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Wigner Ville Distribution in Signal Processing, using Scilab Environment

The Wigner Ville distribution offers a visual display of quantitative information about the way a signal's energy is distributed in both, time and frequency. Through that, this distribution embodies the fundamentally concepts of the Fourier and time-domain analysis. The energy of the signal is distributed so that specific frequencies are localized in time by the group delay time and at specifics instants in time the frequency is given by the instantaneous frequency. The net positive volum of the Wigner distribution is numerically equal to the signal's total energy. The paper shows the application of the Wigner Ville distribution, in the field of signal processing, using Scilab environment.

Keywords*: Wigner Ville distribution, Scilab, time - frequency analysis*

1. Introduction

Numerous tools such as Direct Fourier Transform DFT, maximum entropy method, are used for the spectral analysis of stationary processes [1]. But, in many practical situations (speech, acoustics, biomedicine applications), the assumption of stationarity fails to be true. Specific tools are then to be used when the spectral content of a signal under consideration is time dependent, as the Wigner-Ville representation, which is a particular case of a more general class of spectral estimators [2]. A time – frequency distribution provides simultaneous time resolution and inversely proportional frequency resolution, combining both, time and frequency information, into a single representation.

The Wigner Ville distribution is extensively used in audio signal analyses for spectral estimation, electrocardiogram analysis, etc. This distribution of a signal contains the following properties useful to the signal analyst: frequency response, group delay, instantaneous power and instantaneous frequency.

2. Theoretic bagground of the Wigner – Ville representation

We consider a classic harmonic oscillator, mass and spring, figure 1(a). The system evolution can be completely described through its position x and velocity $\dot{x}$, (the two-dimensional space with the particle's coordinate X and momentum P as dimensions - quadratures) both of them needed to be represented, containing important information about the systems behavior. The position of the simple harmonic oscillator $x(t) = A\cos(\omega t)$ and the velocity $\dot{x}(t) = -A\sin(\omega t)$ can be put on the same graphic representation, figure 1(b).

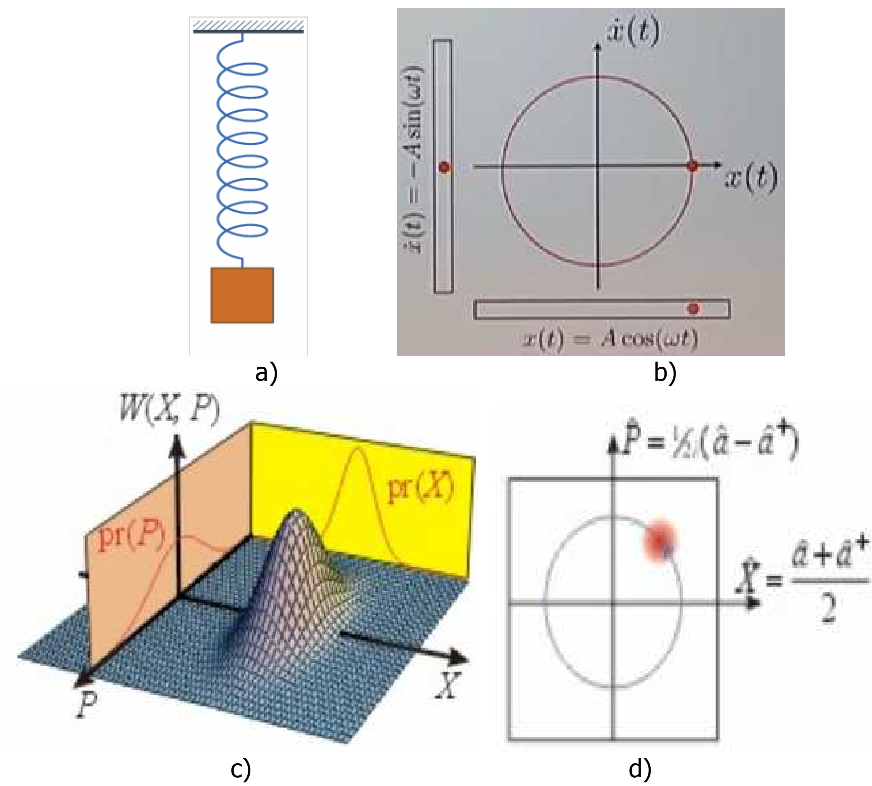

Figure 1. Explain of time – frequency analysis

This representation is the part of the phase space or "state space" representation of a harmonic oscillator, with the variable position and velocity. A particular case is the so called phase-space representation, where in the mechanic, for example, this is given by momentum (P) versus position (X) or, in signal processing, the time (t) versus temporal frequency (ω).

For a large part of classical oscillators, we can talk about the *phase-space probability distribution – a function* W(X,P) which indicates the probability of finding a particle at a certain point in the phase space. This function must be non-negative and normalized: its integral over the entire phase space must be equal to one.

This classical probability distribution has another important property. Consider a series of measurements in which we only measure the oscillator's coordinate but not the momentum. After a large number of such measurements, one obtains the probability distribution associated with the coordinate – the so called *marginal distribution* pr(*X*). This distribution is related to the phase-space probability density in the following way:

$$pr(X) = \int W(X,P)dP\,. \tag{1}$$

In other words, a marginal distribution is just a density projection of *W(X, P)* onto a plane associated with the given quadrature, figure 1(c).

In the quantum world, figure 1(d), the notion of a "certain point in the phase space" does not make sense because the position and the momentum cannot be measured simultaneously (Heisenberg's uncertainty principle). Neither does the phase-space probability density. However, even in the quantum domain one can perform quantum measurements of a *single* quadrature – be it *X*, *P*, or their linear combination. A multiple measurement of a quadrature on a set of identical quantum states will yield a probability density associated with this quadrature, i.e. a marginal distribution.

The fundamental problem of the time- frequency analyses is to have a mathematical device, able to represent simultaneosly a given signal *s(t)* in term of its intensity in time and frequency. Fot this, the density or distribution *P(x)* is a function that express how a given quantity is distributed to a given variable *x* per unit of *x*, such that *P(x)Δx* is the amount that falls in an interval *Δx at x,* while the total amount, that is often normalized to unity, is given by

$$\int_{-\infty}^{\infty} P(x)dx\,. \tag{2}$$

Since most quantities can be expressed as a function of two or more variables, a two-dimensional (and more than two-dimensional). We constrain that a given density *P(x,y)* is such that the total amount is given by:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(x,y)dxdy\,. \tag{3}$$

In case of signal analysis, the most concerned quantity is energy, and the variables along which it distributes are time and frequency. For this the variables *x* and y will be replaced with distributions in form of time (t) and frequency (ω), $P(t,\omega)$.

The time – frequency density has to satisfy a set of certains criterias [6]:

- Has to be positivity, because negative density imply negative energies and cannot be interpreted
$$\forall t, \omega : P(t,\omega) \geq 0, \tag{4}$$
- The total energy of the distribuition should egual to the total energy of the signal
$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} P(t,\omega) dt d\omega = \int_{-\infty}^{\infty} |s(t)|^2 dt = \int_{-\infty}^{\infty} |S(\omega)|^2 d\omega, \tag{5}$$
- The avarage time for a given frequency and the avarage frequency for a given time, should equal the derivate of the signal phase and, respectively spectral phase
$$\langle\omega\rangle_t = \int_{-\infty}^{\infty} \omega P(t,\omega) d\omega = \phi^{'}(t), \qquad \langle t\rangle_\omega = \int_{-\infty}^{\infty} t P(t,\omega) dt = -\Psi^{'}(\omega). \tag{6}$$

The *Wigner-Ville distribution* (WVD), for a given signal $s(t)$ is defined by:
$$W(t,\omega) = \int_{-\infty}^{\infty} s(t+\tau/2) x^{*}(s-\tau/2) e^{-j2\pi\nu\tau} d\tau \tag{7}$$

Considering to be given a discrete-time signal $s(n)$ and is Fourier transform
$$S(\omega) = \sum_{n\in Z} s(n) e^{-j\pi\nu}, \tag{8}$$
the Wigner distribution of $s(n)$ is given by:
$$W(\omega,n) = \frac{1}{\pi}\sum_{k\in Z} s(n+k) s^{*}(n+k) e^{-j2k\nu}. \tag{10}$$

3. Wigner Ville analysis in Scilab environment

The following example represents the result of the Wigner Ville analysis, applied on a sinusoid modulated by a parabola, having a finite duration [5].
$$s(t) = \left\{ \begin{array}{cc} p(t)\sin\left(\frac{2\pi}{16}t + u(t-488)\pi\right) & 400 < t < 570 \\ 0 & 0 \leq t \leq 400; 569 \leq t \leq 950 \end{array} \right\} \tag{11}$$

The Scilab program [9] consists in defining the parabola, the unit step function, to compute the signal to be analyzed (11), to apply the Wigner spectrum and to represent the 3D plot, figure 2.

As it can be seen in figure 2, the represented distribution has the opportunity to highlight the signal energy depending on the different frequency components together with the time evolution.

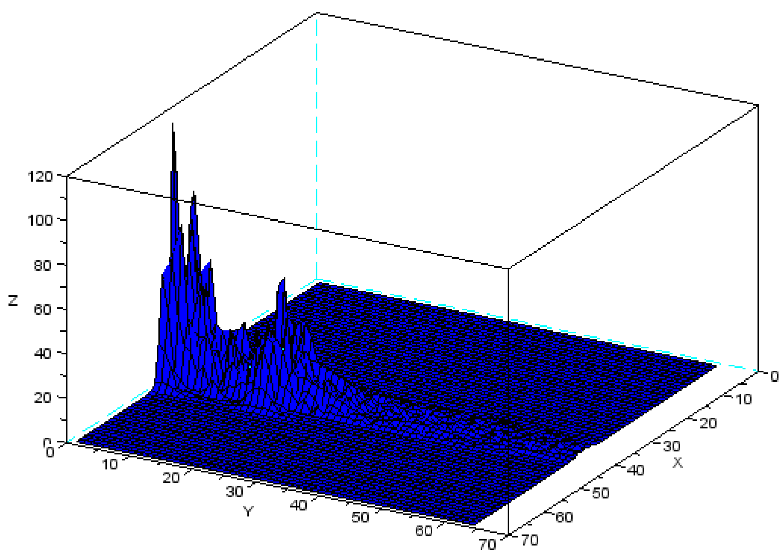

Figure 2. Wigner distribution for the anaysed signal, represented as a surface

4. Conclusion

The problem of time-frequency analysis is to develope a two-dimensional of one dimensional data that can be computed efficiently and that is readily interpreted physically. The Wigner Ville distribution, toghether with the Fourier analysis, combines the traditional concepts associated with the time and frequency domains into a single, joint domain, beeing very useful to visualize processes in position momentum phase space.

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