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On Methods for Estimating the Friction between the Tire and Tread

The influence of the tire-road friction coefficient is studied in the work for the avoidance of collisions of traffic taking part vehicles. The part of the road surface state is made evident.

***Keywords**: friction, tire, rolling surface*

1. Introduction

The active security on the public roads is very important and it benefits of a high attention from the part of the vehicle manufacturers and vehicle users. One of the most important systems of active security is the control system that assures the warning and avoidance of the collision between vehicles and, especially, the chain collision of more vehicles. These control-systems for the road traffic security are dependent of a very important factor, the tire-road friction coefficient. The values of this coefficient can be obtained only by experimental researches, where a lot of factors intervene as the tire construction, internal tire pressure, traffic speeds, braking times, wet or dray road state, motor power etc.

2. Method elaborated on a vehicle model, completely equipped, with a non linear main transmission in the regime of light running

In [1], simulation methods are presented, to estimate the tire-road friction by measuring the number of rotations of the motor, the going out speeds of the main transmissions and wheels. In this way, it is possible to obtain very useful information concerning the tire-road friction, in order to avoid the collisions between the traffic part vehicles.

Recently were effected high accuracy researches, using estimation methods for the variation of rolling surface through variable measurable parameters of vehicle wheels. Other methods are using acceleration sensors and need a detailed knowing of the dynamic loads what are applied to the wheels, during the process of braking.

We are proposing a method elaborated on a vehicle model, completely equipped, with a non linear main transmission and considering that the braking is made without the motor decoupling and with the admission in the regime of light running.

From [2] it is known that the differential equation for the velocity of the transmission driving element is

$$\frac{d\omega_{crt}}{dt} = \frac{1}{\sum_i J_i}\left[\frac{T_t}{R_i} - R_d T_s\right] \tag{1}$$

where $\sum_i J_i = J_{crt,i}$ is the sum of moments of inertia of the organs in rotary motion, in the i step of the gearbox; T_t is the motor torque at the wheel, in the braking period; T_s is the reduced at the wheel resistant torque from the part of the motor and transmission; R_i and R_d are the transmission ratios of the gearbox in the i step, and respectively, of the vehicle differential.

For some vehicles, [3], with the corresponding simplifications, for a driving wheel it can be written:

$$\frac{d\omega_r}{dt} = \frac{1}{\sum J_r}\left(T_s - r_r F_r\right) \tag{2}$$

Where ω_r is the angular velocity of a wheel; $\sum J_r$ is the sum of inertia moments of the wheel composing elements; r_r is the rolling radius of the vehicle wheel; F_x is the longitudinal force, applied to the wheel, from the part of the road.

As in [1] it is used the longitudinal force applied to the wheel, having the expression:

$$F_x = D_x \sin[C_x ctg(B_x \varphi_x)] + S_{vx} \tag{3}$$

Where

$$\varphi_x = (1 - E_x)(\lambda + S_{hx)} + \frac{E_x}{B_x} ctg\left[B_x\left(\lambda + S_{hx}\right)\right] \tag{4}$$

Where the coefficient λ is the relative slipping between the tire and road, and $B_x, C_x, D_x, E_x, S_{hx}, S_{vx}$ are the tire coefficients what are established on the basis of experimental data [4]. Some of these coefficients, like B_x, C_x, D_x, are dependent of the gravity force on the wheel, that is to say, of the tire pressing on the road.

The expression of the longitudinal force F_x dependence of the tire-road friction coefficient is:

$$F_x = F_x\left(F_z, \lambda, \mu\right) = \mu F_{x_0}\left(F_z, \lambda, \mu_0, \alpha\right), \ \alpha = \frac{\mu_0}{\mu}\alpha_0 \quad (5)$$

where α is a shape factor of tire.

It is considered that the shaft of the driving wheel is modeled as a non linear spring, solicited to torsion, so that the differential equation of this shaft is:

$$\frac{dT_s}{dt} = k\left(R_d \omega_{crt} - \omega_r\right) \quad (6)$$

where k is the torsion elastic constant.

In this way, the vehicle motion state can be defined by the vector

$$X = \begin{bmatrix} \omega_{crt} & T & \omega_r \end{bmatrix}^T \quad (7)$$

and, taking account of the equations (1)...(6), it results

$$\dot{x} = F_0(x) + w^T(x)\theta \quad (8)$$

where

$$F_0(x) = \begin{bmatrix} -\frac{R_d}{\sum J_i} x_2 + \frac{1}{R_i \sum J_i}\left(C_i \omega_e^2 + C_{i+1} R_i \omega_e x_1 + C_{i+2} R_i^2 x_1^2\right) \\ k R_d x_1 \\ \frac{1}{\sum J_r} x_2 \end{bmatrix} \quad (9)$$

and

$$w^T(x) = \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\sum J_r} F_{x_0}\left(F_z, \lambda, \mu_0, \alpha\right) \end{bmatrix} \quad (10)$$

and $\theta = \mu$, which is a unknown parameter.

3 The estimation of the unknown parameters

The differential equation is similar to the one in the general case [4]

$$\begin{aligned} \frac{dx}{dt} &= Ax + f(t, x, \theta) + g(t, x, \theta)u(t) \\ y &= Cx \end{aligned} \quad (11)$$

where $x \in R^n, u \in R^m, \theta \in R^p, y \in R^g$, and *f* and *g* are smooth vectorial non-linear functions on R^n and θ is a certain unknown parameter. In a first approximation it can presume that the vectorial functions *f* and *g* linearly depend of the parameter θ, so they have the aspect:

$$f(t,x,\theta) = \sum_{i=1}^{p} \theta_i f_i(t,x)$$
$$g(t,x,\theta) = \sum_{i=1}^{p} \theta_i b_i g_i(t,u,x) \tag{12}$$

where $f_i(t,x) \in R^n$ and $g_i(t,u,x)$ are a non-linear functions and $b_i \in R^n$ are constant vectors.

The identification method, base don the filtered regressor [4] is applicable and it is formulated as follows. It is considered:

$$w^T(t,u,y) = F + BG \tag{13}$$

where $F \in R^{nxp}, B \in R^{nxp}, G \in R^{nxp}$ are defined by:

$$F = [f_1(t,x), f_2(t,x), ..., f_p(t,x)]$$
$$B = [b_1, b_2, ..., b_p]$$
$$G = diag[g_i(t,u,x)]$$

Then, the system (11) becomes:

$$\frac{dx}{dt} = Ax(t) + w^T(t,u,y)\theta \tag{14}$$

For the estimation of the unknown parameters it is proposed an identifier, based on a filtered regressor, so that

$$\frac{dH}{dt} = (A - LC)H + w^T(t,u,y)$$
$$\frac{dH_0}{dt} = (A - LC)H_0 + Ly$$
$$\overline{X} = H\overline{\theta} + H_0 \tag{15}$$
$$\overline{y} = C\overline{x}$$
$$\frac{d\overline{\theta}}{dt} = kH^T C^T (y - \overline{y}); k > 0$$

Where $\overline{x}$ and $\overline{\theta} = [\theta_1, \theta_2, ..., \theta_p]^T$ are the vectors of the estimated states, respectively the vector of the unknown parameters and $L \in R^{nxq}$ is a matrix, chosen so that the matrix $A-LC$ to be a Harwitz matrix. So, the system state can be reconstructed from H and H_0, by

$$x(t)=H(t)\theta+H_0(t)+e^{(A-LC)(t)}[x(0)-H(0)\theta-H_0(0)] \qquad (16)$$

If it is noted by $e=x-\overline{x}$ and by $e_\theta=\theta-\overline{\theta}$, respectively the error of the examined state and the estimation error of the parameter θ, it results:

$$e(t)=H(t)e_\theta+e^{(A-LC)(t)}[x(0)-H(0)\theta-H_0(0)] \qquad (17)$$

The dynamics of estimation of the state errors and of the parameters is obtained from:

$$\frac{de}{dt}=-kHH^TC^TCe+[(A-LC)H-w^T(t,y)]e_0+$$
$$+(A-LC)e^{(A-LC)(t)}[x(0)-H(0)\theta-H_0(0)]$$
$$\frac{de_\theta}{dt}=-kH^TC^TCe \qquad (18)$$

The stability study of this system is made by applying the Lyapunov's method [4]. So, it is chosen a Lyapunov function $V(e_0)=\frac{1}{2}e_\theta^TPe_\theta$ where *P* is a matrix positively defined.

In this way, the conclusions of the stability study are similar to the results from [4].

4. The study of control estimation of the necessary for the valuation of the essential parameter in vehicle dynamics

In the study of control estimation of the necessary for the valuation of the essential parameter in vehicle dynamics, represented by the tire-road friction coefficient, the recursive identification method based on the recursive least square algorithm can be used, as well as the method, based on the filtered recursor.

This thing is possible if the following parameters are supposed as measured: the velocity of the driving element of transmission, $y_1=\omega_{crt}=x_1$, the velocity of the vehicle wheel, $y_2=\omega_w=x_2$ and the angular velocity of the motor $y_3=\omega_e=x_3$.

From the state equation, the reduced observer is defined, based on the measurement of the mechanical elements, as follows:

$$\frac{d\bar{x}_1}{dt} = -\frac{R_d}{\sum J_{crt}} x_2 - \frac{1}{R_i \sum J_{crt}} \xi(y) + k_1(y_1 - \bar{y}_1)$$
$$\frac{d\bar{x}_2}{dt} = kR_d \bar{x}_1 - ky_2 + k_2(y_1 - \bar{y}) \qquad (19)$$

where $\xi(y) = C_i y_3^2 + C_{i+1} y_1 y_3 + C_{i+2} y_1^2$, and k_1, k_{21} are chosen amplification constants [1].

The stability analyses of the identification is made by Lyapunov's method [4]. The system state is reconstituted from the filtered condition given by:

$$x_3 = H\theta + H_\theta + e^{(A-LC_2)t}[x_3(0) + H(0)\theta - H_0(\theta)] \qquad (20)$$

where H_0 is given by

$$\frac{dH_0}{dt} = (L - LC_2)H_0 + Ly_2 + \frac{1}{J_w} x_2 \qquad (21)$$

If $e = x - \bar{x}$, then error dynamics is

$$\dot{e}_1 = -\frac{R_d}{\sum J_{crt}} e_2 - k_1 e_1$$
$$\dot{e}_2 = kR_d e_1 - k_2 e_1$$
$$\dot{e}_3 = \frac{1}{J_w} e_2 - KHH^t c_2^T e_3 + [(A - LC_2)H + w^T(t,u,y)]e_\theta + \qquad (22)$$
$$+ (A - LC_2)e_w + (A - LC_2)e^{(A-LC_2)t}[x(0) - H(0)\theta - H_0(0)]$$

where $e_w = H_0 - H_1$.

The amplifications k_1 and k_2 are chosen so that the errors for $\dot{e}_1$ and $\dot{e}_2$ to be stable, and the error for e_3 is given by

$$e_3 = He_\theta + e_w + e^{(A-LC_2)t}[x_3(0) + H(0)\theta - H_0(0)] \qquad (23)$$

and

$$\dot{e}_w = Le_w + \frac{1}{J_w} e_2$$
$$\dot{e}_\theta = -kH^T C_2^T e_3 \qquad (24)$$

In accordance with [4] it is chosen a Lyapunov function having the expression

$$V(e_\theta, e_w) = e_\theta^T e_\theta + \frac{kC_2}{2} \int_0^\infty e_w^2 \tau d\tau \qquad (25)$$

e_1 and e_2 also k_1 and k_2 are corresponding chosen and they exponentially tend to zero.

From the choosing of the Lyapunov function, the derivative of $V(0,0)$ lengthways of the equation trajectory is:

$$\dot{V} = 2e_\theta^T \dot{e}_\theta - \frac{kc_2}{2} e_w^2 = -2kc_2 (He_\theta + \frac{e_w}{2})^2 - \\ - 2kc_2 He_\theta e^{(A-LC_2)t} [x_3(0) - H(0)\theta - H_0(0)] \quad (26)$$

As it is known, $(A - LC_2)$ is a Hurwitz matrix and e_w asymptotically converges to zero when $t \to \infty$; it results that for any $t \geq T$, $\dot{V}$ is negative. So, then chosen Lyapunov function is monotonously decreasing and from (26) it results that the positive constants α_1 and α_2, so that

$$\dot{V} \leq -2kC_2 e_3^2 + \alpha_1 e^{-\pi} \quad (27)$$

From which taking account of the Gronwall inequality we have $e_3 \in L_2$, so it results that the convergence is asymptotic and so, the parameter θ tends to the real measured value.

5. Conclusions

From the multiple of algorithms of collision warning, , the only ones what were selected, use the measurements concerning the estimation of the tire-road friction coefficient.

In [1], the critical distance, used for a warning algorithm, is defined by:

$$d_{crt} = v_1 t_h - \left(v_1 + \frac{v_{rel}}{2} \right) \frac{v_{rel}}{\eta \mu g} \quad (28)$$

Where v_1 is the vehicle speed; v_{rel} is the relative speed between vehicles; η is the braking efficiency, experimentally determined, μ is the tire-road friction coefficient; g is the intensity of gravitation field, and t_h is the reaction time of the vehicle driver. It is known [2] that in the study of braking process, it is presumed that the brakes act instantaneously, with the entire force, beginning with the considered origin of time.

In reality, there is a time interval necessary for the braking force to get the maximum value and this time interval must be added to the reaction time of the driver. So, the total braking time is influenced by many factors, one of them being the reaction time of the driver; other facrtors are the inertia of the braking-system and the road surface state.

Thus, the critical distance that we propose is

$$d_{crt} = v_1 t_h - \left(v_1 + \frac{v_{rel}}{2} \right) \frac{v_{rel}}{\eta \mu g} \pm \lambda h_1 \quad (29)$$

Where λ is a coefficient that takes account of the road surface state (if this one is wet or dry) and h_1 is a braking distance, determined as in [2]

References

[1] Kyongsu Y.I., Jeong T.: *Obsrver Based Estimation of Tire-Road Friction for Collision Warnin g Algorithm Adaptation*, JSME International Journal, Series C, vol.4.I, no. 1, 1998

[2] Ghiulai C., Vasiliu Ch.: *Vehicle Dynamics*, Ed.Did si Ped., Bucharest 1975 (in romanian)

[3] Untaru M., Peres Gh., Stoicescu A., Potincu Gh., Tabacu I.: *Dynamics of Vehicles on Wheels*, Ed.Did si Ped., Bucharest 1981 (in romanian)

[4] Halanay A.: *Qualitative Theory of Differential Equations*, Ed.Acad., Bucharest 1998 (in romanian)

[5] Niculaiasa V.: *Movement of Vehicles*, Ed. Polirom, Iasi 1996 (in romnian)

[6] Untaru M., Ciolan Gh., Tudor I.: *Vehicle Dynamics*, University of Brasov, 1988 (in romanian)

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