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Peculiar Aspects of Rotor-Bearing Systems Vibrations

The rotor-bearing systems are marked both by an asymmetrical stiffness and by important damping properties, which are causing instability in their rotating motion. The asymmetrical properties presented in this paper are displayed in the rotor motion preferential direction due to the damping internal forces mainly, and also to forces in slide bearings.

***Keywords*:** rotor; eigenfrequency; internal damping; instability

1. Introduction

Any rotor can be discredited by the finite element method. Usually, rotors can be modeled by means of leaning shafts, these oscillating systems having four degrees of freedom (two translations and two rotations).

The rotor-bearings system motion equations can be written as matrix such as:

$$M \cdot \{\ddot{x}\} + C \cdot \{\dot{x}\} + K \cdot \{x\} = F(t)\,, \tag{1}$$

where inertia matrix M is symmetrical, while the stiffness matrix, K, and the damping one, C, are non-symmetrical. The occurring unbalances are due to a preferential rotating direction forthcoming, generated mainly by internal damping forces and forces arising in slide bearings. Secondary causes might be: the forces occuring in different hydrostatic and aerodynamic elements or seals.

2. Rotor eigenfrequencies

Considering the rotor mounted on the shaft by means of elastic supports having the elasticity constant S, corresponding to angular (uncoupled) deviations of bending ϕ_x and ϕ_y.

In the vertical plan (y) we can write:

$$S_{\phi y} = -I \cdot \ddot{\phi}_y - J \cdot \omega \cdot \dot{\phi}_x , \tag{2}$$

and in the horizontal one (*x*):

$$S_{\phi x} = -I \cdot \ddot{\phi}_x + J \cdot \omega \cdot \dot{\phi}_y , \tag{3}$$

Hence it appears that:

$$I \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{\phi}_y \\ \ddot{\phi}_x \end{Bmatrix} + \omega \cdot J \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{Bmatrix} \dot{\phi}_y \\ \dot{\phi}_x \end{Bmatrix} + S \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} \phi_y \\ \phi_x \end{Bmatrix} = 0 , \tag{4}$$

where J is the polar moment of inertia;
I – moment of inertia in related with the diameter (axis passing through the section gravity center).

To better explain the reasons for the occurrence of symmetrical placement within the *[C]* matrix we'll move equation (4) in the complex mode. Because $\phi = \phi_y + i \cdot \phi_x$, we'll get finally:

$$I \cdot \ddot{\phi} - i \cdot J \cdot \omega \cdot \dot{\phi} + S \cdot \phi = 0 , \tag{5}$$

The equation (5) solution is:

$$\phi = A \cdot e^{i \cdot \Omega_{n1} \cdot t} + B \cdot e^{i \cdot \Omega_{n2} \cdot t} , \tag{6}$$

where:

$$\left.\begin{aligned} \Omega_{n1} &= \frac{J \cdot \omega}{2 \cdot I} \cdot \left[1 + \sqrt{1 + \frac{4 \cdot I \cdot S}{J^2 \cdot \omega^2}}\right] & a) \\ \Omega_{n2} &= \frac{J \cdot \omega}{2 \cdot I} \cdot \left[1 - \sqrt{1 + \frac{4 \cdot I \cdot S}{J^2 \cdot \omega^2}}\right] & b) \end{aligned}\right\} , \tag{7}$$

So, the oscillating system free vibratory motion consists of a first high frequency component and of another low frequency (in opposition), both corresponding to the same shaft angular velocity (ω).

For $J > I$, angular velocity ω will never be equal to Ω_{n1} or Ω_{n2}. For $I > J$, equations (7) define two values occurring due to some possible shaft unbalance or to any changes from the vertical position.

3. Rotor instability

3.1. Internal damping forces

Figure 1 describes briefly how the internal damping forces are acting on a rotor bearing system. Rigid outer ring in the figure, fixed around the flexible shaft, rotates with the angular velocity ω, and the mass m elastically hanged up the ring through the elastic spring having the linear elastic constant $\frac{k}{2}$. Also, the interior space of ring and shaft contains viscous fluid with internal damping constant c.

Obviously, the presence of viscous fluid and elastic spring aims to reduce the ring relative mass movement. However, when the mass describes a circular path (corresponding to eigenfrequency $\sqrt{\frac{k}{m}}$) the same way with the ring rotation, but slower (corresponding to a shaft subcritical velocity), the viscous resistance acts on the mass and adds energy on this path.

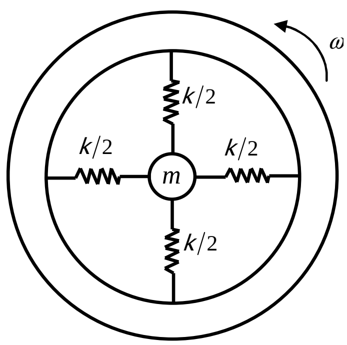

Figure 1. Internal damping effect

Motion equations with shaft internal damping and stiffness can be obtained in stationary coordinates x and y, entering the coordinates u and v along, that are describing the shaft rotation, as in Figure 2. Rotation coordinates are initially required with internal stiffness and damping during shaft rotation.

$$m \cdot \ddot{x} = -[k \cdot u + c \cdot \dot{u}] \cdot \cos \omega t + [k \cdot v + c \cdot \dot{v}] \cdot \sin \omega t \quad (8)$$

$$m \cdot \ddot{y} = -[k \cdot v + c \cdot \dot{v}] \cdot \cos \omega t - [k \cdot u + c \cdot \dot{u}] \cdot \sin \omega t$$

Now, generally

$$v = y \cdot \cos \omega t - x \cdot \sin \omega t \qquad (9)$$
$$u = y \cdot \sin \omega t + x \cdot \cos \omega t$$

So $\dot{v} = \dot{y} \cdot \cos \omega t - \omega \cdot y \cdot \sin \omega t - \dot{x} \cdot \sin \omega t - \omega \cdot x \cdot \cos \omega t$

$\dot{u} = \dot{y} \cdot \sin \omega t + \omega \cdot y \cdot \cos \omega t + \dot{x} \cdot \cos \omega t - \omega \cdot x \cdot \sin \omega t$

Thus, motion equations become:

$$m \cdot \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + c \cdot \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + k \cdot \begin{Bmatrix} x \\ y \end{Bmatrix} + c \cdot \omega \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{Bmatrix} x \\ y \end{Bmatrix} = 0 \qquad (10)$$

They are revealing a matrix type K – unbalanced, providing the potential needed for instability occurrence.

Comparing (10) with (1), the necessary conditions for system instability occurrence are provided and this phenomenon can be studied by Routh-Hourwitz criterion [1], when ω exceeds the $\sqrt{\frac{k}{m}}$ value.

It can be seen that the force $c \cdot \omega$ in equation (10) is orthogonal to the displacement vector. In case a phase angle appears between the resultant velocity and displacement, the $c \cdot \omega$ vector must be at least partially coliniar with the speed vector. This type of force described by the matrix is not conservative, but no the matrix *[K]* is not responsible for this instability.

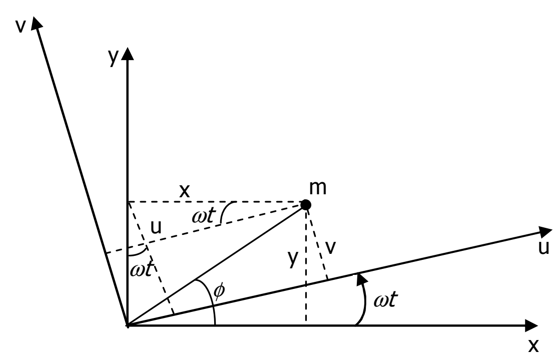

Figure 2. Axis systems used to obtain motion equations

In many part of machinery cases (cardan shaft tube or coupling), there was experimentally [2] noted that the lubricant is possible to just "gather" and building-up a source of destabilization.

The mechanisms of instability occurrence due to viscous fluid are described in Figure 3.

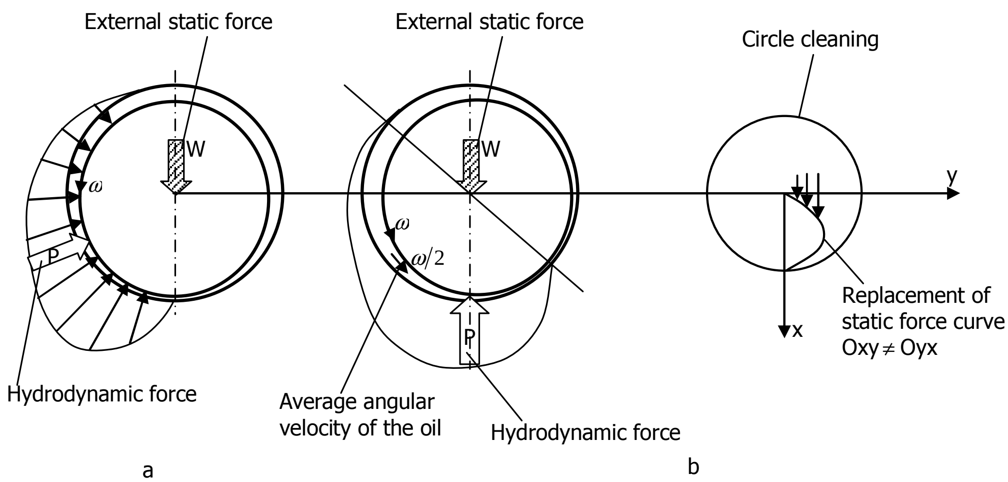

Figure 3. Mechanism of instability due to viscous fluid

3.2. Instability of viscous fluid (oil)

Most common instabilities specific to the mentioned oscillating systems are caused by the bearings lack of flatness that rotors are mounted on.

Taking into account Figure 3, as a result of the external load W, the bearings and lubricant film flexing takes place simultaneously with the system elastic deformation. The deformation takes place in two stages. Bending in load direction (phase I) produces a hydrodynamic pressure that generates the force *P*. This actuates the shaft sideways until the task force *P* becomes coliniar to load W. Referring again to Figure 3 (b) there is a convergence cleft within the lubricant is driven by the spindle rotation. The average angular velocity is approximately equal to half the rotation angular velocity and causes an instant loss of resulting hydrodynamic pressure. The shaft can thus loose its bearing capacity and its movement becomes unstable, turning into a swirl motion, the energy producing instability being taken from its own rotation.

The flexible rotor-bearings system instability and lubricant film experimentally (proved later through calculation) begins at a shaft speed equal to 2 times the first fundamental frequency ($\Omega_{n1} \cong \omega/2$).

The specifically literature [2] assigns that the lubricant film damping properties depend on the cavitation phenomenon appearance.

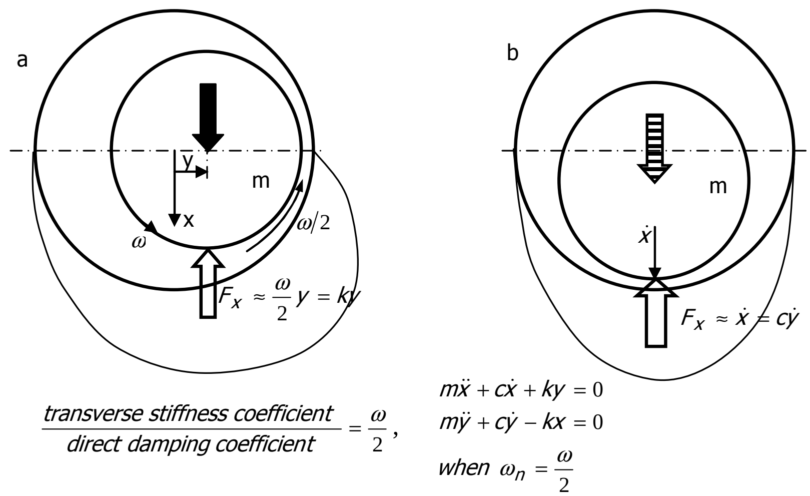

Figure 4. Production of oil film forces

For such a bearing surrounded by a lubricant film, Figure 4 shows the way the two F_x force components of the oil film are arisen. The first component, shown in Figure 4a, arises from the shaft movement y (due to external load) and because of the simultaneous rotation of the shaft speed is proportional to $\omega y/2$. The lubricant film is full kind one ($360°$) and therefore direct stiffness vanishes, the only remained feature of this force component being k - cross stiffness coefficient.

The second component of the force appearing in the lubricant film is shown in Figure 4b and is proportional to the input speed $\dot{x}$. As result, the direct damping coefficient c appears. The ratio $\frac{k}{c}$ begins to be equal to $\frac{\omega}{2}$, as Figure 4 shows.

The vibration equations of spindle center motion are:

$$m \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{x} \\ \ddot{y} \end{Bmatrix} + c \cdot \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + k \cdot \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{11}$$

In the absence of damping, the eigenfrequency of the system will be $\frac{\omega}{2}$ its appearance already showing an incipient instability suggested by previous physical considerations. Unbalance matrix *[K]* is responsible for this phenomenon.

The effects exerted on the bearings are suggestively represented - a general film of lubricant, subjected to cavitational conditions, with four coefficients proportional to the displacement and four coefficients proportional to the speed. System (12) highlights the interaction radial dynamic force, illustrating the relation between the members characterizing the stationary position and that corresponding to rotating movement, as following:

$$\begin{bmatrix} c_{xx} & c_{xy} \\ c_{yx} & c_{yy} \end{bmatrix} \cdot \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \begin{bmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{bmatrix} \cdot \begin{Bmatrix} x \\ y \end{Bmatrix} = \begin{Bmatrix} F_x \\ F_y \end{Bmatrix} \tag{12}$$

4. Conclusions

In future articles, we suggest two purposes. First step we'll study other causes producing rotor-bearings system instability (so-called secondary causes - the sealing ring forces or others appearing in the hydraulic elements).

Considering the bearing positions relatively easy to change and the fact that they can greatly influence the oscillating system dynamic behavior, the team of authors will even try to propose some viable criteria regarding the bearing choice (from its geometrical shape point of view) to meet the goal of better acting under stable operation.

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