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## **Determination of Double Flexible Toothed Wheel Length Function of Elasticity Module (E)**

*This paper presents the length determination modality of the double flexible toothed wheel from a toothed harmonic transmission having a wave generator with  $\pi/2$  phase difference, function of the material's longitudinal elasticity module.*

**Keywords:** *harmonic double transmission, wave generator with  $\pi/2$  phase difference*

### **1. Introduction**

For optimal construction and functioning of a harmonic double toothed wheel transmission with a wave generator having  $\pi/2$  phase difference between the first and the second step, an important role is detained by the length L of the flexible toothed wheel. The right choice of this length L lead to diminish the transmission component parts stresses, growth of lifetime and loading capacity, having also an important role in determining the axial size of the double toothed harmonic transmission.

The flexible toothed wheel length L of the double toothed harmonic transmission is influenced by the elastic force of radial deformation ( $F_e$ ), by the diameter of neutral undeformed fibre, the thickness of wheel wall and the material bending longitudinal elasticity module.

The aim of the paper is determination of the 2L length of the flexible toothed wheel function of the elasticity module (E), fact which implies use of different materials for flexible toothed wheel manufacturing.

## 2. Determination of Flexible Toothed Wheel Length using the Analytical Equation Solutions

From the equation (1) we determine  $L_1$  and  $L_2$  solutions function of material bending longitudinal elasticity module (E), maintaining all others geometrical parameters constants.

$$[\pi DW(n^2 - 1)^2 \frac{n^2}{3}] l^2 - (Fe \cdot n^2 r^3) l + 2\pi DW(n^2 - 1)^2 (1 - \nu) r^2 = 0 \quad (1)$$

In the theoretical formula for flexible toothed wheel length calculus appear the followings: gear module (m), longitudinal elasticity module (E), wall thickness of wheel (h), teeth number (Z) and deformation elastic force ( $F_e$ ), resulted from the action of wave generator's rolls over the flexible toothed wheel. In order to study the dependencies between all these, have been admitted the values presented in table 1.

**Table 1.**

Parameter	Symbol	Number of values	Selected Values
Gear Module	m	1	0.3
Teeth Number	Z	1	200
Wall Thickness	H [mm]	1	0.75
Longitudinal Elasticity Module	E [N/mm <sup>2</sup> ]	4	2.1 ; 2.12 ; 2.14 ; 2.15
Deformation Elastic Force	$F_e$ [N]	8	53; 54; 55; 56; 57; 58; 59; 60

The flexible toothed wheel length L calculation was realised by an Excel macro. The results have been deposited in a calculus sheet called "Results" with the structure from table 2 where, additional to already named parameters, were used:

r - flexible wheel radius

D - bending rigidity, calculated with  $D = \frac{Eh^3}{12(1 - \nu^2)}$

A, B, C - the second degree equation coefficients

$L_1, L_2$  - the two positive solutions of the second degree equation

In order to realise the Macro, has to be create a new workbook, save under „**Rezultate.XLS**”, by pressing **Alt+F8** or from the main menu, following **Tools** → **Macro** → **Macros**; it opens the **Macro** window, where will be written the macro name "**Calculus**" in the "**Macro Name**" field, followed by choosing saving location by option "**This Workbook**". Pushing the "**Create**" button will open the Visual Basic editor, where will be written the Visual Basic instructions as follows:

```

Sub Calcul()
    Sheets("REZULTATE").Select
    Cells.Select
    Selection.ClearContents

    Cells(1, 1) = "Nr.crt." : Cells(1, 2) = "m"
    Cells(1, 3) = "r" : Cells(1, 4) = "h"
    Cells(1, 5) = "E" : Cells(1, 6) = "Fe"
    Cells(1, 7) = "D" : Cells(1, 8) = "A"
    Cells(1, 9) = "B" : Cells(1, 10) = "C"
    Cells(1, 11) = "L1" : Cells(1, 12) = "L2"

    Dim E(100), Fe(100)
    Niu = 0.3
    Coef_n = 2
    m = 0.3
    h = 0.75
    Zet = 200
    E(1) = 2.1: E(2) = 2.12: E(3) = 2.14: E(4) = 2.15
    NE = 4
    Fe(1) = 25: Fe(2) = 30: Fe(3) = 35: Fe(4) = 40
    Fe(5) = 43: Fe(6) = 45 : Fe(7) = 46:
    Fe(8) = 47
    Fe(9) = 48: Fe(10) = 49: Fe(11) = 50:
    Fe(12) = 51
    Fe(13) = 52: Fe(14) = 53: Fe(15) = 54:
    Fe(16) = 55
    Fe(17) = 56 : Fe(18) = 57: Fe(19) = 58:
    Fe(20) = 59
    Fe(21) = 60
    NFe = 21

    Pi = Atn(1) * 4
    lin = 2

    For iE = 1 To NE
        For iFe = 1 To NFe
            Cells(lin, 1) = lin - 1
            Cells(lin, 2) = m
            r = m * Zet / 2: Cells(lin, 3) = r
            Cells(lin, 4) = Zet
            Cells(lin, 5) = h
            E1 = E(iE): Cells(lin, 6) = E1
            Fe1 = Fe(iFe): Cells(lin, 7) = Fe1

            Dmare = E1 * 100000 * h ^ 3 / (12 * (1 - Niu ^ 2))
            Coef_A = (Coef_n ^ 2 - 1) ^ 2 * Pi * Dmare * m * Coef_n ^ 2 / 3
            Coef_B = -Fe1 * Coef_n ^ 2 * r ^ 3
            Coef_C = 2 * Pi * Dmare * m * ((Coef_n ^ 2 - 1) ^ 2) * (1 - Niu) * r ^ 2
            delta = Coef_B * Coef_B - 4 * Coef_A * Coef_C
            Cells(lin, 8) = Coef_A
            Cells(lin, 9) = Coef_B
            Cells(lin, 10) = Coef_C
            If delta >= 0 Then
                L1 = (-Coef_B + Sqr(delta)) / (2 * Coef_A)
                L2 = (-Coef_B - Sqr(delta)) / (2 * Coef_A)
                Cells(lin, 11) = L1
                Cells(lin, 12) = L2
            End If
            lin = lin + 1
        Next iFe
    Next iE
End Sub

```

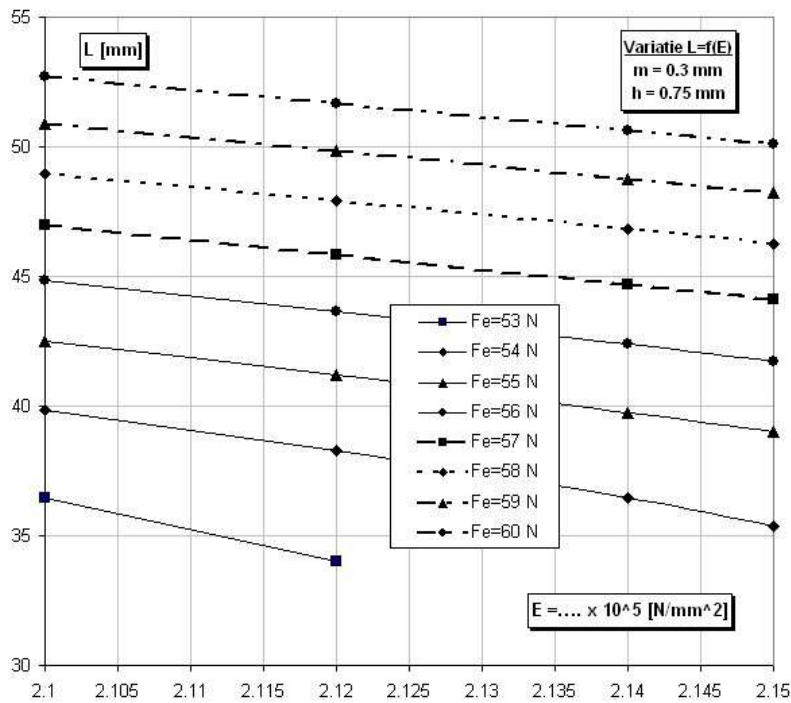
After activating the **Results** sheet, will be created the table containing program results and memorise in vectors the possible values of the elasticity module respectively of the classical deformation elastic force. After, by two cycles, will generate all the possible values of the elasticity module respectively of the classical deformation elastic force, for each set being calculated the flexible wheel radius  $r$ , constant  $D$ , the coefficients and the determinant of the second degree equation „A”, „B”, „C”, „D”, and - only for a positive determinant – the two valid solutions of

the flexible toothed wheel length „L<sub>1</sub>” respectively „L<sub>2</sub>”. The results are deposited in the **Results** sheet in the corresponding columns, as showed in table 2.

**Table 2.**

r [mm]	h [mm]	E [N/mm <sup>2</sup> ]	F <sub>e</sub> [N]	D	A	B	C	L <sub>1</sub>	L <sub>2</sub>	Z
30	0.75	2.1	53	8113	91756	-5724000	86709090	36.47	25.91	200
30	0.75	2.12	53	8190	92630	-5724000	87534891	34.00	27.79	200
30	0.75	2.1	54	8113	91756	-5832000	86709090	39.84	23.72	200
30	0.75	2.12	54	8190	92630	-5832000	87534891	38.26	24.70	200
30	0.75	2.14	54	8268	93503	-5832000	88360692	36.44	25.94	200
30	0.75	2.15	54	8306	93940	-5832000	88773592	35.35	26.73	200
30	0.75	2.1	55	8113	91756	-5940000	86709090	42.50	22.23	200
30	0.75	2.12	55	8190	92630	-5940000	87534891	41.18	22.95	200
30	0.75	2.14	55	8268	93503	-5940000	88360692	39.76	23.77	200
30	0.75	2.15	55	8306	93940	-5940000	88773592	39.00	24.23	200
30	0.75	2.1	56	8113	91756	-6048000	86709090	44.84	21.08	200
30	0.75	2.12	56	8190	92630	-6048000	87534891	43.64	21.66	200
30	0.75	2.14	56	8268	93503	-6048000	88360692	42.39	22.29	200
30	0.75	2.15	56	8306	93940	-6048000	88773592	41.74	22.64	200
30	0.75	2.1	57	8113	91756	-6156000	86709090	46.97	20.12	200
30	0.75	2.12	57	8190	92630	-6156000	87534891	45.85	20.61	200
30	0.75	2.14	57	8268	93503	-6156000	88360692	44.69	21.14	200
30	0.75	2.15	57	8306	93940	-6156000	88773592	44.10	21.43	200
30	0.75	2.1	58	8113	91756	-6264000	86709090	48.97	19.30	200
30	0.75	2.12	58	8190	92630	-6264000	87534891	47.89	19.73	200
30	0.75	2.14	58	8268	93503	-6264000	88360692	46.80	20.19	200
30	0.75	2.15	58	8306	93940	-6264000	88773592	46.25	20.43	200
30	0.75	2.1	59	8113	91756	-6372000	86709090	50.87	18.58	200
30	0.75	2.12	59	8190	92630	-6372000	87534891	49.82	18.97	200
30	0.75	2.14	59	8268	93503	-6372000	88360692	48.77	19.38	200
30	0.75	2.15	59	8306	93940	-6372000	88773592	48.24	19.59	200
30	0.75	2.1	60	8113	91756	-6480000	86709090	52.69	17.94	200
30	0.75	2.12	60	8190	92630	-6480000	87534891	51.67	18.29	200
30	0.75	2.14	60	8268	93503	-6480000	88360692	50.64	18.66	200
30	0.75	2.15	60	8306	93940	-6480000	88773592	50.13	18.85	200

In figure 1 is graphically represented the flexible toothed wheel length L function of the elasticity module at different values of the elastic force ( $F_e$ ).



**Figure 1.** Toothed wheel length L function of the Elasticity Module

#### 4. Conclusion

By the use of flexible double toothed wheel length L equation, the paper present the solutions which have physical sense for establishing the flexible toothed wheel lengths function of the Elasticity Module at certain constant values for the geometrical parameters of the harmonic double toothed transmission.

From the presented graphic in figure 1 can be chose the proper manufacturing material for a certain lenght of the flexible toothed wheel.

## References

- [1] Vela D. *PC Simulation of the flexible toothed composing harmonic transmissions*, PhD report, 2008.
- [2] Vela I. *Researches regarding functionality and construction of mechanisms with elastic toothed elements*, PhD thesis, 1991.
- [3] Volkov D.V. and Akulov V. P., JETP Lett. **16**, 438, 1972.
- [4] De Steven R. Daniewicz, J.C. Newman, Karl-Heinz Schwalbe *Fatigue & Fracture Mechanics*, STP 1461, ASTM Committee E-8 on Fatigue and Fracture, European 1, 2003.
- [5] Nedelcu D., *Microsoft Excel – Theoretical concepts and applications*, Ed. Orizonturi universitare, Timisoara, 2003.

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