small angles and using Vx=R · Ω [2]: $\delta - \alpha_f + \alpha_r = L/R$,

Eliminating slip angles:

$F_{yf} = (Ir/L) \cdot m \cdot Vx^2 / R$ and $F_{yr} = (If/L) \cdot m \cdot Vx^2 / R,$

$$\delta - F_{yf} / C_{\alpha f} + F_{yr} / C_{\alpha r} = L / R,$$

Eliminating lateral forces:

$$d = L / R + \left[(Ir / L) / C_{\alpha f} - (If / L) / C_{\alpha r}\right] \cdot m \cdot Vx^2 / R, \qquad (1)$$

a) Understeer coefficient

Equation (1) can also be expressed as:

$$\delta = L / R + \left[Wf / C_{\alpha f} - Wr / C_{\alpha r}\right] \cdot Vx^2 / (g \cdot R),$$

- Wf and Wr are vertical weight load at each axle

$Kus = \dfrac{Wf}{C_{\alpha f}} - \dfrac{Wr}{C_{\alpha r}},$ and is called understeer coefficient [3]

Using 'Saab 9-3' data file we can calculate the understeer coef.

m = 1675 [kg]   Curb weight (full tank, no driver or pass.)

L = 2.675 [m]   Wheel base

lf = 1.070 [m]   Distance along X-axis from CoG to front axle

lr = 1.605 [m]   Distance along X-axis from CoG to rear axle

Wf = m · g · lr/L = 1675 · 9.81 · 1.605/2.675 = 9859.05[N]

Wr = m · g · lf/L = 1675 · 9.81 · 1.070/2.675 = 6572.7 [N]

Caf=9.3e4 [N/rad]   Cornering stiffness / tire at front axle load

Car=7.5e4 [N/rad]   Cornering stiffness / tire at rear axle load

Kus = Wf/(2 · Caf) – Wr/(2 · Car) = 9859.05/(2 · 9.3e4) – 6572.7/(2 · 7.5e4) =
=0.053 – 0.047 = 0.009,

- Using understeer coef. we can simplify eq. (1) which represents the relationship between steer angle (δ), speed (v) and cornering radius (R), to:

$$\delta = L / R + Kus \cdot Vx^2 / (g \cdot R), \qquad (2)$$

- Modifications for a neutral steered vehicle

Kus = 0   Wf/Caf – Wr/Car = 0   =>   Wf/Caf = Wr/Car   =>

Caf/Car = Wr/Wf.

Caf = 0.66 · Caf

for Caf = 9.3e4 [N/rad]   =>   Car = 13.9e4 [N/rad]

Cornering radius, **R = 50 [m]**

Vx = 40 [km/h] = 11.11 [m/s]

for a neural steered vehicle: δ = L/R = 2.675/50 = 0.0535 [°]

for the Saab 9-3 data: $\delta = L/R + Kus \cdot Vx^2/(g \cdot R)$ =>

$R = L/[\delta - Kus \cdot Vx^2/(g \cdot R)]$

$R = 2.675/[0.0535 - 0.009 \cdot 11.11^2/(9.81 \cdot 50)] = 52.21$ [m]

Cornering radius, **R = 52.21 [m]**

We can see that the cornering radius for a 'understeered' vehicle is greater that the one for a 'neutral steered' vehicle with the same specified inputs.

2. Results and Discussion

Comparison of under- and oversteer vehicles

- Original specifications for Saab 9-3

$C\alpha f$=9.3e4 [N/rad] Cornering stiffness per tire at front axle load

$C\alpha r$=7.5e4 [N/rad] Cornering stiffness per tire at rear axle load

- Changes to get the oversteered vehicle

$C\alpha f$=9.3e4 [N/rad] Changed value of $C\alpha f$

$C\alpha r$=5.5e4 [N/rad] Changed value of $C\alpha r$

By changing the Cornering stiffness of the tires we change the side slip angles. This can lead up to different response of the vehicle during cornering, different lateral forces that recive a yaw moment. If rear tires side slip angle is greater then the front ones the vehicle will be 'oversteered', if it's smaller it will be 'understeered' and if they are equal the vehicle will be 'neutrally' steered.

$Kus = \frac{Wf}{C_{\alpha f}} - \frac{Wr}{C_{\alpha r}}$, if we decrease $C\alpha r$ till $Wf/C\alpha r < Wr/C\alpha f$ =>

Kus<0 and the vehicle is oversteered

a) Constant radius maneuver

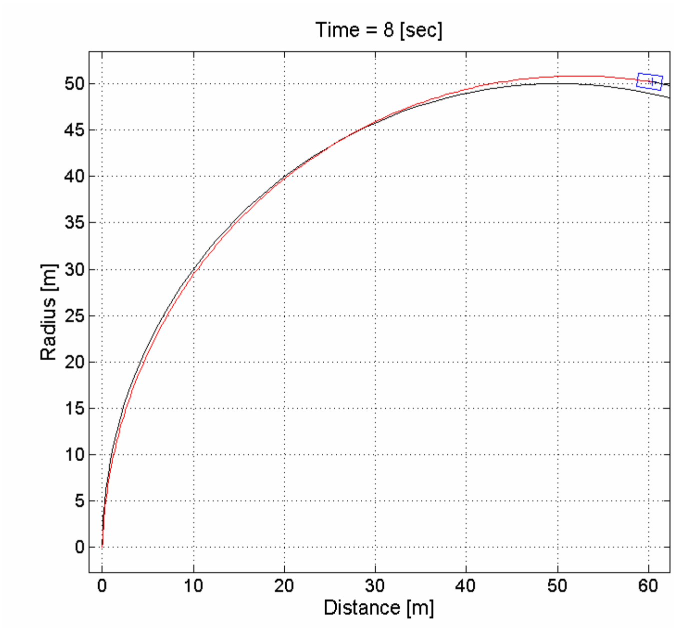

Figure 2. Understeered vehicle (Vx = 40 km/h)

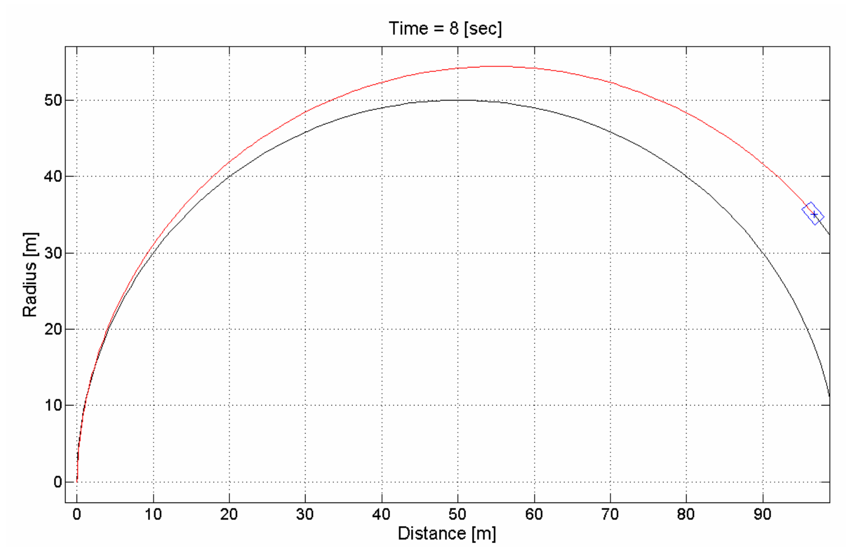

Figure 3. Understeered vehicle (Vx = 60 km/h]

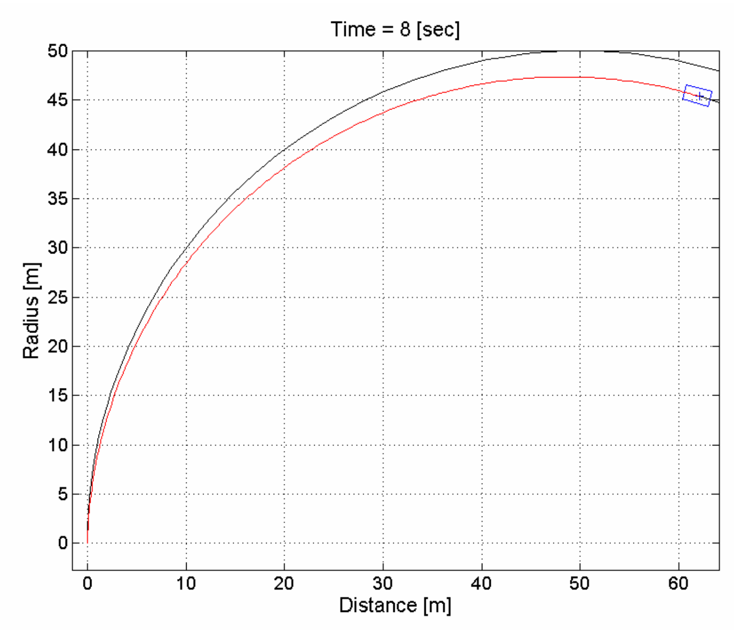

Figure 4. Oversteered vehicle (Vx = 40 km/h]

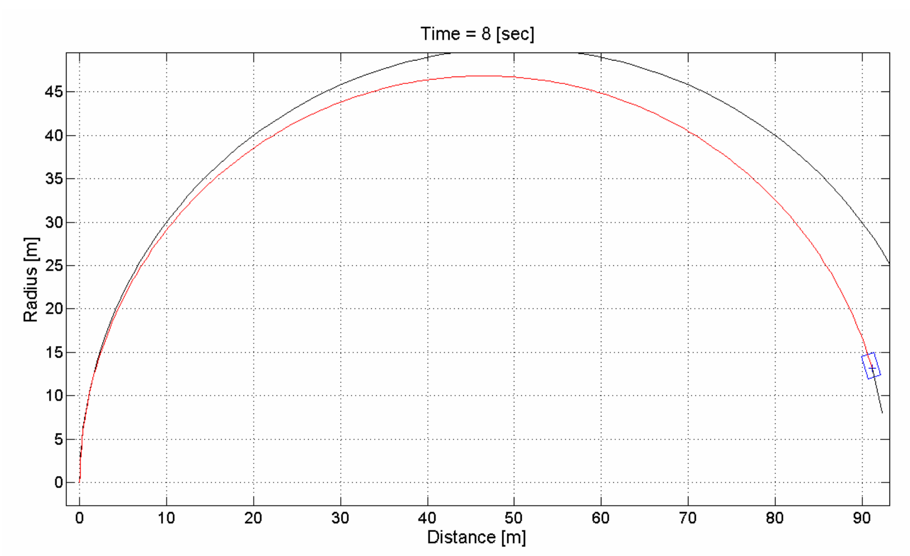

Figure 5. Oversteered vehicle (Vx = 60 km/h)

Figures 2 - 5 show the under- and oversteered vehicle cornering at different velocities. From the first two diagrams, cornering of an understeered vehicle, we can see that the cornering radius is greater then the constant R = 50 [m]. For smaller yaw rate (at the beginning of the cornering) the vehicle is following neutral path, but for the following cornering the vehicle becames 'understeered'. For lower velocities the side slip angles can not generate high enough lateral forces that will create the yaw moment.

Figures 4 - 5 illustrate the oversteered vehicle. It is cornering through a path with cornering radius that is smaller then the constant. Rear lateral forces are higher than the front ones which creates the yaw moment in the direction of the corner. This moment 'helps' the vehicle to turn and the vehicle becomes 'oversteered'. With further cornering the driver will have to rotate backwise the steering wheel so that the vehicle can follow the desired path.

In both cases, with cornering the understeered coefficient (Kus) increases because of the increase of the yaw rate.

3. Conclusion

Kus is understeered coefficient and is expressed in radians. The steer angle required to follow a given curve depends of the wheelbase, turning radius, longitudinal velocity (or lateral acceleration), and the understeer coefficient of the vehicle. It is a function of weight distribution and tire cornering stiffness. Depending on the understeer coefficient Kus, or the relationship between the slip angles of the front and rear tires, the steady state handling characteristics can be classified into: neutral, under- and oversteer.

Understeer behaviuor is when Kus > 0 which is equivalent to the slip angle of the front tire α_f beeing greater than the one of the rear α_r (Wf/Cαf > Wr/Cαr). For an understeered vehicle, when it is driven in a constant radius turn, the driver must increase the steer angle. Or when driving with the same steering wheel position and vehicle longitudinal velocity, the turning radius of an understeered vehicle is greater then that of a neutral steer vehicle. The front tires will develop a slip angle greater than that of the rear tires, as a result a yaw moment is initiated, turning the vehicle away from the direction of the side force.

Oversteer coefficient Kus < 0, which is equivalent to the slip angle of the front tire α_f beeing smaller then the rear tire α_r (Wf/Cαf < Wr/Cαr). A vehicle with this handling property is known as 'oversteered'. For an oversteered vehicle, when it is driven in a constant radius turn, the driver must decrease the steer angle. Or when it is driven with the steering wheel fixed, the turning radius decreases. Also the yaw moment is created as a result of a different side slip angles ($\alpha_f < \alpha_r$), turning the vehicle in the direction of the side force.

References

[1] Wong J.Y., *Theory of Ground Vehicles*, third edition, 2008.

[2] Gillespie T.D., *Fundamentals of vehicle dynamics*, 1998.

[3] Furuichi T., Sakai H., *Dynamic Cornering Properties of Tires,* 2008.

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