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Velocity Domain Determination of a Waterline Plane Using Continuous Sources

A calculation algorithm of velocity domain using continuous sources method was developed from successive deduction of complex potential and current function for "n" continuous sources. This paper presents the applicability of this method to a ship waterline. For the waterline curve are known the widths, and the continuous sources are placed in the longitudinal plane of the ship. Result by calculations, the velocity components helps to determine the velocity module along the curve.

Keywords: *velocity, continuous sources, waterline*

1. Introduction

It consider a continuous sources disposed on the Ox axis between the abscise points x_{k-1} and x_k (see fig. 1).

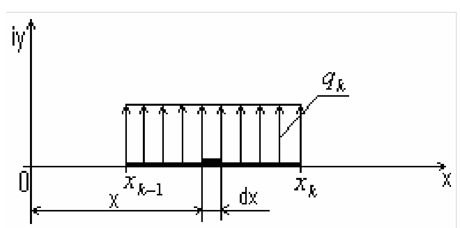


Figure 1. Continuous source.

Be q_k=constant, the source specific flow (i.e. the flow provided by a source segment of length equal to unity). An elementary continuous source extended between the abscise points x and x+dx ($x \in [x_{k-1}, x_k]$) will be treated as a point source of flow $Q = q_k \cdot dx$ and particle center x, so it will have the elementary potential:

$$dW_k = \frac{Q_k}{2\pi}\ln(z - z_0) = \frac{q_k \cdot dx}{2\pi}\ln(z - x) \quad (1)$$

The source complex potential of finite length $l_k=x_k-x_{k-1}$ is obtained by integration:

$$W_k(z) = \int_{x_{k-1}}^{x_k} dW = \int_{x_{k-1}}^{x_k} \frac{q_k}{2\pi}\ln(z - x)dx \quad (2)$$

Noting z-x=t, it results:

$$\begin{aligned} W_k(z) &= \int_{z-x_{k-1}}^{z-x_k} -\frac{q_k}{2\pi}\ln t \cdot dt = \\ &= \frac{q_k}{2\pi}\left[(z - x_{k-1})\cdot\ln(z - x_{k-1}) - (z - x_k)\cdot\ln(z - x_k) - l_k\right] \end{aligned} \quad (3)$$

Giving up the term independent by z (so, constant) from relation (3) the continuous source complex potential become:

$$W_k(z) = \frac{q_k}{2\pi}\left[(z - x_{k-1})\cdot\ln(z - x_{k-1}) - (z - x_k)\cdot\ln(z - x_k)\right] \quad (4)$$

2. The complex potential of a system with "n" continuous sources.

Be "n" continuous sources system of specify constants flows qk, k=1,…,n disposed on the Ox axes; the source by rank k is delimited by the abscise points x_{k-1} and x_k; k=1,…,n. The complex potential of resultant motion will be:

$$W(z) = \sum_{k=1}^{n} W_k(z) \quad (5)$$

where $W_k(z)$ has the expression (4).

Results:

$$W(z) = \frac{1}{2\pi}\sum_{k=1}^{n} q_k\left[(z - x_{k-1})\cdot\ln(z - x_{k-1}) - (z - x_k)\cdot\ln(z - x_k)\right] \quad (6)$$

3. The resultant complex potential of an axial current and a "n" continuous sources system

For an axial current of velocity v_0 conducted on the positive direction and way of x axes, the complex potential has the expression:

$$W_0(z) = v_0 \cdot z \quad (7)$$

Superposing this current over the "n" continuous sources system, results a motion described by complex potential [1]:

$$
\begin{aligned}
&W^*(z) = W_0(z) + W(z) = \\
&= v_0 \cdot z + \frac{1}{2\pi}\sum_{k=1}^{n} q_k \cdot \left[(z - x_{k-1})\ln(z - x_{k-1}) - (z - x_k)\ln(z - x_k)\right]
\end{aligned} \tag{8}
$$

4. Equation determining the "zero" current line

The current function ψ* included in the complex potential W*(z) from relation (8) is determining performing the replacements below:

$$W^*(z) = \varphi^* + i\Psi^*$$

$$z = x + iy$$

$$
\begin{aligned}
&z - x_{k-1} = x - x_{k-1} + iy = \sqrt{(x - x_{k-1})^2 + y^2}\left(\frac{x - x_{k-1}}{\sqrt{(x - x_{k-1})^2 + y^2}}\right) = \\
&= r_{k-1}(\cos\theta_{k-1} + i\sin\theta_{k-1}) = r_{k-1}e^{i\theta_{k-1}} \\
&\qquad r_{k-1} = \sqrt{(x - x_{k-1})^2 + y^2} \\
&\qquad \cos\theta_{k-1} = \frac{x - x_{k-1}}{r_{k-1}} \\
&\qquad \sin\theta_{k-1} = \frac{y}{r_{k-1}} \\
&z - x_k = x - x_k + iy = \sqrt{(x - x_k)^2 + y^2}\left(\frac{x - x_k}{\sqrt{(x - x_k)^2 + y^2}}\right) = \\
&= r_k(\cos\theta_k + i\sin\theta_k) = r_k e^{i\theta_k} \\
&\qquad r_k = \sqrt{(x - x_k)^2 + y^2} \\
&\qquad \cos\theta_k = \frac{x - x_k}{r_k} \\
&\qquad \sin\theta_k = \frac{y}{r_k}
\end{aligned} \tag{9}
$$

The relation (8) become:

$$\Psi*+i\Psi*=v_0(x+iy)+\frac{1}{2\pi}\cdot\sum_{k=1}^{n}q_k\cdot$$
$$\cdot\left[\left(r_{k-1}\cdot\theta_{k-1}\cdot\cos\theta_{k-1}+r_{k-1}\cdot\ln r_{k-1}\cdot\sin\theta_{k-1}\right)-\left(r_k\cdot\theta_k\cdot\cos\theta_k+r_k\cdot\ln r_k\cdot\sin\theta_k\right)\right] \quad (10)$$

Results:

$$\Psi^*=v_0\cdot y+\frac{1}{2\pi}\cdot\sum_{k=1}^{n}q_k\cdot\left[\left(r_{k-1}\cdot\theta_{k-1}\cdot\cos\theta_{k-1}+r_{k-1}\cdot\ln r_{k-1}\cdot\sin\theta_{k-1}\right)-\right.$$
$$\left.-\left(r_k\cdot\theta_k\cdot\cos\theta_k+r_k\cdot\ln r_k\cdot\sin\theta_k\right)\right] \quad (11)$$

The "zero" current line is obtained by imposing the condition ψ*=0:

$$v_0\cdot y+\frac{1}{2\pi}\cdot\sum_{k=1}^{n}q_k\cdot$$
$$\cdot\left[r_{k-1}\left(\theta_{k-1}\cdot\cos\theta_{k-1}+\ln r_{k-1}\cdot\sin\theta_{k-1}\right)-r_k\left(\theta_k\cdot\cos\theta_k+\ln r_k\cdot\sin\theta_k\right)\right]=0 \quad (12)$$

It is symmetric about the Ox axis, because replacing y with –y and θ_i with $-\theta_i$ the equation (12) is verified.

The "zero" current line is a closed curve if the condition is valid:

$$\sum_{k=1}^{n}q_k\cdot l_k=0 \quad (13)$$

5. Velocity determination in the motion plan

To obtain the velocity components v_x and v_y, the complex velocity is used:

$$\frac{dW^*}{dz}=v_x-iv_y$$

$$\frac{dW^*}{dz}=v_0+\frac{1}{2\pi}\sum_{k=1}^{n}q_k\left[\ln\left(z-x_{k-1}\right)+1-\ln\left(z-x_k\right)\right]=$$
$$=v_0+\frac{1}{2\pi}\sum_{k=1}^{n}q_k\ln\frac{z-x_{k-1}}{z-x_k}=v_x-iv_y \quad (14)$$

Using relations (9), results [2]:

$$v_x-iv_y=v_0+\sum_{k=1}^{n}\frac{q_k}{2\pi}\left[\ln\frac{r_{k-1}}{r_k}+e^{i\theta_{k-1}-\theta_k}\right] \quad (15)$$

$$v_x=v_0+\sum_{k=1}^{n}\frac{q_k}{2\pi}\ln\frac{r_{k-1}}{r_k};$$
$$v_y=\sum_{k=1}^{n}\frac{q_k}{2\pi}\left(\theta_k-\theta_{k-1}\right) \quad (16)$$

6. Velocity determination in the waterline plane

Velocity domain of a waterline plane is represented by the relations group (16), assuming that the "zero" current line has the equation (12), satisfies the relation (12) and is confused with the waterline on.

In particular, the coordinates of the waterline distinct points must verify the equation (12). Considering the equation (12) it is obtained a linear system of "n" equations with q_k (k=1,...n) unknown. By solving this system it is obtain the q_k (k=1,...n) unknown specific flows, with different algebraic signs, and the velocity domain of the waterline is determinate by relations (16).

7. Conclusions

In a ship case v_0 represents the regime speed expressed in m/s.

The Ox axis from figure 1 represents the symmetry axis of the waterline.

The axis system origin presented from figure 1, belong to the stem of abscise $x_0=0$; this point belong to the "zero" current line (relation (12)).results:

$x=y=0$

$r_0=x_0=0$, $r_1=x_1$, $r_2=x_2$,..., $r_n=x_n$;

$\theta_1=\theta_2=\dots\ \theta_n=\pi$, $\cos\theta_1=\cos\theta_2=\dots=\cos\theta_n$; $\sin\theta_1= \sin\theta_1=\dots=\sin\theta_n$.

Performing these replacements, results:

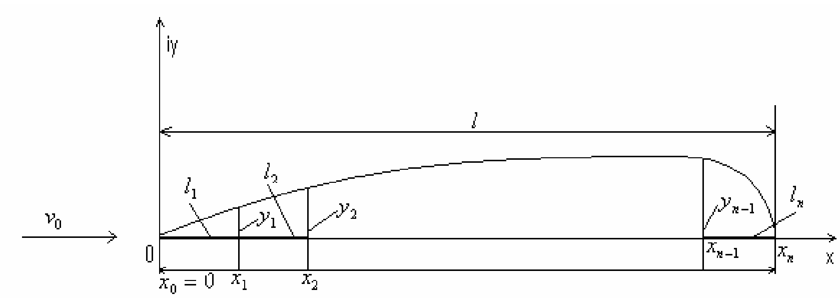
$$\frac{\pi}{2\pi}\cdot\sum_{k=1}^{n} q_k\left(x_k - x_{k-1}\right)=0$$

Or, (17)

$$\frac{1}{2}\cdot\sum_{k=1}^{n} q_k l_k = 0$$

The axis point Ox of abscise x_n belong to the stern and like the origin this point belong to the "zero" current line.

In figure 2 is represented the mid-waterline corresponding to positive coordinates.



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References

[1] Andrei I.V., Mecanica fluidelor, vol. II, Miscări potenţiale, Editura Fundaţiei Universitare "Dunarea de Jos, Galaţi, 2004.
[2] Andrei I.V., Mecanica fluidelor, vol. I, Fluide ideale, Editura Fundaţiei Universitare "Dunărea de Jos, Galaţi, 2003.

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