ANALELE UNIVERSITĂŢII
"EFTIMIE MURGU" REŞIŢA
ANUL XVII, NR. 2, 2010, ISSN 1453 - 7397

Amalia Ţîrdea, Cristina Opriţescu, Mihai Toader

On Dynamic Systems with Piecewise Linear Feature

Impact dynamics is considered to be one of the most important problems which arise in vibrating systems. Such impact oscillator occurs in the motion with amplitude constraining stop. In the past years, this simple model has been found rich phenomena and given benefit for understanding of impact systems. Different types of impacting response, such as periodic and non-periodic oscillations, can be predicted by using bifurcation diagrams. Many mechanical systems in engineering applications represent systems which are driven in some way and which undergo intermittent or a continuous sequence of contacts with limiting motion by constraints. For example, the principles of the operation of vibration hammers, impact dampers, inertial shakers, milling and forming machines etc, are based on the impact action for moving bodies. With other equipment, machines with clearances, heat exchangers, steam generator tubes, fuel rods in nuclear power plants, rolling railway wheel sets, piping systems, gear transmissions and so on, impacts also occur, but they are undesirable as they bring about failures, strains, and increased noise levels.

1. Introduction

It is important the modeling of the dynamics of an impacting system, so as to enlarge the profitable effects such as optimum design and reliability analysis and to minimize the adverse effects such as pitting, scoring, and high noise levels. Compared with a single impact, the nonlinear dynamics of vibroimpact systems is more complicated. The trajectories of such systems in phase space have discontinuities caused by the impacts. Consequently, the presence of the nonlinearity and discontinuity complicates the dynamic analysis of such systems considerably, but they can be described theoretically and numerically by discontinuities in good agreement with reality. The broad interest in analyzing and understanding the performance of such systems is reflected by an ever increasing amount of researches devoted to this area. Several methods of theoretical analysis have been developed and different models of impacts have been assumed in the past several years.

In the study of dynamical systems driven by percussive forces exists numerous methods for analytical transposition of the shock interactions.

2. Vibropercutant system

Is considering the mechanical system composed of mass m fixed with springs of elastic constant k and viscous damper with damping coefficient c (Figure 1)

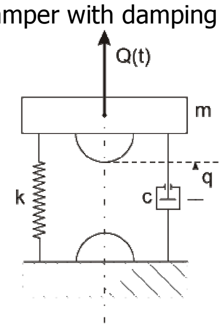

Figure 1. Model of the mechanical system considered

On the mass *m,* acting the disturbing force Q(t), periodically, with period T. In system exists also a percussive couple, the mass being bound by a fixed system. The mass position is given by coordinates q and q_0 represents the distance between fixed point and measuring origin of coordinate q, can associate a percussive couple

$$f(q)=q+q_0 \geq 0 \qquad (1)$$

The differential equation of the motion of the percussive system is

$$m\ddot{q}+c\dot{q}+\varphi(q)=Q(t) \qquad (2)$$

Where $\varphi(q)$ is the characteristic of the system.

For the percussive system considered the feature is represented in the figure 2, and can observe its linearity on the piecewise

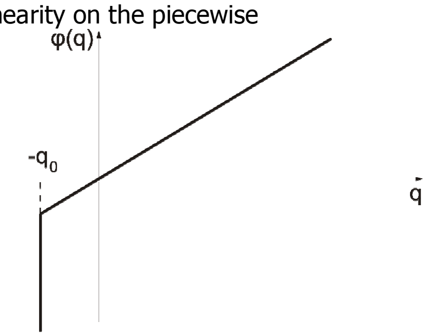

Figure 2. Feature for the percussive system

The nonlinear characteristic follows also from movement speeds breaks between the shocks. For k-order shock witch occurs at time $t=t_k$, the speeds at the beginning and at the end of shock will be $\dot{q}_k$ and $q'_k=-R\dot{q}_k$, where R is the coefficient of restitution. The system is non-autonom and the motion intervenes in phase space, using the cylindrical coordinates: $\rho=\rho_0+q+q_0$, $\theta=t$ and $z=\dot{q}$. In this representation on the surface of the cylinder $\rho=\rho_0$ are represented shocks $q=-q_0$ while the exterior has $\rho>\rho_0$ the possible movements $q>-q_0$ (Figure 3)

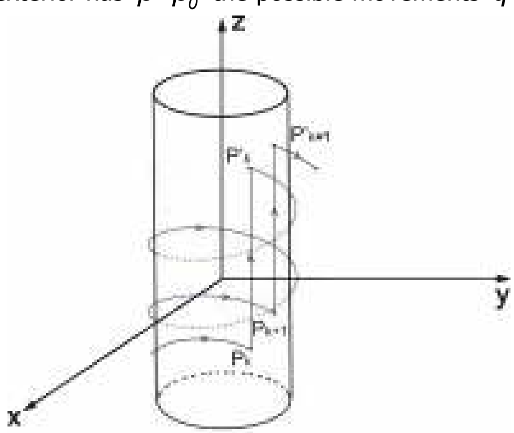

Figure 3. Phases of trajectory

So, the beginning and the end of the k-shock order, corresponds of points $P_k(\rho_0,t_k,\dot{q}_k)$ and $P'_k(\rho_0,t_k,-R\dot{q}_k)$. The trajectories of the phase realize the transformation point of the cylinder $\rho=\rho_0$ in the same time. For each periodic movement correspond a closed phase trajectory and the representative point can make a large number of complete rotations. So in the periodic motion it must consider $t_{k+1}=t_k+rT$ (r=1,2...) and $P_{k+1}=P_k=P_C$

Is found that periodical motions are possible for different values of parameter *r,* which specialized in multiform regime of motion possible. This characteristic property of non linear system will be checked for the case considered

3. The study of the motion

For make a systematic study of the motion of the percussive systems with one degree of freedom is proposed a method based on complex number.

Noting

$$p=\frac{k}{m}-\left(\frac{c}{2m}\right)^2,\ w=\frac{c}{2m}+ip,\ (i^2=-1) \tag{3}$$

and the complex conjugate for w number is $\bar{w}$, the differential equation of k (and k+1) order can be write by relations

$$\begin{aligned}&\dot{q}+\frac{c}{2m}q=S\\&\dot{S}+\frac{c}{2m}S+p^2q=\frac{1}{m}Q(t)\end{aligned} \tag{4}$$

and if is introduced in the complex function

$$z=S+ipq=\dot{q}+wq \tag{5}$$

the system of differential equations becomes

$$\dot{z}+\bar{w}z=\frac{1}{m}Q(t) \tag{6}$$

The solution to this differential equation will be as $z=C^{'}e^{-wt}$ and the general solution will be as

$$z=ce^{-wt}+\frac{1}{m}\int_0^t Q(\tau)e^{-w(t-\tau)}d\tau$$

We determine the constant c (from) the initial conditions that correspond to the end of the impact of order *k*, for $t=t_k$.

$$z^{'}_{k}=-R\dot{q}_k-wq_0 \tag{7}$$

So, the general solution for equation (6) can be write as

$$\varphi(q,q_k,t,t_k)=z-z^{'}_{k}e^{-w(t-t_k)}-\frac{1}{m}\int_0^t Q(\tau)e^{-w(t-\tau)}d\tau=0 \tag{8}$$

To the end of interval for $t=t_k$, so to the beginning of the shock of k+1 order get the final condition

$$z_{k+1}=\dot{q}_{k+1}-wq \tag{9}$$

This condition is introduced in equation (8) and results the equation:

$$\varphi(\dot{q}_{k+1},\dot{q}_k,t_{k+1},t_k)=z_{k+1}-z^{'}_{k}e^{-w(t_{k+1}-t_k)}-\frac{1}{m}\int_0^t Q(\tau)e^{-w(t-\tau)}d\tau=0 \tag{10}$$

This equation determining the elements of successive shock during the motion is equivalent to two scaling relations.

This equation is fundamental to the method accessed for the study of the periodical motions and for the stability study, too.

Periodic motion of the r order of multiplicity is given by equation

$$\varphi(q_C,\dot{q}_C,t_k+rT,t_k)=0 \tag{11}$$

From this complex equation, equivalent to two algebraic equations can be determined the parameters of the periodic motion $\dot{q}_C$ and t_k

Result that periodic movement has real solutions. Is required any time the law of motion needs to accomplish the condition:

$$Imz=0 \tag{12}$$

Stability conditions can be obtained simply from complex equation 10, where $\dot{q}_k$ and t_k is replaced with $\dot{q}_C+\Delta q_k$ and $t_k+\Delta t_k$ where q_k and t_k are considered as being minor perturbations. Can write the equation

$$\varphi=\varphi(q_C+q_{k+1},q_C+\Delta q_k,t_{k+1}+\Delta t_{k+1},t_k+\Delta t_k)-\varphi(q_C,q_C,t_{k+1},t_k)=0 \tag{13}$$

which after linearization with relation to disturbances becomes

$$\Delta_k\varphi=\Delta\dot{q}_{k+1}+Re^{-\bar{w}rT}\Delta\dot{q}_k-\left[\frac{Q(t_k)+kq_C}{m}-w\dot{q}_C\right]\Delta t_{k+1}+$$
$$+\left[\frac{Q(t_k)+kq_0}{m}+R\,\bar{w}\dot{q}_C\right]e^{-\bar{w}rT}\Delta t_k=0 \tag{13'}$$

From this complex equation is obtained a homogeneous linear system which has solutions $\Delta\dot{q}_k=v\beta^{k'},\Delta t_k=\theta\beta^k\,(k=1,2...)$

Is obtaining also a linear system with two equations, with unknowns v *and* θ. This system has non-zero solutions if the system determinant is zero. The characteristic equation of the percussive system, will be

$$\beta^2+Ne^{-\frac{C}{2m}rT}\beta+R^2e^{-\frac{C}{m}rT}=0 \tag{14}$$

Where $N=2Rcos(prT)-\dfrac{(1+R)\left[Q(t_k)+kq_0\right]}{mpq_C}sin(prT)$

Necessary and sufficient conditions for the periodic motion to be stable are getting by requiring that the solutions of the characteristic equation have subunit module and the resulting Schur creiterium. Must be verified in addition inequality

$$-(1+R^2e^{-\frac{C}{m}rT})\le Ne^{-\frac{C}{m}rT}\le 1+R^2e^{-\frac{C}{2m}rT} \tag{15}$$

For real collisions these inequalities are the conditions for stability of periodic motions with collisions of the vibropercutant system.

4. Conclusions

The shock interaction effect is the appearance of some velocity-jump. In case of actuation by shock, in the vibropercutats systems also appears limitation of motion, a limitation which does not occur in the systems which have impulses. For such percussive interaction, the highest form of study method, is based on the introduction of binding unilateral analytical as a form of presentation of a percussive couple.

In this purpose using complex numbers, with the aid of which the calculus performed in a manner simpler and more convenient and the results can be easily interpreted

REFERENCES

[1] Silaş Gh., *O nouă metodă de studiu a sistemelor vibropercutante liniare cu un grad de libertate şi o cuplă percutantă*. Bul.St.şi Tehnic al IP Timişoara, Seria Mat.-Fiz.-Mec.teor. şi applicata, tom.16(30), fasc.1, 1971

[2] Silaş Gh., Brîndeu L., *Behandlung periodischer Bewegungen von Ruttelsystemen mit den Lagarngeschen Gleichungen*. VDI Berichte, Nr.135, 1969.

[3] Silaş Gh., Brîndeu L.: *Sisteme vibropercutante*, Ed.Tehnică, Bucureşti 1986.

[4] Y.Yue, J.H.Xie, *Symmetry and Bifurcations of a Two Degree of Freedom Vibro-Impact System*, J. Sound and Vibration 2008(314).

[5] L.Jin, Q.S.Lu, E.H. Twizell, *A Method for Calculating the Spectrum of Lyapunov Exponents, by Local Maps in Non-Smooth Impact-Vibrating Systems*, J.Sound and Vibration, 2006(298).

[6] Andrzej Stefanski: *Estimation of the Lyapunov Exponent in Systems with Impact*, Chaos Solitons and Fractals, 2000(11).

[7] W.P.M.H. Heemels, B.Brogliato: *The complemetarity classof hybrid dynamical systems*, Minicourse at the European Control Confe

[8]P.Ballard: T*he dynamics of discrete mechanical systems with perfect unilateral constraints*, Arch. Rat. Mech. Anal 2000(154).

[9] June-Yule Lee, Jun-Jun Yan: *Control of Impact Oscillator*, Chaos Solitons and Fractals, 2006(28).

Addresses:

- Eng. Amalia Ţîrdea ICECON Bucureşti Departamentul de Cercetare Timişoara, e-mail: amalia_tirdea@yahoo.com
- Dr. Eng. Cristina Opriţescu ICECON Bucureşti Departamentul de Cercetare Timişoara, e-mail: opritescucristina@yahoo.com
- Prof. Dr. Eng. Mihai Toader ICECON Bucureşti Departamentul de Cercetare Timişoara, e-mail: *toader@mec.upt.ro*