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Mathematical Model for the Contact Stress Calculation of Fluoropolymerized Toothed Wheels

In this paper is presented a contact stress calculation algorithm for a new type of cylindrical toothed wheel. This new type consists in the application of a Xylan 1052 fluoropolymer film on the surface of a metal toothed wheel. Due to the fact that the steel strength, in any case is better than the plastic strength, the main issue which appears is at the contact between the two homologous flank surfaces.

Keywords*: toothed wheel, fluoropolymer, stress contact calculation*

1. Introduction

The association plastics-metal materials in cylindrical toothed wheels construction has various forms: metal toothed wheel engaging plastic toothed wheel and vice-versa, plastic toothed wheel fixed to a metal shaft, plastic crown gear fixed to the metal support and, more recently, a film coating made of stress resistive fluoropolymer material applied to the surface of a metal toothed wheel.

The last variant is a new concept used in construction of cylindrical toothed wheels. This concept has been studied up to date in terms of vibration damping and mechanical efficiency, the results being encouraging. The fluoropolymerisation toothed wheels behave like a class processing accuracy which decrease from VIII to VI and improves efficiency by 10% after the last performed experiments.

The fluoropolymerisation treatment is equally applicable to a new or one used cylindrical gear.

If the standards provide detailed information about metallic materials, regarding plastics, the standards are extremely low in prizes and no guarantee of experimental research shows the full results.

Particular, Xylan 1052 fluoropolymer material shows high stress contact and very low friction.

The Xylan 1052 film coating was performed by a company that usually do fluoropolymeric covers for various usage. By fluoropolymerisation technology the thickness of a coating film can be between 12-25 μm and it could be applied in one or more films on a surface. For two applied layers they indicated to us a value for the film stress contact about 343 N/mm^2.

Keeping in mind that the toothed wheel is practically made from steel and it's consequently dimensioned, the main issue is to determine the stress contact of the Xylan 1052 film coating.

It’s well known that the stress contact is described in many calculation algorithm structures. How and in what way we calculate it depends of the specific data and functional conditions.

In this case, we will present a calculation model for the stress contact that depends of the some coefficients, applied contact strength and existing geometrical data.

The verification relations and calculation methods of cylindrical gear tooth flanks to the contact stress are shown below in few steps.

2. Mathematic relations for the stress contact fatigue calculation.

For the stress contact calculation the following relation is adopted:

$$\sigma_H = Z_H \cdot Z_E \cdot Z_\varepsilon \cdot Z_\beta \cdot \sqrt{\frac{F_{tH} \cdot K_A \cdot K_V \cdot K_{H\beta} \cdot K_{H\alpha} \cdot}{b_w \cdot d_{w1}} \cdot \frac{u \pm 1}{u}} \quad (1)$$

where the minus sign is adopted for gear inside.

$F_{tH} \cdot K_A \cdot K_{H\beta} \cdot K_{H\alpha} = F_{tHef}$ - tangential force used in calculating the effective tooth flank contact stress.

K_A - utilization factor;

$K_{H\beta}$ - load distribution factor on the teeth width at the stress contact solicitation;

$K_{H\alpha}$ - frontal load distribution factor at the stress contact solicitation;

K_V - dynamic factor;

b_w - gear working width (mm);

d_W - pitch circle diameter (mm)

u - teeth number ratio (z_2/z_1).

The material factor Z_E (MPa$^{1/2}$):

$$Z_E = \left\{ 1 \Big/ \left[\pi \left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right) \right] \right\}^{1/2} \quad (2)$$

or determined from tables.

E - modulus of elasticity (MPa);

v - kinematic viscosity, Poisson's constant (mm^2/s).

The area factor of contact Z_H:

$$Z_H = \left[2 \cdot \cos\beta_b / (\cos\alpha_t \cdot \sin\alpha_{wt})\right]^{1/2} \tag{3}$$

β_b – helix angle on the root cylinder;
α_t – presure angle in the frontal plan;
α_{wt} – gearing angle in the frontal plan;

The minimum length factor of contact Z_ε:

$$Z_\varepsilon = [(4 - \varepsilon_\alpha)/3]^{1/2} \tag{4}$$

for spur wheel

$$\varepsilon_\beta < 1 \quad Z_\varepsilon = [\frac{4 - \varepsilon_\alpha}{3}(1 - \varepsilon_\beta) + \frac{\varepsilon_\beta}{\varepsilon_\alpha}]^{1/2} \tag{5}$$

$$\varepsilon_\beta \geq 1 \quad Z_\varepsilon = (1/\varepsilon_\alpha)^{1/2} \tag{6}$$

for herringbone and bevelling toothed wheels.
ε_α – frontal overlap ratio;
ε_β – overlap ration depending on bevelling of teeth.

The inclination factor of teeth Z_β:

$$Z_\beta = (\cos\beta)^{1/2} \tag{7}$$

3. The fatigue limit of stress contact on the teeth flanks $\sigma_{H\ lim\ 1(2)}$ (MPa).

The fatigue limit of stress contact on the teeth flanks $\sigma_{H\ lim}$ is separately calculated for pinion ($\sigma_{H\ lim\ 1}$) and wheel ($\sigma_{H\ lim\ 2}$).

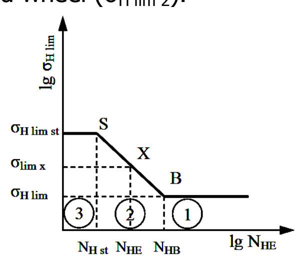

Figure 1. The stress contact solicitation limit depending on functioning period

For the requested functioning period we'll check in which of the marked fields 1, 2 or 3, the gear works and, accordingly, the following relations are used:

1. $N_{HE\ 1(2)} \geq N_{HB\ 1(2)}$;$\sigma_{H\ lim\ 1(2)} = \sigma_{H\ lim\ b1(2)} \cdot Z^{*}_{L1(2)} \cdot Z^{*}_{V1(2)} \cdot Z^{*}_{R1(2)} \cdot Z^{*}_{W} \cdot Z^{*}_{X1(2)}$, (8)

Z_L – lubrification influence factor for the contact stress solicitation;

Z_V – influence of periphery speed factor to the contact stress solicitation;

Z_R – roughness factor of the teeth flank;

Z_W – factor of the flanks hardness ratio;

Z_X – dimension factor for the contact stress solicitation;

$N_{HE\ 1(2)}$ – base cycles number at the contact stress solicitation;

$N_{HB\ 1\ (2)}$ - functioning equivalent number of cycles to reduce a variable load to a constant one in the case of the contact stress solicitation

*At the gears with hardened pinion and improved toothed wheel, the Z_V factors will be established for the low hardness wheel and corresponding values will be used for the hardened pinion.

2. $N_{H\ st\ 1(2)} < N_{HE\ 1(2)} < N_{HB\ 1(2)}$; $\sigma_{H\ lim\ x1(2)} = \sigma_{H\ lim\ 1(2)} \cdot [\ N_{HB\ 1(2)}/N_{HE\ 1(2)}]^{1/m^{*}_{H}}$, (9)

where:

$$m^{*}_{H1(2)} = \lg\left[\frac{N_{HB1(2)}}{N_{Hst1(2)}}\right] / \lg\left[\frac{\sigma_{H\lim st1(2)}}{\sigma_{H\lim 1(2)}}\right]$$

$m^{*}_{H\ 1(2)}$ – reduced mass on the contact line;

$N_{H\ st\ 1(2)}$ – number of static loads to the contact solicitation;

$\sigma_{H\ lim\ x1(2)}$ – stress limit of contact in the range S - B;

$\sigma_{H\ lim\ st\ 1(2)}$ – static limit of contact stress of the teeth flank;

$\sigma_{H\ lim\ 1(2)}$ - determined according to the case 1 and $\sigma_{H\ lim\ st1(2)}$, acording to the case 3.

3. $N_{HE\ 1(2)} \leq N_{H\ st\ 1(2)}$; $\sigma_{H\ lim\ st\ 1(2)} = \sigma_{H\ lim\ b1(2)} \cdot Z_{N\ st\ 1(2)} \cdot Z_W$, (10)

$Z_{N\ st\ 1(2)}$ – durability factor at the static solicitation of the teeth flank.

* The case *3* represents the case of the static solicitation contact (at the plastic deformation) of the teeth flank, $\sigma_{H\ lim\ st\ 1(2)}$ being the static contact stress limit of teeth flank.

The influential factor of the lubrication at the contact solicitation Z_L:

$$Z_L = C_{ZL} + \frac{4 \cdot (1 - C_{ZL})}{(1{,}2 + 80 / v_{50^o})^2}\ , \qquad (11)$$

where: $C_{ZL} = 0{,}83 + 0{,}08 \dfrac{\sigma_{H\lim b} - 850}{350}$

C_{ZL} – coefficient that takes into account the influence of lubrication to the contact solicitation;

$\sigma_{H\ lim\ b}$ – base stress limit of the contact fatigue of teeth flank.

If $\sigma_{H\ lim\ b} < 850$ MPa, $\sigma_{H\ lim\ b}$=850 MPa will be adopt, and if $\sigma_{H\ lim\ b}$>1200 MPa, $\sigma_{H\ lim\ b}$=1200 MPa will be taken.

The influential factor of the peripheral speed to the contact solicitation Z_V:

$$Z_V = C_{ZV} + \frac{2(1 - C_{ZV})}{(0,8 + 32/v_l)^{1/2}}, \quad (12)$$

where: $C_{ZV} = 0,85 + 0,08\frac{\sigma_{H\,\lim b} - 850}{350}$.

If $\sigma_{H\,\lim\,b}$ < 850 MPa, $\sigma_{H\,\lim\,b}$=850 MPa will be adopt, and if $\sigma_{H\,\lim\,b}$>1200 MPa, $\sigma_{H\,\lim\,b}$=1200 MPa will be taken.

The teeth flank roughness factor Z_R:

$$Z_R = (3/R_{z100})^{C_{ZR}}, \quad (13)$$

where: $C_{ZR} = 0,12 + \frac{1000 - \sigma_{H\,\lim b}}{5000}$

C_{ZR} – coefficient that takes into account the effect of roughness to the contact solicitation.

If $\sigma_{H\,\lim\,b}$ < 850 MPa, $\sigma_{H\,\lim\,b}$=850 MPa will be adopt, and if $\sigma_{H\,\lim\,b}$>1200 MPa, $\sigma_{H\,\lim\,b}$=1200 MPa will be taken.

$$R_{Z100} = R_Z(100/a_w)^{1/3},$$

where : $R_Z = 0,5(R_{Z1} + R_{Z2})$ si ($R_Z \approx 5R_a$)

R_Z – roughness (μm).

The flanks hardness ratio factor Z_w:

If the gear consists of a wheel with 1300 <HB <400 MPa and the other wheel is hardened and rectified (RZ ≤ 6 mm), then:

$$Z_w = 1,2 - \frac{HB - 1300}{17000} \quad (14)$$

In other cases Z_w=1.

The dimension factor for the contact solicitation Z_X:

Generally, Z_X = 1. If exist precisely data about the tooth dimensions influence to the contact fatigue, appropriate Z_X is adopted.

4. The contact fatigue checking of the teeth flank

It is separately checked for pinion and wheel:

a. By checking the safety factor to the contact fatigue

$$S_{H1(2)} = \frac{\sigma_{H\,\lim 1(2)}(sau\,\sigma_{H\,\lim x1(2)})}{\sigma_H} \geq S_{HP1(2)}, \quad (15)$$

$S_{HP1(2)}$ - from table.

The safety factors for minimum permissible resistance of the cylindrical toothed wheels are given below.

Table 1.

Operation requirements	S_{HP}; $S_{HP\ st}$	S_{FP}; $S_{FP\ st}$
> 99% (very big, special cases)	1,25 ... 1,5	2 ... 2,5
99% (normal)	**1,15**	**1,25**
90% (low)	1,00	1,00

* The bolded values are normally used.

b. By comparison with the allowable stress to the contact fatigue σ_{HP}:

$$\sigma_{H\ 1(2)} \leq \sigma_{HP\ 1(2)}, \tag{16}$$

where $\sigma_{HP\ 1(2)} = \sigma_{H\ lim\ 1(2)}$ (or $\sigma_{H\ lim\ x\ 1(2)}$) / $S_{HP\ 1(2)}$.

5. Conclusion

Applying Xylan 1052 fluoropolymer film improves the tribological properties, reduce the vibrations and grows-up the corrosion resistance of toothed wheels.

Once the factors and coefficients are determined by software simulation and experiments on the test bench, they can be used into the calculation model.

The accuracy of result, obtained by mathematical calculation, depends on how precisely are data for factors and coefficients.

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