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Optimal Solutioning Mode of the Assignment Problems

In the present paper we are presenting an optimal way to solve the Asignment problems using the Network Modeling Module of the WinQSB software, by the Hungarian algorithm. Meantime, the given problem was solved also regarded as a transport problem, due to the fact that the assignment problems may be took in consideration as extensions of the transport problems.

***Keywords**: Hungarian algorithm, distribution, balanced problem, optimal solution.*

1. Introduction.

Assignment consists in the optimum distribution or allotment of a certain entity to a certain location.

The assignment problems are encountered in the domain of the organisation of supply with a certain product, of assigning certain plan tasks to existing pieces of equipment, of forming research teams in which each person elaborates alone a part of the studied theme, etc.

There are $n \in N^*$ necessities and for each there are at least one and maximum n possibilities of achievement, it is required to find n possibilities to distribute the n necessities corresponding to a maximum efficiency.

If we need to distribute n applicants onto n vacant positions, in the hypothesis an applicant has the competency to fill more than one position, the assignment may take into account the global satisfaction of preferences or the obtaining of a maximum global efficiency. If n persons must divide n indivisible assts of quasi-equal values, one will aim at performing an equitable partition.

2. Case Study.

Three beneficiaries must be supplied with merchandise from 5 distribution centres. The transportation distances (in km) are shown in the table below.

It is required to determine the optimum assignment of beneficiaries to distribution centres to which corresponds a minimum total transportation distance, on condition each beneficiary is supplied from minimum one centre and from maximum two.

From \ To	Y1	Y 2	Y 3	Y 4	Y 5
X1	10	15	22	18	11
X2	12	12	20	15	10
X 3	11	12	21	16	8

Figure 1. Problem Data

1. Using the Hungarian algorithm, a square matrix is formed adding 2 fictitious beneficiaries (X_4 and X_5), to have a balanced problem.

Table 1.

Centres Beneficiaries	Y_1	Y_2	Y_3	Y_4	Y_5	Minorant
X_1	10	15	22	18	11	$\alpha_1 = 10$
X_2	12	12	20	15	10	$\alpha_2 = 10$
X_3	11	12	21	16	8	$\alpha_3 = 8$
X_4	0	0	0	0	0	$\alpha_4 = 0$
X_5	0	0	0	0	0	$\alpha_5 = 0$

We continue the algorithm with the marking of rows and columns.

Hungarian Method for Anale UEM 2011 - Iteration 1

From \ To	Y1	Y 2	Y 3	Y 4	Y 5
X1	0	5	12	8	1
X2	2	2	10	5	0
X 3	3	4	13	8	0
Dummy	0	0	0	0	0
Dummy	0	0	0	0	0

Figure 2. First Iteration

The minorant of the elements remained uncut is 2, it is subtracted from the uncut elements and added to the doubly cut elements, whereas the simply cut elements remained unchanged.

Hungarian Method for Anale UEM 2011 - Iteration 2 (Final)

From \ To	Y1	Y2	Y3	Y4	Y5
X1	0	5	12	8	3
X2	0	0	8	3	0
X3	1	2	11	6	0
Dummy	0	0	0	0	2
Dummy	0	0	0	0	2

Figure 3. Final iteration

In the above matrix, there is one framed ˝0˝, and thus the coupling is maximum. The optimum solution found is (figure 4): (X_1,Y_1); (X_2, Y_2); (X_3, Y_5); (X_4, Y_3); (X_5, Y_4)

(Assignment Problem)

08-27-2011	From	To	Assignment	Unit Cost	Total Cost	Reduced Cost
1	Source 1	Destination 1	1	10	10	0
2	Source 2	Destination 2	1	12	12	0
3	Source 3	Destination 5	1	8	8	0
4	Unfilled_Demand	Destination 3	1	0	0	0
5	Unfilled_Demand	Destination 4	1	0	0	0
	Total	Objective	Function	Value =	30	

Figure 4. The final solution in matrix form

The Centres Y_3 and Y_4 supply the fictitious beneficiaries X_4 and X_5. From these centres the "real" beneficiaries (X_1, X_2, X_3) will have to get supplied.

From \ To	Y3	Y4
X1	22	18
X2	20	15
X3	21	16

Figure 5. The defined problem

We shall form a square matrix, introducing a fictitious warehouse.

Hungarian Method for Anale 2 - Iteration 1

From \ To	Y3	Y4	Dummy
X1	2	3	0
X2	0	0	0
X3	1	1	0

Figure 6. Iteration 1

The minorant of each row is 0. So it will subtract only the minorant of each column. The minorant of the elements remained uncut is 1. It is added to the doubly cut elements, subtracted from the uncut elements, leaving the simply cut elements unchanged.

Hungarian Method for Anale 2 - Iteration 2 (Final)

From \ To	Y3	Y4	Dummy
X1	1	2	0
X2	0	0	1
X3	0	0	0

Figure 7. Final iteration

Each row and column contains one framed zero, and thus the solution found is optimum: (X_2, Y_3); (X_3, Y_4), or the proposed version by WinQSB software (figure 8) - (X_2, Y_4); (X_3, Y_3).

08-24-2011	From	To	Assignment	Unit Cost	Total Cost	Reduced Cost
1	X1	Unused_Supply	1	0	0	0
2	X2	Y4	1	15	15	0
3	X3	Y3	1	21	21	0
	Total	Objective	Function	Value =	36	

Figure 8. The final solution

But taking into account also to previously determined solution, the solution of the problem is:

1.) $X_1 \rightarrow Y_1$; $X_2 \rightarrow Y_2$, Y_3 and $X_3 \rightarrow Y_5$, Y_4

$[min]f_1 = 10 + 12 + 20 + 8 + 16 = 66$ km

2.) The proposed solution by WinQSB $X_1 \rightarrow Y_1$; $X_2 \rightarrow Y_2$, Y_4 and $X_3 \rightarrow Y_5$, Y_3

$[min]f_2 = 10 + 12 + 15 + 21 + 8 = 66$ km

2. The Assignment problem, being an extension of the transport problem, can be solved using the same module of WinQSB software, meaning Network Modeling, but selecting this time the transport problem form the File menu window instead of the Assigment problem (figure 9).

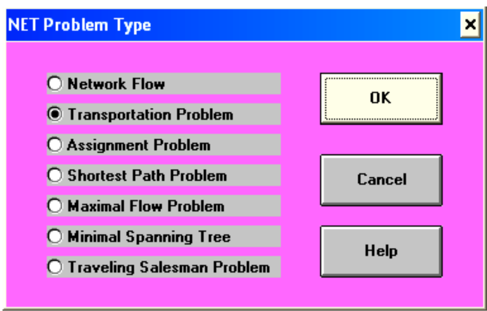

Figure 9. Dialog window NET Problem Specification

(Transportation Problem)

08-27-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	Source 1	Destination 1	1	10	10	0
2	Source 2	Destination 2	1	12	12	0
3	Source 3	Destination 5	1	8	8	0
4	Unfilled_Demand	Destination 3	1	0	0	0
5	Unfilled_Demand	Destination 4	1	0	0	0
	Total	Objective	Function	Value =	30	

Figure 10. The solution matrix form

It can observe the same distribution as in the case of solving by the hungarian algorithm (fig.4).

(Transportation Problem)

09-04-2011	From	To	Shipment	Unit Cost	Total Cost	Reduced Cost
1	Source 1	Unused_Supply	1	0	0	0
2	Source 2	Destination 4	1	15	15	0
3	Source 3	Destination 3	1	21	21	0
	Total	Objective	Function	Value =	36	

Figure 11. Final optimal solution

3. Conclusion

No matter which is the chosen solving method, the optimal solution offered by WinQSB software is the same, in particular:

- X_1 Beneficiary is supplied form the Y_1 Distribution center, the distance between being 10 km;
- X_2 Beneficiary is supplied form the Y_2 and Y_4 Distribution centers, the distance between being 12 km, respectively 15 km;
- X_3 Beneficiary is supplied form the Y_5 and Y_3 Distribution centers, the distance between being 8 km, respectively 21 km;
- The Objectiv Function is so 66 km.

By aplying the Hungarian method results one optimal solution, meaning:

- X_1 Beneficiary is supplied form the Y_1 Distribution center, the distance between being 10 km;
- X_2 Beneficiary is supplied form the Y_2 and Y_4 Distribution centers, the distance between being 12 km, respectively 20 km;
- X_3 Beneficiary is supplied form the Y_5 and Y_3 Distribution centers, the distance between being 8 km, respectively 16 km;
- The Objectiv Function is the same 66 km.

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