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## Hysteresis Models, State of the Art

*Hysteresis represents a new challenge for scientists in last years. Several models of hysteresis were developed in order to understand the delay between input and output. This article does a classification of vectorial hysteresis models and presents simulation results obtained for some specimens using variable separation, and Preisach algorithm.*

**Keywords:** *vectorial hysteresis models, Preisach models, Stoner-Wohlfarth models, Preisach-Stoner-Wohlfarth models*

### 1. Introduction

Hysteresis is a phenomenon encountered in many domains: in physics, in engineering, in socio-economical systems. Starting from its original meaning from Greek, hystesis represents a delay between input and output. In engineering a very usual example is represented by thermostats. Other examples [1] are met in granular motion; spin glasses, porous media filtration.

Hysteresis can be defined also as nonlinearity, where only some past input extrema (not the entire input variations) influence future states [2]. Starting from the original meaning this phenomenon represents a delay between input and output signal [3].

First hysteresis models developed were scalar. The main limitation of these classes of models is that only response to an applied field a given axis and magnetization is considered.

From scalar models, vectorial hysteresis models were developed, that have the ability to respect some general physical principles [4]. Main characteristic of these models is the capacity to determine magnetization in more than one direction.

Vectorial hysteresis models can be divided in three main categories. First category consists in Preisach models represented by hysteresis models that start from original Preisach model. The second category is represented by Stoner-Wohlfarth models, developed from original model. Another class of vectorial

hysteresis models is represented by Preisach-Stoner-Wohlfarth, a mixture between the previous two categories.

This delay is determined by magnetic walls displacement because of Barkhausen jumps [5]. Because of this, study of hysteresis is an interesting challenge for scientists.

## 2. Preisach models

F. Preisach developed a model [6], in 1935, in which considers the material as being composed of elementary particles called hysterons (Preisach dipoles) with a rectangular asymmetric cycle. A hysteron magnetized in +1 or -1 is associated in every point. Its share in overall is given by  $p(a, b)$  which represents the probability to find an elemental volume in a certain region in space.

Overall system is modeled with weighted parallel of nonideal relays named Preisach elemental operators [7].

Numerous attempts of improving the classical Preisach model were done recently for all kind of materials like ferromagnets, magnetostrictive materials and shape memory alloys [8, 9].

Real hysteresis processes besides changing the value of field intensity, can also determine rotation of magnetization if there is no isotropic medium.

The product model, developed in 1999, by Edward Della Torre starts from scalar Preisach model [10]. Congruency property is eliminated because it is supposed that dependence of magnetic susceptibility on magnetization is determined by experimental data [11].

In [12], a simplified vectorial hysteresis model is introduced based on classical Preisach model. Between magnetization and rotational field vectors there is a delay for every value of the angle between these 2 vectors. Magnetization is determined in 2 independent steps: first step consists in amplitude determination and second in achieving magnetization phase determination. The model needs alternative and rotational cycles, with an analytical formula of the Preisach function, in particular Lorentzian used in computation.

Fast vector hysteresis model is interesting because of its mathematical properties and is easy to implement on PC. Also this model satisfies saturation property but it doesn't respect losses property.

For measurements a rotational and alternative loops are needed. When there is a magnetization for a material in a specific direction it becomes demagnetized in normal direction.

Numerical calculation was done using Lorentzian distribution of Preisach function and it was observed that time needed for calculation has the same order as that needed for scalar Preisach model.

Another extension of the classical Preisach model is the simplified vector Preisach model [13]. Rotational correction and Preisach integrals are used for determination of irreversible magnetization using the formula:

$$m_{ij} = R(I_x, I_y, I_z) I_j \quad (1)$$

where  $m_{ij}$  represents irreversible component of magnetization,  $R(I_x, I_y, I_z)$  represents rotational correction which is determined using this formula:

$$R(I_x, I_y, I_z) = \frac{|I_x| + |I_y| + |I_z|}{\sqrt{I_x^2 + I_y^2 + I_z^2}} \quad (2)$$

where **Eroare! Obiectele nu se creează din editarea codurilor de câmp..**

Also  $I_j$  represents Preisach integral which is determined with:

$$I_j = \iint_{v_j < u_j} Q_j p(u_j, v_j) du_j dv_j \quad (3)$$

where we have  $j=x, y$  or  $z$  and  $Q_j$  represents state vector determined by selection rules. In this formula  $u_j$  and  $v_j$  represent positive and negative tipping thresholds.

This model obeys saturation and losses property but the rotational correction determines cross coupling and makes it impossible to obtain an analytical solution close to magnetic susceptibility.

An important extension of simplified vector Preisach model is the reduced vector Preisach model [14]. This model eliminates the rotational correction needed in case of simplified vector Preisach model.

Reversible normalized magnetization is determined using:

$$m_{ij} = \iint Q_{Rj} p(u_j, v_j) du_j dv_j \quad (4)$$

where  $u_j$  and  $v_j$  are tipping thresholds (positive and negative),  $p(u_j, v_j)$  is normalized Preisach function and  $Q_{Rj}$  represents a state function determined by selection rules (different from that used in simplified vector Preisach model).

A differential method for calculation of magnetization can be used because rotational correction is eliminated. Simulations results show that isotropic media is simulated accurately with this model.

Identification of Preisach function using Gaussian distribution is presented in [15] for a Fe-Si steel. A detailed presentation of reduced vector Preisach model and simulations are done for symmetrical cycles and first order demagnetization curves.

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A closed, ellipsoidal surface is used to describe hysteron vector in H space. When a magnetic field is applied outside critical surface, irreversible magnetization component becomes radial. If applied field enters the critical surface, irreversible magnetization component is frozen until it exits the surface. After this, magnetization suffers a Barkhausen jump and it is redirected in radial direction.

Radial vector model is used in simulations for isotropic and anisotropic mediums, in 2D simulations.

Congruency and removal property are respected, but the model doesn't satisfy the symmetry property.

Also this hysteresis model has the ability of predicting angle reduction between magnetization and field intensity vectors but it predicts a higher value of hysteresis losses if magnetic field intensity is raised. The model behaves the same in every direction because it is isotropic.

Based on the idea of radial vector model [16] a general vector hysteresis model is developed.  $\Omega$  vector is used to describe magnetic field in a tridimensional space. If magnetization is outside critical surface, it suffers a Barkhausen jump and it is redirected in a direction that depends on magnetic field. Magnetization is placed between applied field and easy magnetization axis.

Irreversible magnetization is determined using this formula:

$$M_i = \int_{u>v} Q(\Omega)P(\Omega)d\Omega \quad (5)$$

and normalized magnetization value is calculated with:

$$m_i = \int Q(\Omega)p(\Omega)d\Omega \quad (6)$$

where  $P(\Omega)$  represents the number of histererons associated to each critical surface and  $Q(\Omega)$  is the magnetization vector.

Reversible magnetization is obtained by replacing interaction field with an operative field  $H+a \cdot M$  like in Weiss theory.

For determination of this component of magnetization, the following formula it is used:

$$m_r = \left[ \frac{m_H \cdot 1_H + 1}{2} F(|H|) + \frac{m_l \cdot 1_H - 1}{2} f(-|H|) \cdot 1_H \right] \quad (7)$$

where  $1_H$  is unitar vector in applied field direction and  $f$  is a monoton raising function of  $H$  which goes asymptotic to 1 for a high value of  $H$ .

When reversible magnetization changes continuously  $m_r$  should be a continuous function of applied field and magnetization should be a closer to the direction of applied field.

The model can be used in simulations for isotropic and anisotropic mediums. Also other advantages are that it eliminates deletion property and it isn't dependent of system coordinates.

Variations of magnetisation are not considered outside critical surface and magnetization is not perpendicular on critical surface.

In an extension for the bi-dimensional case, a presupposition that Preisach functions can be factorized in individual Gaussian [17] functions is made. Critical surfaces are ellipses in this case.

An important advantage in this model is that defazation angle between applied field and magnetization is predicted. Also symmetry in magnetization plane is proved.

Magnetization is calculated by moving the point in which magnetization is determined instead of moving the ellipsoid.

In a 3D extension [18] of general hysteresis operator, magnetization vector is mathematically defined in spherical coordinates.

One important disadvantage of this extension is that the model is capable of representing asymmetric cycles. A relative magnetization can be constructed in every point and saturation, congruency, deletion and losses property are satisfied.

An important disadvantage is that results are validated only for the classical model of hysteron.

Numerous techniques of improving, measuring and applying the vectorial properties [19, 20] of Preisach models have been used. Comparison between measured magnetic induction and induction in center of sample is done with using finite element method and Femm.

In Jiles-Atherton vectorial model [21], magnetization is decomposed in a reversible and an irreversible component. The model is based on classical Langevin theory. Proposed model consists in 3 identical scalar models placed along principal axes. The inverse model proposed [22] is adopted along with a scaling factor. This model is used in simulations for an isotropic medium and it is proved that it respects basic vectorial properties. Although finite element method is used in simulations an interesting method treating differential susceptibility is considered.

### **3. Stoner-Wohlfarth models**

Main characteristic of previous vectorial hysteresis models is that they try to "vectorize" a scalar relationship. Therefore, previous models can not effectively integrate experimental observation of vector hysteresis.

A hysteresis model based intrinsically on fundamental vectorial hysteresis ideas is the Stoner-Wohlfarth model [23] developed by E. C. Stoner and E. P. Wohlfarth in 1948. The model considers magnetic material composed of ellipsoidal shaped particles with uniaxial anisotropy and magnetized to saturation.

Easy magnetization axis corresponds to one of the ellipsoid axes and the magnetization  $M$  remains constant in module, but can rotate under the action of an external field  $H$  (coherent rotation) in the plane formed by easy axis and magnetic field intensity  $H$ . This particle, called the Stoner-Wohlfarth particle, will have an energy component dependent on magnetization orientation.

Stoner-Wohlfarth particle is not accurately describing systems with more than one particle. The model is built from an simple case.

The Stoner-Wohlfarth particle isn't offering an accurate swirching field. If applied field is normal to easy axis the elemental particle reaches saturation [24]. Also it is remarked that if applied field is normal to easy axis, the elemental particle

goes to saturation. Another inconvenience is that the model is computationally slow.

Extensions of classical model for ferromagnetic materials [25] interested scientists over the years, because this is an intrinsically vectorial model of hysteresis. This model it is yet studied because of it's vectorial nature.

A detailed presentation of Stoner-Wohlfarth model is done also for 3D media [26]. Material is considered composed of small particles without any interaction between them, uniformly magnetized in a single direction. These particles have the ability to rotate and anisotropy energy is taken into consideration easy. In this natural tridimensional model a single mechanism is responsible for reversible or irreversible magnetization. This model doesn't take into consideration joint effects as well as interaction between magnetic domains.

Another extension of Stoner-Wohlfarth model [27] for 3D case is used to incorporate magnetic interactions between particles. For an isotropic material, in case of an alternating field, the magnetization and applied field strength are collinear. Magnetization vector lags the magnetic field for a certain angle in case of circulating rotating magnetic field. Model is usefull in simulations of magnetic process in case of complicated flux electromagnetic devices.

#### **4. Mixed Preisach-Stoner-Wohlfarth models**

Mixed models, a combination between Preisach and Stoner-Wohlfarth models, have been investigated recently by scientists. In one of the first mixed models developed [28], it is suggested an algorithm to model an isotropic magnetic material. The Stoner-Wohlfarth model is used as theoretical basis of algorithm and in order to consider coercitive force, interaction field, the Preisach density function is adopted. A permanent rubber ferite magnet of is considered and projection of magnetic field on that axis is taken into account. The effect of interaction field is to move and rotate the Stoner-Wohlfarth astroid curve. Necessary data used in simulations is represented by transition curves measured with a Vibrating Sample Magnetometer. One important disadvantage is not applicable to anisotropic materials. The errors are considered to vanish if density function is defined properly.

An accurate description of this mixed model [29], is presented along with it's basic ideas (from radial vector model). Several properties are developed and demonstrated in order to show generality and complexity of these model. .

In classical Preisach-Stoner-Wohlfarth model, magnetization has the same variation for all critical surfaces [30], because net field (sum of interaction and applied field), is the same for all particles. Gaussian distribution is considered for Preisach function and measurements are done with a Vibrating Sample Magnetometer. Numerical implementation is done for stored values of magnetization angle. Simulations are done for magnitude and magnetization

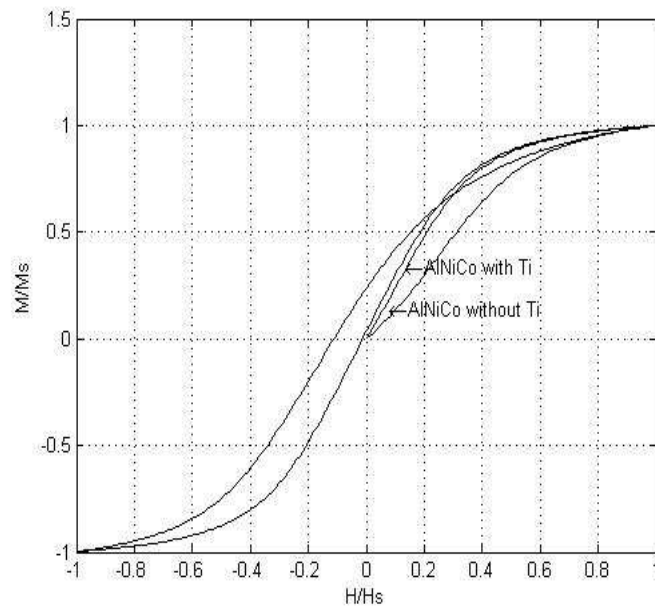
direction when a rotational magnetic field is applied. Only irreversible magnetization component is numerically determined, in this article.

## 5. Experimental results

A vibrating sample magnetometer, VSM 7304 provided by the UPB team, was used in measurements of magnetization and reversal curves for a number of specimens using Preisach model, for hard magnetic materials.

Variable separation method was used in representation of measured data. Initial magnetization and reversal magnetization curves were represented using variable separation method and Preisach formalism.

A comparison between Alnico sample with saturation magnetization force  $H_s=795745$  A/m, saturation magnetization  $M_s=208740$  A/m and Ti alloy Alnico with  $H_s=795744$  A/m and  $M_s=350960$  A/m is simulated in figure 1 for 300 measured points, using Biorci method.



**Figure 1.** Comparison between Alnico with Ti and Alnico without Ti using Preisach formalism

Simulation results show that representation depends on quality and quantity of measured data.

## 6. Conclusions

A classification of vectorial hysteresis models was done starting from 2 main model categories and necessary data extracted from minimal measured data. Minimal data needed for identification of Preisach-Stoner-Wohlfarth models is a future task for my articles.

A comparison between Alnico with and without Ti alloy is done starting from minimal measured data using the Preisach model. Results show that alloying with Ti influences the form of hysteresis cycle.

The reconstructed initial and reversal magnetization curves are in good agreement with measured data.

Identification approach, using Preisach formalism, based on Biorci model is a way to obtain numerical solutions from with minimum measured data. Obtained results will be compared with other implementations of hysteresis models from other categories.

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