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Natural Frequencies Distribution on the Damaged Beams

This paper presents the natural frequencies distribution on the damaged beams. The frequencies distributions was analyzed for three types of beams: cantilever, double clamped and simple supported. The analysis was performed using the finite element method (FEM), for the undamaged beam and for the beam having damages at around 190 locations, considered one by one, and nine levels of depth. The authors deduced analytical relationships for damaged beam frequency shift and presents the results.

***Keywords**: natural frequencies distribution, damage, beam*

1. Introduction

Damage detection using natural frequency shifts is largely presented in literature [2], [3] and [4]. The methods based on frequency change can be classified in two categories: methods limited to damage detection and methods destined to detect, locate and quantify damages. Literature reviews, [3] and [5], affirm that all methods based on natural frequency changes belong to the second category are model-based, typically relying on the use of finite element models. Due to low sensitivity of frequencies shifts to damage very precise measurements are required. Often the changes in the frequency caused by structural damage are smaller than those observed between the undamaged structure and the mathematical model. This makes almost impossible to discern between inadequate modeling and changes due to damage, consequently the use of models is difficult to be used, [6] and [7]. Other problems in using natural frequencies shifts to detect damages reside in the fact that natural frequencies are sensitive to changes in temperature and loads applied on the structure.

Another class of damage identification methods is based on fitting the behavior of analytical models as closely as possible with that of the real structure by adjusting some elements of the model. Both direct methods, one of the first of this class used in damage detection, as well as sensitivity-based methods are discussed in detail in [8]. Direct methods use finite elements models, where elements in stiffness, mass or damping matrices are changed in order to tune the models with the real structure. In most cases a large number of elements in the matrices may be changed; this is a major problem for damage location. Sensitivity-based methods use continuous or finite elements models, allowing a wide choice of physically meaningful parameters. The idea is to fit the parameters in order to minimize the difference between modal quantities like natural frequencies or mode shapes of the measured data and model predictions. One of the problems in model-based vibration-based damage detection is the need for a very accurate mathematical model, differences between the real structure and the model should be significantly lower than changes occurring due to damages in the structure. Other methods are of course available; a comprehensive classification and description is made in [3].

In this paper a new method to detect, locate and evaluate damages in a beam is presented. The method is based on an extensive study regarding the behaviour of damaged beams, which allowed us to establish how the natural frequency varies depending on the position of the damage on the beam for a high number of frequency modes. Consequently, unlike other authors who analysed the behaviour of damaged beam considering a low number of frequency modes (generally one or two), we could involve in this analysis the first ten vibration modes. Our complex research showed that the shift in natural frequency for a certain vibration mode depends on the position of the damage on the beam, being influenced by the mode shape vector for a given location, while the defect depth only amplifies this phenomenon.

2. Numerical investigations

Since the aim of the research was to develop a simple method to detect, localize and quantify the severity of damages with the least equipment possible, weak-axis bending vibrations were considered and the theory has been developed for this case. In our research we have studied the natural frequency changes on cantilever, simple supported and double clamped beams.

For finite element method (FEM) analyses we used structural steel beam, having the following geometrical characteristics: length L = 1000 mm; width B = 50 mm and height H = 5 mm. Consequently, for the undamaged state the beam has the cross-section $A = 250 \cdot 10\text{-}6\ m^2$ and the moment of inertia $I = 520.833 \cdot 10^{-12}\ m^4$. The material parameters of the specimens are: mass density $\rho = 7850\ kg/m^3$; Young's modulus $E = 2.0 \cdot 10^{11}\ N/m^2$ and Poisson's ratio $\nu = 0.3$. The first ten natural frequencies of the weak-axis bending modes for the undamaged beam were determined.

Afterwards, a series of damages placed separately one after the other on 190 locations along the whole length of the beam were modelled. We selected a simple damage geometry, easy to reproduce on the real structure by saw cuts, with the constant width of 2 mm (fig. 1) and 9 levels of depth, reducing the cross-section by 8%, 17%, 25%, 33%, 42%, 50%, 58%, 67% and 75% respectively. For all the resulting 1,710 damage cases the first ten natural frequencies were determined

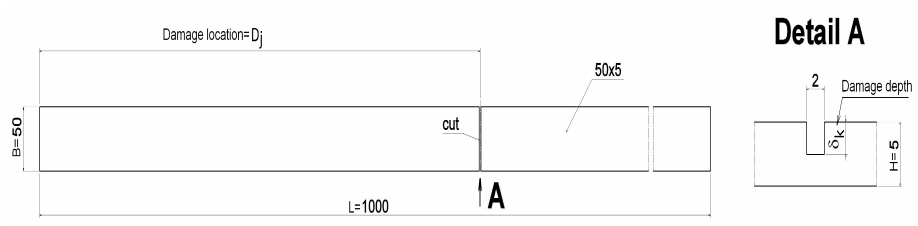


Figure 1. Analyzed beam with a detail of considered damage

For all the 190 cases analyzed and for each damage depth considered, at each vibration mode, have traced the curves of frequencies shift. The figure 2 presents the first and second vibration mode frequencies shift for damaged cantilever beam, for different levels of depth, figure 3 presents the third and fourth vibration mode frequencies shift for damaged double clamped beam, different levels of depth and figure 4 presents the fifth and sixth vibration mode frequencies shift for damaged simple supported beam with different levels of depth, versus the natural frequency for undamaged beam.

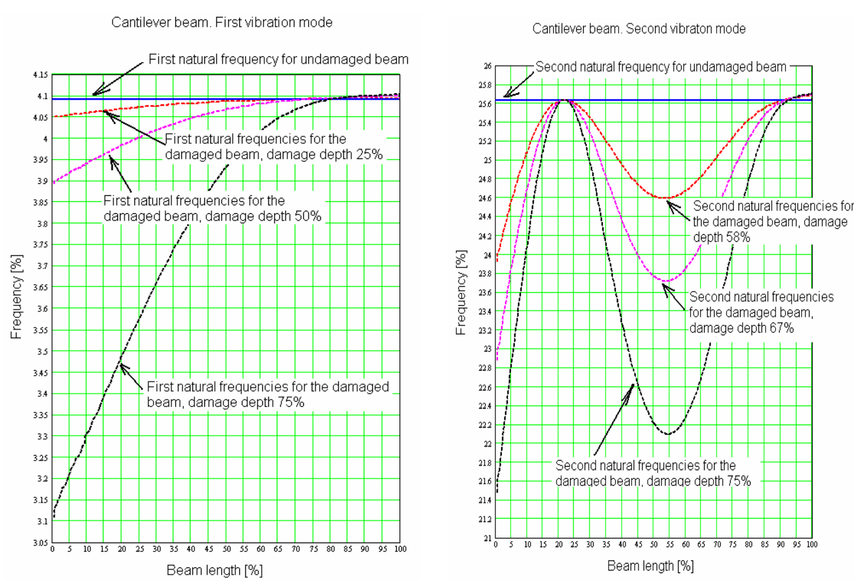


Fig. 2. First and second vibration mode frequencies shift for cantilever beam

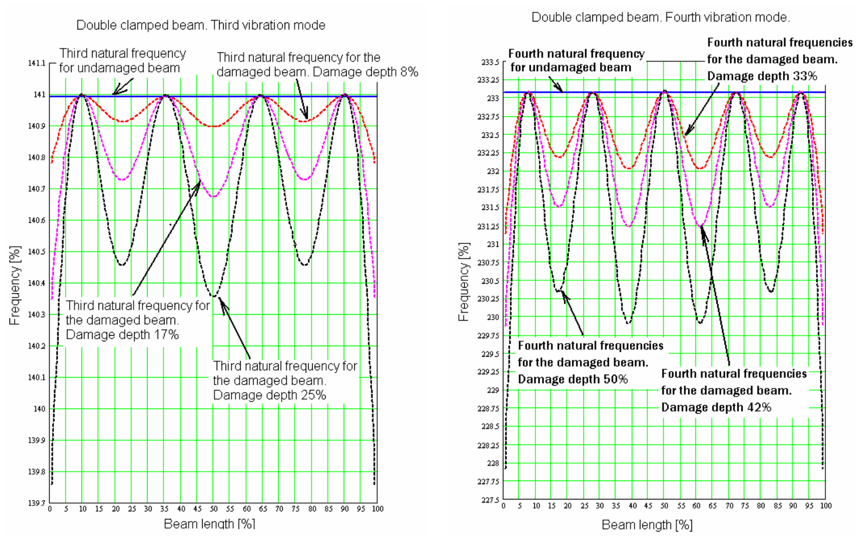


Figure 3. Third and fourth vibration mode frequencies shift for double clamped beam

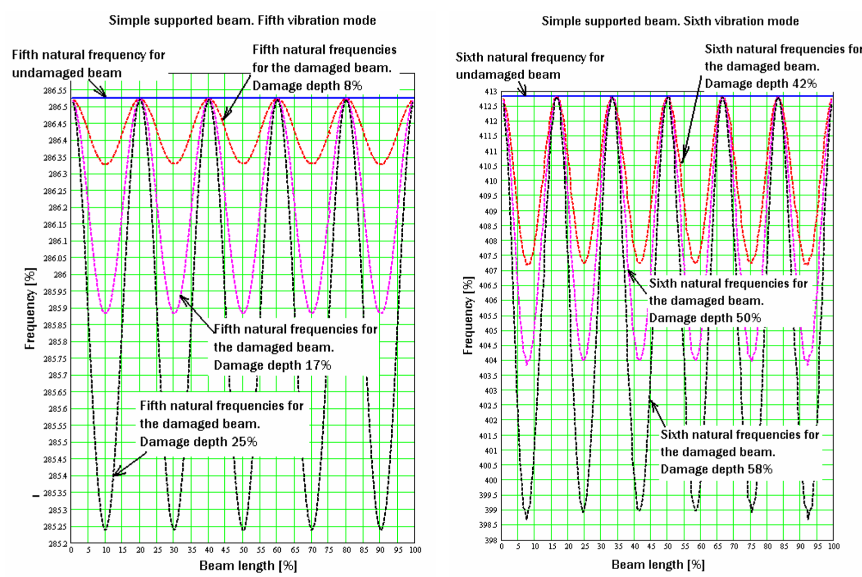


Figure 4. Fifth and sixth vibration mode frequencies shift for simple supported beam

3. Analytical formulation

Based on numerical results we were able to determine the analytical expression for frequencies shift of damaged beam [1], [11], generally valid for all cases of boundary condition, beam lengths, widths, heights and damage depths.

Analytical expression (1) depends on stored energy and depending on the location of the damage on the beam.

$$f_D(x,\delta)_n = f_{U\,n} \cdot \left[1 - C_1(\delta) \cdot C_2 \cdot \frac{\pi^4}{L} \cdot \left(\frac{d^2}{dx^2} \Phi(x)_n \right)^2 \right], \quad (1)$$

where,

$f_D(x,\delta)_n$ - represents the frequency shift for damaged beam, with damage at x location on the beam and δ damage depth for vibration mode No. n;

$f_{U\,n}$ - natural frequency for undamaged beam at vibration mode n;

$C_1(\delta)$ - a coefficient depending the damage depth. The values of this coefficient are presented in figure 5;

C_2 - a constant dependenging the boundary condition. For simple supported beam this coefficient has value 1, for other boundary condition, the value is 0.5;

L - length of the beam;

$\Phi(x)$ - mode shape for vibration mode n.

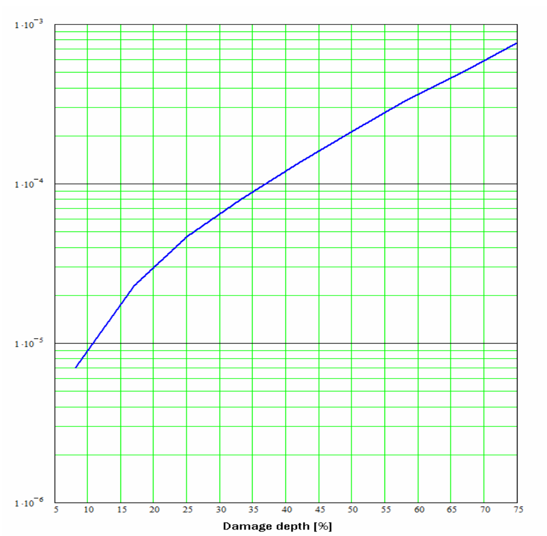
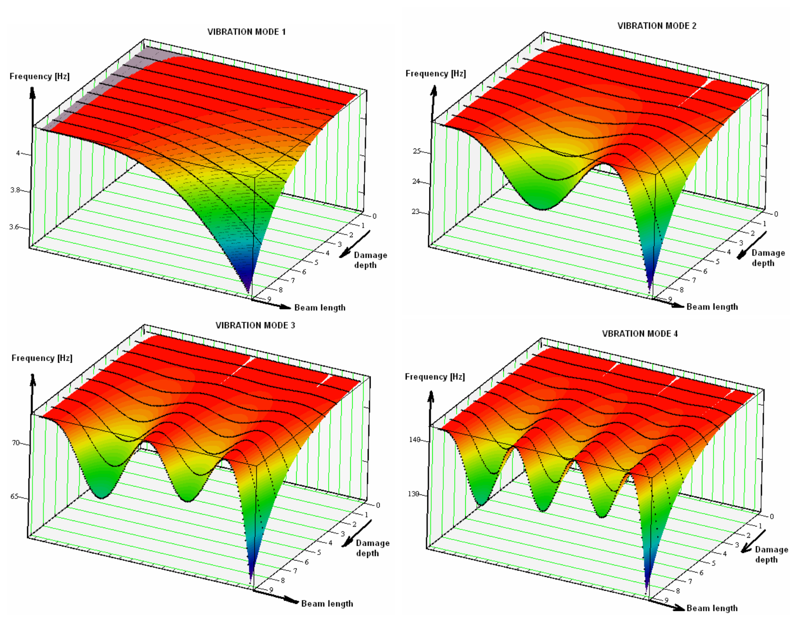


Figure 5. Diagram for defining the coefficient $C_1(\delta)$

For other cross sectional beams, in relation (1) appears a third coefficient C_3 that takes into account of the height of the beam. This coeficient must be multiplied in the last tem of relation (1) For example, for the considered beam (50x5 mm), $C_3=1$; for a beam with a cross sectional of 50x3 mm, $C_3 = \sqrt{\frac{3}{5}}$; for a beam with a cross sectional of 40x4 m, $C_3 = \sqrt{\frac{4}{5}}$; for a beam with a cross sectional of 30x4.4 m, $C_3 = \sqrt{\frac{4.4}{5}}$, etc.

Based on realtionship (1) we can cover all cases of damaged beam, with damage depth between 0% and 75%. In the figures below, are presented in 3D coordinate the frequencies shift for the damaged beam for the first ten vibration mode for cantilever beam (fig. 6), double clamped beam (fig. 7) and simple supported beam (fig. 8).



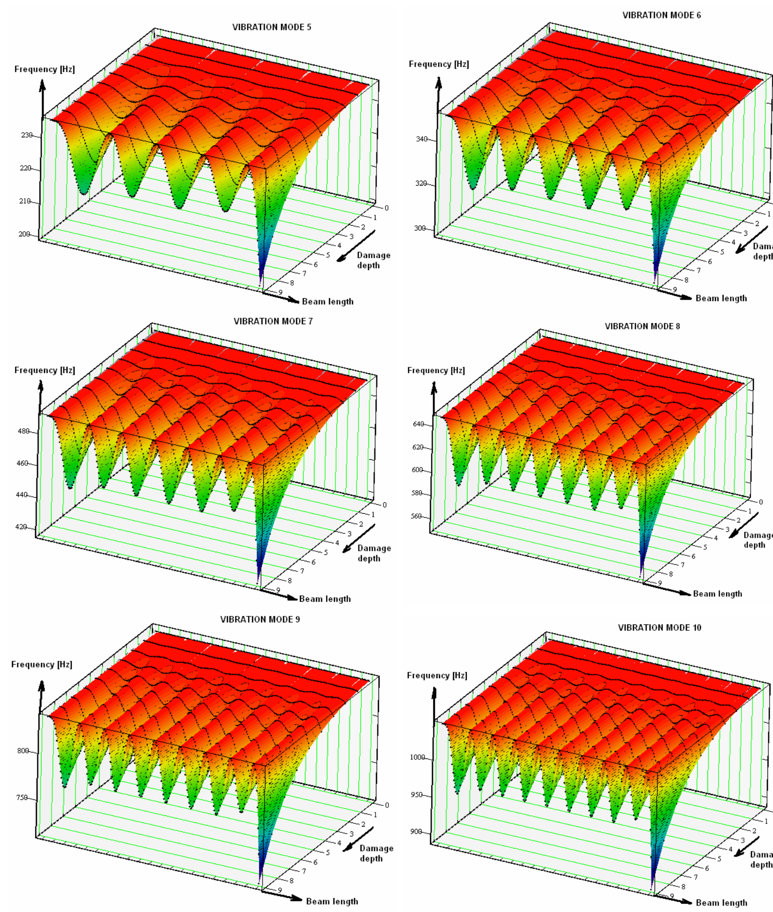
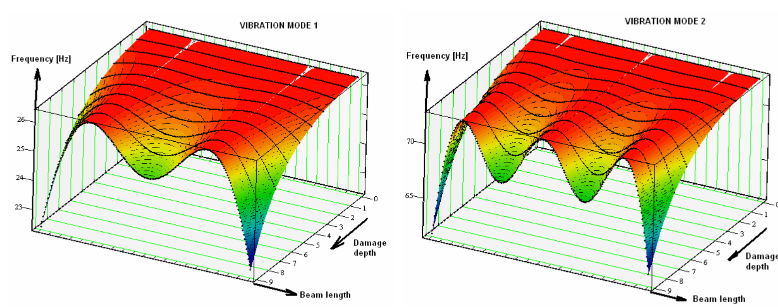


Figure 6. First ten vibration mode for damaged cantilever beam frequencies shift



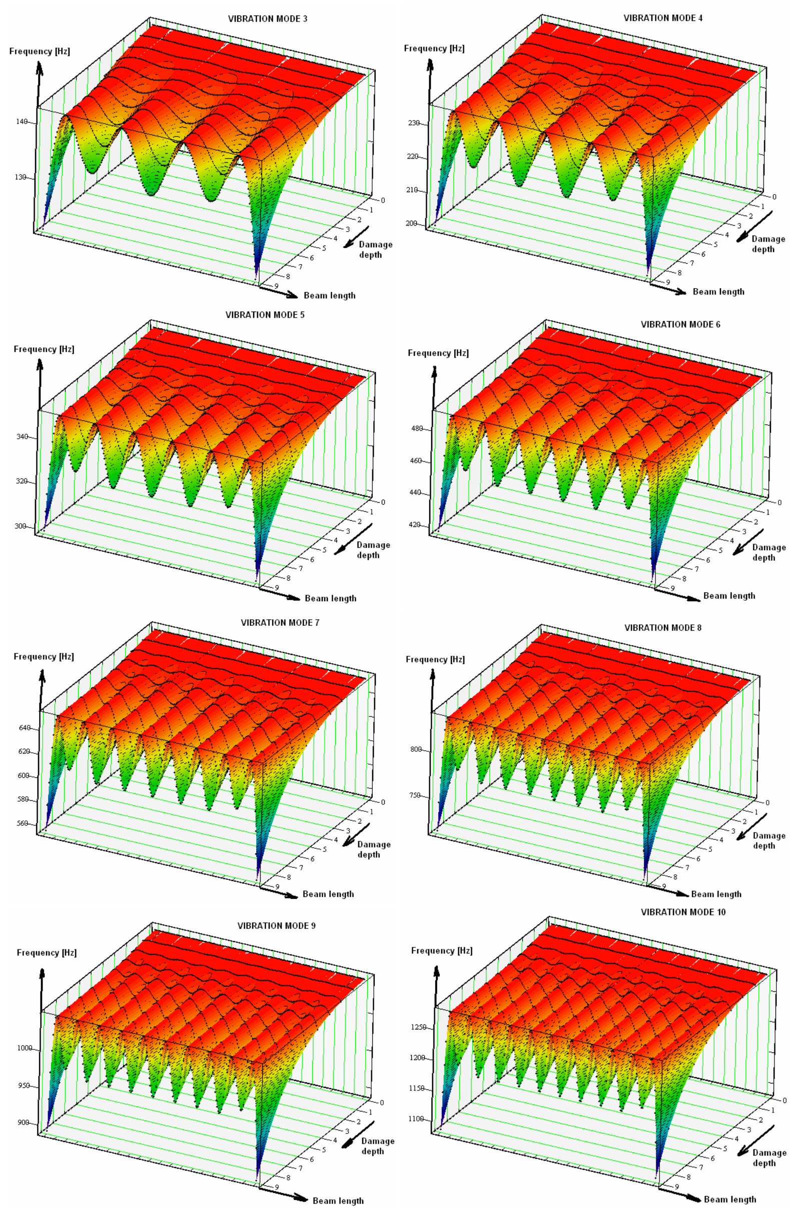
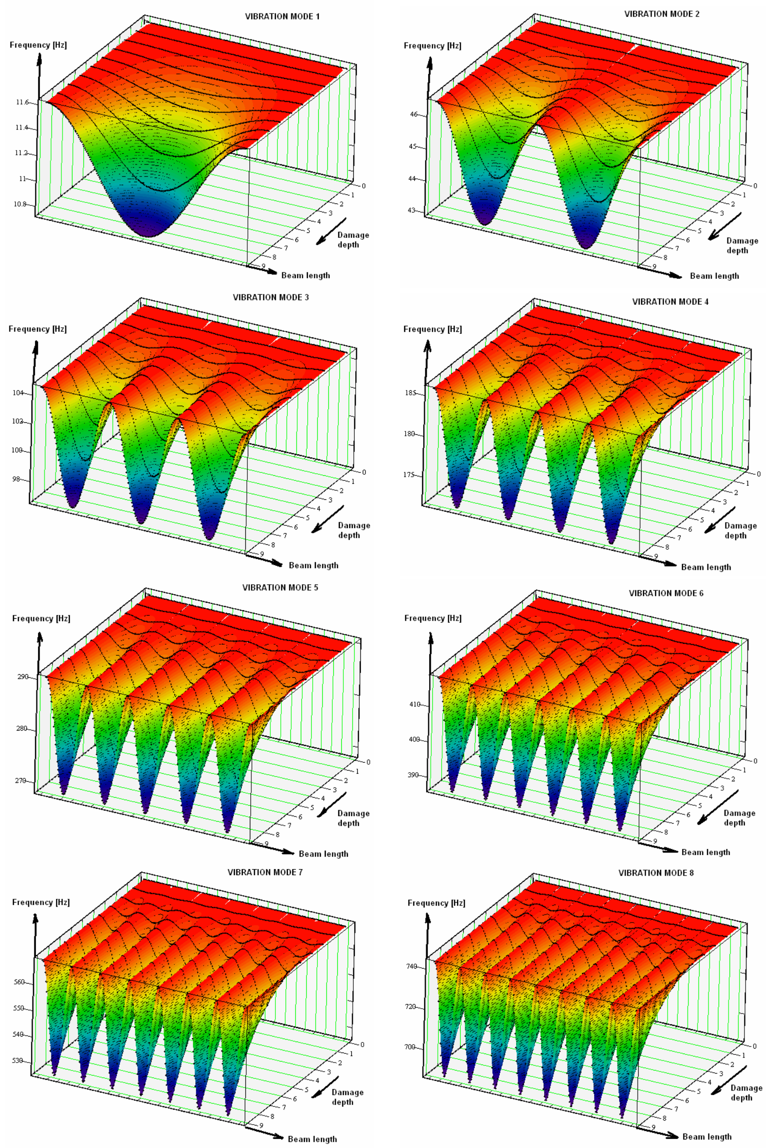


Figure 7. First ten vibration mode for damaged double clamped beam frequencies shift


VIBRATION MODE 1
Frequency [Hz]
Damage depth
Beam length
VIBRATION MODE 2
Frequency [Hz]
Damage depth
Beam length
VIBRATION MODE 3
Frequency [Hz]
Damage depth
Beam length
VIBRATION MODE 4
Frequency [Hz]
Damage depth
Beam length
VIBRATION MODE 5
Frequency [Hz]
Damage depth
Beam length
VIBRATION MODE 6
Frequency [Hz]
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Frequency [Hz]
Damage depth
Beam length

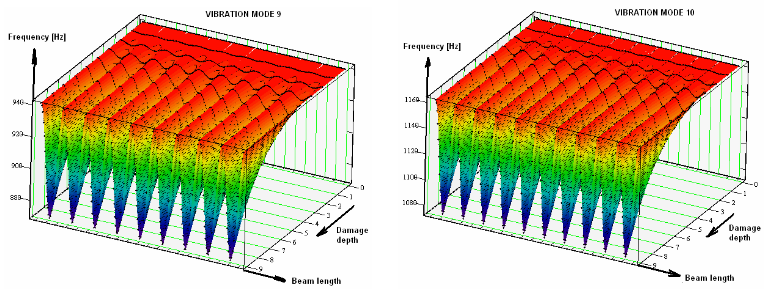

Figure 8. First ten vibration mode for damaged simple supported beam frequencies shift

4. Conclusions

Based on laborious work by authors, we managed an analytical expression of the phenomenon that occurs in damaged beams.

Analyzing the figures 6, 7 and 8, it can be observed that the changes in natural frequencies of beams, due to damages, are significantly influenced by the damage's location, while its depth just amplifies this effect.

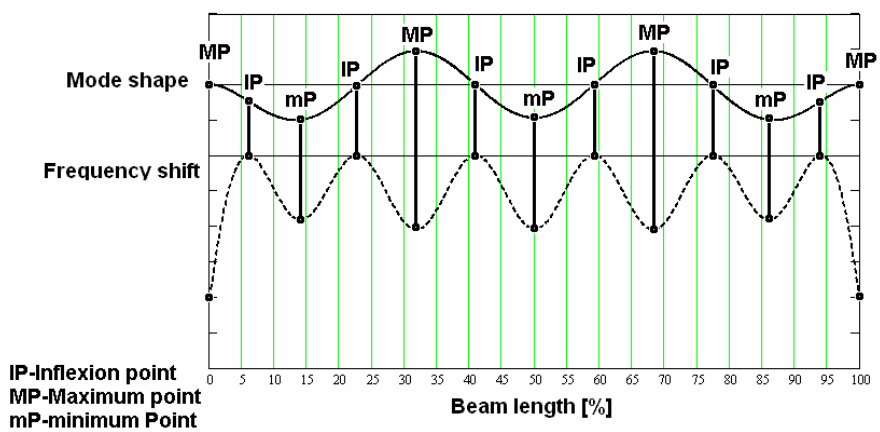

Figure 9. Fifth vibration mode, double clamp beam. Mode shape's critical point versus the frequencies changes for the damaged beam

For all types of supporting beam and each vibration mode, there are locations on the damaged beam that natural frequency remains unchanged, irrespective of the damage depth. These locations correspond to the inflexion points (IP) from the mode shape (fig. 9).

Also, analyzing the figures 6, 7 and 8, it can be observed that, for all types of supporting beam, at each vibration mode, there are locations on the damaged beam that natural frequency exhibits local minima, amplified by the depth of the damage. These locations correspond to the maximum points (MP) and minimum points (mP) from the mode shape (see figure 9).

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