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Pipeline Analyzer using the Fractional Fourier Transform for Engine Control and Satellites Data

The aim of this paper is to present an algorithm for computing the fractional Fourier transform integrated into the pipeline of processing multi-variate and distributed data recorded by the engine control unit (ECU) of a car and its satellites. The role of this transform is vital in establishing a time-variant filter and therefore it must be computed in a fast way. But for large scale time series, the application of the discrete fractional Fourier transform involves the computations of a large number of Hermite polynomials of increasingly order. The parallel algorithm presented will optimally compute the discrete Fourier-type transform for any given angle.

Keywords: *ECU, fractional Fourier transform, pipeline, parallelization*

1. Introduction

The main observation that can be considered as the starting point of this work is the fact that the noise which is recorded by the ECU and the satellites together with acceleration data is not usually correlated with the useful signal and it can be filtered in the best way using an convex domain shape in the time-frequency place. Therefore, the use of the fractional Fourier transform in the noise filtering step of the pipeline analyzer for the acceleration data is an appropiate tool. The fractional Fourier transform (FRFT) is a generalization of the ordinary Fourier transform, allowing a variable rotation in the frequency plan by any angle. By a standard convention ([18]), it is denoted the a-th order fractional Fourier transform F^a as the a-th power of the ordinary Fourier transform operation F. The first order (a = 1) of the fractional transform corresponds to the ordinary Fourier transform operation, and the zeroth-order fractional transform is the identity operation. Thus the first-order fractional transform of a function is exactly its ordinary Fourier transform, and the zeroth-order transform is the function itself.

Although the FRFT is an intuitive tool, its discrete counterpart is not as straightforward to implement in an fast way. In ([7]), it was proposed a type of DFRFT(discrete FRFT) by searching the eigenvectors and eigenvalues of the DFT matrix and then computing the fractional power of the DFT matrix. This type of DFRFT will work similar to the continuous FRFT and will also satisfy the properties of orthogonality, additivity and reversibility. The problem is that the eigenvectors cannot be expressed in closed form and they also lack the fast computational algorithms ([12]). Therefore a parallel algorithm will be a key solution especially when dealing with large time series for filtering reasons.

2. Discrete Fractional Fourier Transform

The DFT matrix has only four distinct eigenvalues (λ_k = $exp(-j\pi k/2)$ 1,-1,j,-j) ([1]). The eigenvalues are in general degenerated so that the eigenvector set is not unique. For this reason, it is necessary to specify a particular eigenvector set to be used. In the continuous case, this ambiguity is resolved by choosing the Hermite-Gaussian functions as the eigenfunctions. Since our aim is to use a definition of the discrete transform that is completely analogous to the continuous transform, we will resolve this ambiguity in the classical manner ([2,5]), by choosing the common eigenvector set of the DFT matrix and the discrete counterparts of the Hermite-Gaussian functions. Another ambiguity arises in taking the fractional power of the eigenvalues since the fractional power operation is not singular valued. This ambiguity will again be resolved by analogy with the continuous case ([2,5]), by taking λ_k^a = $exp(-i\pi ka/2)$. Distinct definitions based on other choices are discussed in ([14]). The particular choice we are concentrating on is the one that has been most studied and has overwhelmingly found the largest number of applications. Denoting the discrete Hermite-Gaussians as $u_k[n]$, the definition of the discrete fractional Fourier transform becomes:

$$F^a[m,n] = \sum_{k=0}^{N-1} u_k[m] e^{-j\frac{\pi}{2}ka} u_k[n] \quad (1)$$

Now, we must explicitly define the discrete counterparts of the Hermite-Gaussian functions.

3. Parallel algorithm

We will consider that we have available on our system k processors. The parallel algorithm will be described for a distributed memory system, meanwhile for a shared memory machine, the implementation is quite straightforward with a concurrent of data accessing due to the independence of functional tasks and the minimum data exchange among processors. Therefore, this case can be directly obtained from a simple modification of the other one and we will concentrate our attention on the more complex situation. The implementation of the algorithm wants to reduce at maximum the communication time. We recall that our discrete framework is made of *k* processors each working simultaneous on *n/k* samples. We will describe in details in the follow each step of the algorithm using MATLAB^{TM} notations.

Algorithm 1 - Parallel algorithm for DFRFT

Input: x(n) – signal of length n, k-number of processors, angle – a vector of angles

GLOBAL STEP

1. Broadcast signals of length *n/k,* and the angles to the available *k*-processors

LOCAL STEPS:

1. Apply locally the iterative formula for the Hermite-Gauss polynomials

2. Apply locally the DFRFT matrix using the vector with the angles

GLOBAL STEP

2. Integrate into the pipeline the data blocks on each processor

Note that the efficiency of this algorithm is not much influenced by the transmission time due to fact that we are only sending once and we are only receiving once (global broadcast) and this communication time could be associated with a globally storage procedure running sequentially on one core processing element. Since all parallel computations are done at the local level the coefficient of speed-up in comparison with the serial and while the efficiency increases with a factor of *n/k*. We used for tests MATLAB^{TM} on a single-core standard PC for the case of sequential implementation and MATLAB MPI (from MIT) on a multi-core processing machine.

In the table 1 below we summarize the statistical findings of numerical experiments, processed several times. Also, the speed-up versus the sequential implementation is displayed in Figure 1. In most cases less than half execution time is needed. This confirms the efficiency and the scalability of parallel method also from the run time point of view.

Table 1. Run time timings

	k = 3		*k* = 5		*k* = 12	
	Seq.	Par.	Seq.	Par.	Seq.	Par.
N=480	1.600	0.694	4.123	2.101	45.664	17.679
N=480	0.723	0.307	1.163	0.583	4.988	2.976
N=480	0.160	0.083	0.349	0.159	1.702	0.865
N=1200	18.770	6.432	49.647	12.651	597.034	186.765
N=1200	4.541	1.792	10.751	4.806	52.635	30.549
N=1200	1.496	0.378	2.680	1.218	11.363	7.709
N=240	0.077	0.065	0.551	0.350	3.972	2.524
N=240	0.018	0.017	0.171	0.117	0.810	0.478
N=240	0.011	0.013	0.048	0.023	0.197	0.198

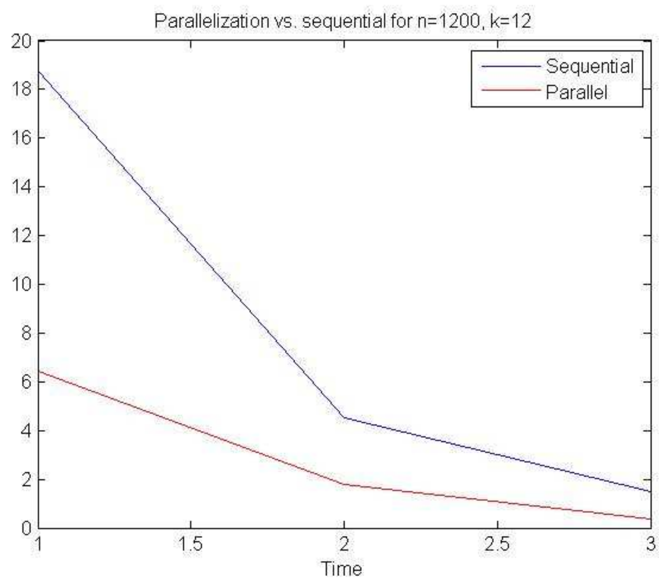

Figure 1. Speed-up

3. Integration into the pipeline

The complicated structure of the pulses from the ECU, involve mix stationary components and transients, transmitted over several overwritten channels. During crash tests or real crashes, the data are known only on a nonuniformly spaced sampling set. This nonuniformity is due to the partial or total destruction of the sensors during the crash and prevents the use of the standard methods from Fourier analysis. We will consider four sensors located on the four extremities of the car (north, south, west, east) and a central processing unit situated in the geometric center of the car at equal distance from the distributed sensors. Lets consider for example that each sensor will measure with some disturbances the acceleration signal but due to the expected impact and the presence of noise the sampling set is irregular. Moreover, even if the assumption that a function belongs to a particular space is valid, the samples of the acquisition are not exactly measured due to digital inaccuracy, or the samples are corrupted by noise when they are obtained by a real measuring device. Therefore, will use for testing purposes an application of the proposed method to accelerometer-type data where it is crucial to have a fast processing method (if possible close to real time) of the recorded samples during a crash. The purpose of the numerical quality of the measurement is not within the scope of this paper and we will consider here only the possibility of applying the algorithm, the acceleration of the method and the possiblity of integrating in a pipeline analyzer. We will consider a multi-variate systems of four simulated signals, generated by a car crash simulator with the standard parameters. The pipeline processing for the crash test data, recorded by an accelerometer, starts with a pre-filtering before sampling at a roll off frequency of 4,000 Hz. The pre-filtered data, referred to as wideband data, contains the same signal as the raw data (the impact stress recorded by an accelerometer). This data is then sampled at a rate of 12,500 points per second (or 0.08 milli-seconds per data point) and yields an input acceleration. To obtain the signal in its useful frequency range, a digital filtering technique which satisfies the frequency response corridor specified by SAE J211 (SAE Recommended Practice on the Instrumentation for Impact Tests) should be used. After this step we can use the DFRFT for the parallel filtering of the frequency channels. This step is essential for crash signal applications where in most of the cases the noise is not correlated with the data.

4. Conclusions

The pipeline parallel method for computing the fractional Fourier transform, improves the treatment of the data filtering inside the pipeline by a n/k factor of efficiency and cuts the processing time in a scalable way determined by the direct proportion between the growth of the number of samples and the number of atoms. The tests performed in the case of car crash data show the potential of the method on realistic applications where the data is corrupted by noise or it is non-uniform sampled.

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